

104 - 4.3 Rational Exponents

Part 1 - Review of Exponent Laws

Exponent Law	
Note that a and b are rational or variable bases and m and n are integral exponents.	
1	Product of Powers $(a^m)(a^n) = a^{m+n}$
2	Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
3	Power of a Power $(a^m)^n = a^{mn}$
4	Power of a Product $(ab)^m = (a^m)(b^m)$
5	Power of a Quotient $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
6	Zero Exponent $a^0 = 1, a \neq 0$

Q1: $(x^5)(x^3) =$

$$x^{5+3}$$

$$x^8$$

Q2: $(x^3)^4 =$

$$x^{(3)(4)}$$

$$x^{12}$$

Q3: $\frac{x^8}{x^6} =$

$$x^{8-6}$$

$$x^2$$

Part 2 - Fractions

Q4: $5 + 3 = 8$

Q5: $\frac{5}{2} + \frac{3}{2} = \frac{8}{2} = 4$

Q6: $(3)(4) = 12$

Q7: $\left(\frac{3}{2}\right)\left(\frac{4}{3}\right) = \frac{12}{6} = 2$

Q8: $8 - 6 = 2$

Q9: $\frac{8}{7} - \frac{6}{7} = \frac{2}{7}$

Part 3 – Exponent Laws with Fractions

$$\begin{aligned} \text{Q10: } (x^5)(x^3) &= x^{5+3} \\ &= x^8 \end{aligned}$$

$$\begin{aligned} \text{Q11: } \left(x^{\frac{5}{2}}\right)\left(x^{\frac{3}{2}}\right) &= x^{\frac{5}{2} + \frac{3}{2}} \\ &= x^{\frac{8}{2}} \\ &= x^4 \end{aligned}$$

$$\begin{aligned} \text{Q12: } (x^3)^4 &= x^{(3)(4)} \\ &= x^{12} \end{aligned}$$

$$\begin{aligned} \text{Q13: } \left(x^{\frac{3}{2}}\right)^{\frac{4}{3}} &= x^{\left(\frac{3}{2}\right)\left(\frac{4}{3}\right)} \\ &= x^{\frac{12}{6}} \\ &= x^2 \end{aligned}$$

$$\begin{aligned} \text{Q14: } \frac{x^8}{x^6} &= x^{8-6} \\ &= x^2 \end{aligned}$$

$$\begin{aligned} \text{Q15: } \frac{x^{\frac{8}{7}}}{x^{\frac{6}{7}}} &= x^{\frac{8}{7} - \frac{6}{7}} \\ &= x^{\frac{2}{7}} \end{aligned}$$

Part 4 – Rational Exponents – What Does It Mean?

$$\text{Q16: } \left(9^{\frac{1}{2}}\right)\left(9^{\frac{1}{2}}\right) = 9^{\frac{1}{2} + \frac{1}{2}} = 9^1$$

But wait!
 $(3)(3) = 9$

Also
 $(\sqrt{9})(\sqrt{9}) = 9$

So
 $9^{\frac{1}{2}} = \sqrt{9} = 3$

$$\text{Q17: } \left(8^{\frac{1}{3}}\right)\left(8^{\frac{1}{3}}\right)\left(8^{\frac{1}{3}}\right) = 8^{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 8^1$$

But wait!
 $(2)(2)(2) = 8$

Also
 $(\sqrt[3]{8})(\sqrt[3]{8})(\sqrt[3]{8}) = 8$

So
 $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

Confirm the values of $9^{\frac{1}{2}}$ and $8^{\frac{1}{3}}$ using your calculator.

Part 5 - Exponents to Radicals

Q18: Write $x^{\frac{3}{2}}$ as an entire radical.

Option #1

$$x^{\frac{3}{2}} = (x^3)^{\frac{1}{2}} = \sqrt{x^3}$$

Option #2

$$x^{\frac{3}{2}} = (x^{\frac{1}{2}})^3 = (\sqrt{x})^3$$

Q19: Write $y^{\frac{3}{5}}$ as an entire radical.

Option #1

$$y^{\frac{3}{5}} = (y^3)^{\frac{1}{5}} = \sqrt[5]{y^3}$$

Option #2

$$y^{\frac{3}{5}} = (y^{\frac{1}{5}})^3 = (\sqrt[5]{y})^3$$

Part 6 - Rational Exponents with Numerical Response

Q20: The expression $\frac{\left(x^{\frac{2}{3}}\right)\left(x^{\frac{4}{3}}\right)}{\left(x^{\frac{1}{2}}\right)^{-\frac{1}{3}}}$ simplifies to $x^{\frac{ab}{c}}$, where a , b , and c are __, __, and __.

(Record your three digit answer in the Numerical Response boxes below)

1	3	6	
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$$\frac{\left(x^{\frac{2}{3}}\right)\left(x^{\frac{4}{3}}\right)}{\left(x^{\frac{1}{2}}\right)^{-\frac{1}{3}}} = \frac{x^{\frac{2}{3} + \frac{4}{3}}}{x^{\left(\frac{1}{2}\right)\left(-\frac{1}{3}\right)}} = \frac{x^{\frac{6}{3}}}{x^{-\frac{1}{6}}} = \frac{x^2}{x^{-\frac{1}{6}}}$$

$$= \frac{\left(x^2\right)\left(x^{\frac{1}{6}}\right)}{1} = x^{\frac{12}{6} + \frac{1}{6}} = \boxed{x^{\frac{13}{6}}}$$

so $ab = 13$
 $c = 6$

Part 7 – Harder Practice

Q21: Use the exponent laws to simplify each expression.

$$\left(x^{\frac{1}{2}}\right)\left(x^{\frac{7}{2}}\right) = x^{\frac{1}{2} + \frac{7}{2}} = x^{\frac{8}{2}} = x^4$$

$$\left(\frac{5x^3}{20x}\right)^{\frac{1}{2}} = \left(\frac{x^2}{4}\right)^{\frac{1}{2}} = \frac{(x^2)^{\frac{1}{2}}}{(4)^{\frac{1}{2}}} = \frac{x}{2}$$

$$\begin{aligned} \left(\frac{x^{\frac{1}{4}}}{16x^{\frac{3}{4}}}\right)^{\frac{1}{2}} &= \left(\frac{1}{16x^{\frac{1}{2}}}\right)^{\frac{1}{2}} = \frac{1}{(16)^{\frac{1}{2}}(x^{\frac{1}{2}})^{\frac{1}{2}}} \\ &= \frac{1}{4x^{\frac{1}{4}}} \end{aligned}$$

$$\begin{aligned} \left(x^{\frac{1}{3}}y^{\frac{4}{5}}\right)^0 \left(x^{\frac{1}{3}}\right)^6 &= (1)(x^{\frac{1}{3}})^6 \\ &= x^{6/3} \\ &= x^2 \end{aligned}$$

Simplify first, then evaluate.

$$\frac{5^{-2}}{125^{\frac{1}{3}}} = \frac{5^{-2}}{(5^3)^{\frac{1}{3}}} = \frac{5^{-2}}{5^1} = \frac{1}{5^3} = \frac{1}{125}$$

$$(4^3)\left(4^{\frac{3}{2}}\right) = 4^{3 + \frac{3}{2}} = 4^{\frac{6}{2} + \frac{3}{2}} = 4^{\frac{9}{2}}$$

$$\begin{aligned} \left(8^{\frac{2}{3}}\right)\left(16^{\frac{3}{2}}\right) &= (2^3)^{\frac{2}{3}} \cdot (2^4)^{\frac{3}{2}} \\ &= (2^2) \cdot (2^6) \\ &= 2^8 \end{aligned}$$

$$\begin{aligned} \frac{\left(\left(\frac{1}{9^2}\right)\left(\frac{1}{9^3}\right)\right)^2}{27^{\frac{2}{3}}} &= \frac{\left[(3^2)^{-\frac{1}{2}}(3^2)^{-\frac{1}{3}}\right]^2}{(3^3)^{\frac{2}{3}}} = \frac{\left[3 \cdot 3^{\frac{2}{3}}\right]^2}{3^2} \\ &= \frac{\left[3^{\frac{5}{3}}\right]^2}{3^2} = \frac{3^{\frac{10}{3}}}{3^{\frac{6}{3}}} = 3^{\frac{4}{3}} \end{aligned}$$