LO4 – 6.2 Multiplying and Dividing Rational Expressions

Key Ideas

Multiplying Key Ideas:

- Factor as much as you can.
- Reduce only Common Factors on both the Numerator and the Denominator.
- Identify all Non-Permissible Values (NPV).

Dividing Key Ideas:

- Remember that when dividing an expression, you simply need to multiply by the reciprocal.
- Factor as much as you can.
- Reduce only Common Factors on both the Numerator and the Denominator.
- Identify all Non-Permissible Values (NPV).
- **Don't forget the EXTRA NPV's**

Part 1 – Multiplying Fractions

Q1: Multiply the following fractions:

$$\frac{5}{3} * \frac{3}{2}$$

$$= \frac{(5)}{(3)} \cdot \frac{(3)}{(2)}$$

$$= \frac{5}{2}$$

$$\frac{4}{15} * \frac{35}{22}$$

$$= \frac{(2)(2)}{(3)(5)} \cdot \frac{(5)(7)}{(2)(11)}$$

$$= \frac{(2)(7)}{(3)(11)}$$

$$= \frac{14}{33}$$

$$\frac{14}{9} * \frac{21}{10}$$

$$= \frac{(2)(7)}{(3)(3)} \cdot \frac{(3)(7)}{(2)(5)}$$

$$= \frac{(7)(7)}{(3)(5)}$$

$$= \frac{49}{15}$$

$$\frac{30}{7} * \frac{14}{15}$$

$$= \frac{(3)(2)(5)}{(7)} \cdot \frac{(2)(7)}{(3)(5)}$$

$$= (2)(2)$$

$$= 4$$

$$\frac{10}{0} * \frac{6}{5}$$

Undefined number.
Cannot have a denominator equal to zero.

$$\frac{6}{5} * \frac{5}{0}$$

Undefined number.
Cannot have a denominator equal to zero.

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Part 2 – Multiplying Rational Expressions

Q2: Multiply, simplify, and determine any Non-Permissible Values (NPV).

$$\frac{(x+5)(x-2)}{(x-3)(x+3)} * \frac{(x+3)(x+2)}{(x-2)(x+4)}$$

$x \neq 3$ ⟵ ⟶ $x \neq -3$ ⟶ $x \neq 2$ ⟶ $x \neq -4$

$$= \frac{(x+5)(x+2)}{(x-3)(x+4)}$$

where $x \neq -4, -3, 2, 3$

$$\frac{2(x-3)(x+5)}{(x-4)(x+1)} * \frac{(x+1)(x-3)}{(x+5)(x+4)}$$

$x \neq 4$ $x \neq -1$ $x \neq -5$ $x \neq -4$

$$= \frac{2(x-3)(x-3)}{(x-4)(x+4)} \text{ or } \frac{2(x-3)^2}{(x-4)(x+4)}$$

where $x \neq -5, -4, -1, 4$

Q3: Multiply, simplify, and determine any Non-Permissible Values (NPV).

$$\frac{x^2+5x+6}{x^2-1} * \frac{x^2+5x+4}{x^2+x-6}$$

$$= \frac{(x+2)(x+3)}{(x+1)(x-1)} \cdot \frac{(x+1)(x+4)}{(x-2)(x+3)}$$

$x \neq -1$ $x \neq 1$ $x \neq 2$ $x \neq -3$

$$= \frac{(x+2)(x+4)}{(x-1)(x-2)} \text{ where } x \neq -3, -1, 1, 2$$

$x^2 + 5x + 6$ +2   +3
$$\square + \square = 5$$
$$\square \times \square = 6$$
$(x+2)(x+3)$

$x^2 - 1$ +1   -1
$$\square + \square = 0$$
$$\square \times \square = -1$$
$(x+1)(x-1)$

$x^2 + 5x + 4$ +1   +4
$$\square + \square = 5$$
$$\square \times \square = 4$$
$(x+1)(x+4)$

$x^2 + x - 6$ -2   +3
$$\square + \square = 1$$
$$\square \times \square = -6$$
$(x-2)(x+3)$

Q4: Multiply, simplify, and determine any Non-Permissible Values (NPV).

$$\frac{x^2+3x+2}{x^2+8x+15} * \frac{x^2+4x-5}{x^2+8x+7}$$

$$= \frac{(x+1)(x+2)}{(x+3)(x+5)} \cdot \frac{(x+5)(x-1)}{(x+1)(x+7)}$$

$$x \neq -3 \qquad x \neq -5 \qquad x \neq -1 \qquad x \neq -7$$

$$= \frac{(x+2)(x-1)}{(x+3)(x+7)} \quad \text{where } x \neq -7, -5, -3, -1$$

$x^2 + 3x + 2$

$$\square_{+1} + \square_{+2} = 3$$
$$\square \times \square = 2$$

$$(x+1)(x+2)$$

$x^2 + 8x + 15$

$$\square_{+3} + \square_{+5} = 8$$
$$\square \times \square = 15$$

$$(x+3)(x+5)$$

$x^2 + 4x - 5$

$$\square_{+5} + \square_{-1} = 4$$
$$\square \times \square = -5$$

$$(x+5)(x-1)$$

$x^2 + 8x + 7$

$$\square_{+1} + \square_{+7} = 8$$
$$\square \times \square = 7$$

$$(x+1)(x+7)$$

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Q5: Multiply, simplify, and determine any Non-Permissible Values (NPV).

$$\frac{2x^2+5x-3}{x^2-x-6} * \frac{x^2+x-2}{2x^2+x-1}$$

$$= \frac{(2x-1)(x+3)}{(x+2)(x-3)} \cdot \frac{(x-1)(x+2)}{(2x-1)(x+1)}$$

↙ ↓ ↓ ↘ $x \neq -1$

$x \neq -2$ $x \neq 3$ $2x-1 \neq 0$

$2x \neq 1$

$x \neq \frac{1}{2}$

$$= \frac{(x+3)(x-1)}{(x-3)(x+1)}$$

where $x \neq -2, -1, \frac{1}{2}, 3$

$2x^2 + 5x - 3$

-1 $+6$

$\square + \square = 5$

$\square \times \square = -6$

$2x^2 - 1x + 6x - 3$

$(2x^2 - 1x) + (6x - 3)$

$x(2x-1) + 3(2x-1)$

$(2x-1)(x+3)$

$x^2 - x - 6$

$+2$ -3

$\square + \square = -1$

$\square \times \square = -6$

$(x+2)(x-3)$

$x^2 + x - 2$

-1 $+2$

$\square + \square = 1$

$\square \times \square = -2$

$(x-1)(x+2)$

$2x^2 + x - 1$

-1 $+2$

$\square + \square = 1$

$\square \times \square = -2$

$2x^2 - 1x + 2x - 1$

$(2x^2 - 1x) + (2x - 1)$

$x(2x-1) + 1(2x-1)$

$(2x-1)(x+1)$

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Part 3 – Dividing Fractions

Q6: Determine the quotient in simplest form.

$$\frac{5}{3} \div \frac{2}{3}$$

$$= \frac{5}{3} \cdot \frac{3}{2}$$

$$= \frac{5}{2}$$

$$\frac{6}{35} \div \frac{9}{14}$$

$$= \frac{6}{35} \cdot \frac{14}{9}$$

$$= \frac{(2)(3)}{(5)(7)} \cdot \frac{(2)(7)}{(3)(3)} = \frac{4}{15}$$

$$\frac{10}{12} \div \frac{14}{15}$$

$$= \frac{10}{12} \cdot \frac{15}{14}$$

$$= \frac{(2)(5)}{(2)(2)(3)} \cdot \frac{(3)(5)}{(2)(7)} = \frac{25}{28}$$

$$\frac{5}{0} \div \frac{2}{3}$$

Undefined number.
Cannot have a
denominator equal
to zero.

$$\frac{5}{3} \div \frac{2}{0}$$

Undefined number.
Cannot have a
denominator equal
to zero.

$$\frac{5}{3} \div \frac{0}{3}$$

Cannot divide 5/3 by zero.
So we have a __NEW__ non-permissib
value... the numerator of the
second term.

Part 4 – Dividing Rational Expressions

Q7: Determine the quotient in simplest form and state all Non-Permissible Values (NPV).

$x \ne 4$ $x \ne 7$

$$\frac{(x+2)(x-4)}{(x+5)(x+1)} \div \frac{(x-4)(x-7)}{(x+1)(x+2)}$$

$x \ne -5$ $x \ne -1$ $x \ne -1$ $x \ne -2$

$$= \frac{(x+2)(x-4)}{(x+5)(x+1)} \cdot \frac{(x+1)(x+2)}{(x-4)(x-7)}$$

$$= \frac{(x+2)(x+2)}{(x+5)(x-7)}$$

where $x \ne -5, -2, -1, 4, 7$

$2x-5 \ne 0$
$x \ne 5/2$

$x \ne 1$

$$\frac{3(x+1)(2x-5)}{2(x+7)(x-5)} \div \frac{6(2x-5)(x-1)}{5(x+7)(x-7)}$$

$x \ne -7$ $x \ne 5$ $x \ne -7$ $x \ne 7$

$$= \frac{3(x+1)(2x-5)}{2(x+7)(x-5)} \cdot \frac{5(x+7)(x-7)}{6(2x-5)(x-1)}$$

$$= \frac{(3)(5)(x+1)(x-7)}{(2)(x-5)(6)(x-1)} = \frac{5(x+1)(x-7)}{4(x-5)(x-1)}$$

where $x \ne -7, 1, 5/2, 5, 7$

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Q8: Determine the quotient in simplest form and state all Non-Permissible Values (NPV).

$$\frac{x^2+5x+6}{4x^2-1} \div \frac{x^2-x-6}{2x^2+x-1}$$

$x \ne -2$ $x \ne 3$

$$= \frac{(x+2)(x+3)}{(2x+1)(2x-1)} \div \frac{(x+2)(x-3)}{(2x-1)(x+1)}$$

$2x+1 \ne 0$ $x \ne \frac{1}{2}$ $x \ne \frac{1}{2}$ $x \ne -1$
$2x \ne -1$
$x \ne -\frac{1}{2}$

$$= \frac{(x+2)(x+3)}{(2x+1)(2x-1)} \cdot \frac{(2x-1)(x+1)}{(x+2)(x-3)}$$

$$= \frac{(x+3)(x+1)}{(2x+1)(x-3)}$$

where $x \ne -2, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, 3$

$x^2 + 5x + 6$

+2   +3
$\square + \square = 5$
$\square \times \square = 6$

$(x+2)(x+3)$

$x^2 - x - 6$

+2   -3
$\square + \square = -1$
$\square \times \square = -6$

$(x+2)(x-3)$

$4x^2 - 1$

$(2x+1)(2x-1)$

$2x^2 + x - 1$

-1   +2
$\square + \square = 1$
$\square \times \square = -2$

$2x^2 - 1x + 2x - 1$

$(2x^2 - 1x) + (2x - 1)$

$x(2x-1) + 1(2x-1)$

$(2x-1)(x+1)$

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Part 5 – Multiplying and Dividing

Q9: Simplify the rational expression and determine any Non-Permissible Values (NPV).

$$\frac{2x^2+6x}{x^2-25} \div \frac{x^2-x-6}{2x+2} * \frac{x^2+7x+10}{4x^2+12x}$$

$$= \frac{(2x)(x+3)}{(x+5)(x-5)} \div \frac{(x+2)(x-3)}{(2)(x+1)} \cdot \frac{(x+2)(x+5)}{(4x)(x+3)}$$

$x \neq -2$ $x \neq 3$

$x \neq -5$ $x \neq 5$ $x \neq -1$ $x \neq 0$ $x \neq -3$

$$= \frac{(2x)(x+3)}{(x+5)(x-5)} \cdot \frac{(2)(x+1)}{(x+2)(x-3)} \cdot \frac{(x+2)(x+5)}{(4x)(x+3)}$$

$$= \frac{(2x)(2)(x+1)}{(x-5)(x-3)(4x)}$$

$$= \frac{(4x)(x+1)}{(4x)(x-5)(x-3)}$$

$$= \frac{(x+1)}{(x-5)(x-3)} \quad \text{where } x \neq -5, -3, -2, -1, 0, 3, 5$$