

1.05 – 6.3 Adding and Subtracting Rational Expressions**Key Ideas**

Adding and Subtracting Rational Expressions:

- In order to add or subtract rational expressions, they must have the same (common) denominator.
- Don't forget to state the NPV's.

**Part 1 – Adding and Subtracting Fractions**

Q1: Simplify the following expressions.

$$\frac{2}{3} + \frac{5}{3}$$

$$= \frac{(2) + (5)}{3}$$

$$= \frac{7}{3}$$

$$\frac{7}{4} - \frac{3}{4}$$

$$= \frac{(7) - (3)}{4}$$

$$= \frac{4}{4} = 1$$

$$\frac{4}{4} + \frac{3}{4}$$

$$= \frac{(4) + (3)}{4} = \frac{7}{4}$$

Q2: Simplify the following expressions.

$$\frac{2}{3} + \frac{5}{4}$$

$$= \frac{2}{3} \left( \frac{4}{4} \right) + \frac{5}{4} \left( \frac{3}{3} \right)$$

$$= \frac{8}{12} + \frac{15}{12}$$

$$= \frac{(8) + (15)}{12} = \frac{23}{12}$$

$$\frac{1}{8} - \frac{2}{4}$$

$$= \frac{1}{8} - \frac{2}{4} \left( \frac{2}{2} \right)$$

$$= \frac{1}{8} - \frac{4}{8}$$

$$= \frac{(1) - (4)}{8} = -\frac{3}{8}$$

$$\frac{5}{8} + 2 = \frac{5}{8} + \frac{2}{1}$$

$$\frac{5}{8} + \frac{2}{1} \left( \frac{8}{8} \right) = \frac{5}{8} + \frac{16}{8}$$

$$= \frac{(5) + (16)}{8} = \frac{21}{8}$$

Q3: Simplify the following expressions.

$$\frac{x}{5} + \frac{3}{5}$$

$$= \frac{(x) + (3)}{5}$$

$$= \frac{x+3}{5}$$

$$\frac{3x}{4} + \frac{5}{4}$$

$$= \frac{(3x) + (5)}{4}$$

$$= \frac{3x+5}{4}$$

$$\frac{2x}{3} + \frac{1}{4}$$

$$= \frac{2x}{3} \left( \frac{4}{4} \right) + \frac{1}{4} \left( \frac{3}{3} \right)$$

$$= \frac{8x}{12} + \frac{3}{12} = \frac{(8x) + (3)}{12}$$

$$= \frac{8x+3}{12}$$

## Part 2 – Adding and Subtracting Rational Expressions (Easy)

Q4: Simplify the following expressions. State NPV's.

$$\frac{5}{(x+2)} + \frac{2}{(x+3)}$$

$\downarrow$   $\downarrow$   
 $x \neq -2$   $x \neq -3$

$$= \frac{5}{(x+2)} \left( \frac{x+3}{x+3} \right) + \frac{2}{(x+3)} \left( \frac{x+2}{x+2} \right)$$

$$= \frac{5x+15}{(x+2)(x+3)} + \frac{2x+4}{(x+2)(x+3)}$$

$$= \frac{(5x+15) + (2x+4)}{(x+2)(x+3)}$$

$$= \frac{7x+19}{(x+2)(x+3)} \text{ where } x \neq -3, -2$$

$$\frac{3}{(x-5)} + \frac{7}{(x+2)}$$

$\downarrow$   $\downarrow$   
 $x \neq 5$   $x \neq -2$

$$= \frac{3}{(x-5)} \left( \frac{x+2}{x+2} \right) + \frac{7}{(x+2)} \left( \frac{x-5}{x-5} \right)$$

$$= \frac{3x+6}{(x-5)(x+2)} + \frac{7x-35}{(x+2)(x-5)}$$

$$= \frac{(3x+6) + (7x-35)}{(x+2)(x-5)}$$

$$= \frac{10x-29}{(x+2)(x-5)} \text{ where } x \neq -2, 5$$

Q5: Simplify the following expressions. State NPV's.

$$\frac{4}{(x-2)} - \frac{1}{(x+1)}$$

$\downarrow$   $\downarrow$   
 $x \neq 2$   $x \neq -1$

$$= \frac{4}{(x-2)} \left( \frac{x+1}{x+1} \right) - \frac{1}{(x+1)} \left( \frac{x-2}{x-2} \right)$$

$$= \frac{4x+4}{(x-2)(x+1)} - \frac{x-2}{(x+1)(x-2)}$$

$$= \frac{(4x+4) - (x-2)}{(x+1)(x-2)}$$

$$= \frac{3x+6}{(x+1)(x-2)} \text{ or } \frac{3(x+2)}{(x+1)(x-2)}$$

where  $x \neq -1, 2$

$$\frac{8}{(x+5)} - \frac{2}{(x-3)}$$

$\downarrow$   $\downarrow$   
 $x \neq -5$   $x \neq 3$

$$= \frac{8}{(x+5)} \left( \frac{x-3}{x-3} \right) - \frac{2}{(x-3)} \left( \frac{x+5}{x+5} \right)$$

$$= \frac{8x-24}{(x+5)(x-3)} - \frac{2x+10}{(x-3)(x+5)}$$

$$= \frac{(8x-24) - (2x+10)}{(x+5)(x-3)}$$

$$= \frac{6x-34}{(x+5)(x-3)} \text{ or } \frac{2(3x-17)}{(x+5)(x-3)}$$

where  $x \neq -5, 3$

Q6: Simplify the following expressions. State NPV's.

$$\begin{aligned} & \frac{(x+2)}{(x-4)} + \frac{(x-5)}{(x+1)} \\ & \quad \hookrightarrow x \neq 4 \quad \hookrightarrow x \neq -1 \\ = & \frac{(x+2)(x+1)}{(x-4)(x+1)} + \frac{(x-5)(x-4)}{(x+1)(x-4)} \\ = & \frac{x^2+3x+2}{(x-4)(x+1)} + \frac{x^2-9x+20}{(x+1)(x-4)} \\ = & \frac{(x^2+3x+2) + (x^2-9x+20)}{(x-4)(x+1)} \\ = & \frac{2x^2-6x+22}{(x-4)(x+1)} \text{ or } \frac{2(x^2-3x+11)}{(x-4)(x+1)} \\ & \text{where } x \neq -1, 4 \end{aligned}$$

$$\begin{aligned} & \frac{(x+1)}{(x+2)} + \frac{(x-1)}{(x+5)} \\ & \quad \hookrightarrow x \neq -2 \quad \hookrightarrow x \neq -5 \\ = & \frac{(x+1)(x+5)}{(x+2)(x+5)} + \frac{(x-1)(x+2)}{(x+5)(x+2)} \\ = & \frac{x^2+6x+5}{(x+2)(x+5)} + \frac{x^2+x-2}{(x+2)(x+5)} \\ = & \frac{(x^2+6x+5) + (x^2+x-2)}{(x+2)(x+5)} \\ = & \frac{2x^2+7x+3}{(x+2)(x+5)} \text{ or } \frac{(2x+1)(x+3)}{(x+2)(x+5)} \\ & \text{where } x \neq -5, -2 \end{aligned}$$

Q7: Simplify the following expressions. State NPV's.

$$\begin{aligned} & \frac{(x-3)}{(x+1)} - \frac{(x-4)}{(x+2)} \\ & \quad \hookrightarrow x \neq -1 \quad \hookrightarrow x \neq -2 \\ = & \frac{(x-3)(x+2)}{(x+1)(x+2)} - \frac{(x-4)(x+1)}{(x+2)(x+1)} \\ = & \frac{x^2-x-6}{(x+1)(x+2)} - \frac{x^2-3x-4}{(x+2)(x+1)} \\ = & \frac{(x^2-x-6) - (x^2-3x-4)}{(x+1)(x+2)} \\ = & \frac{2x-2}{(x+1)(x+2)} \text{ or } \frac{2(x-1)}{(x+1)(x+2)} \\ & \text{where } x \neq -2, -1 \end{aligned}$$

$$\begin{aligned} & \frac{(x-6)}{(x+1)} - \frac{(x-2)}{(x+2)} \\ & \quad \hookrightarrow x \neq -1 \quad \hookrightarrow x \neq -2 \\ = & \frac{(x-6)(x+2)}{(x+1)(x+2)} - \frac{(x-2)(x+1)}{(x+2)(x+1)} \\ = & \frac{x^2-4x-12}{(x+1)(x+2)} - \frac{x^2-x-2}{(x+1)(x+2)} \\ = & \frac{(x^2-4x-12) - (x^2-x-2)}{(x+1)(x+2)} \\ = & \frac{-3x-10}{(x+1)(x+2)} \text{ where } x \neq -2, -1 \end{aligned}$$

**Part 3 – Adding and Subtracting Rational Expressions (Harder)**

Q8: Simplify the following expression. State NPV's.

$$\begin{aligned}
 \frac{x^2+5x+6}{x^2+4x+3} + \frac{x^2-3x-10}{x^2+6x+8} &= \frac{(x+2)\cancel{(x+3)}}{\cancel{(x+3)}(x+1)} + \frac{(x-5)\cancel{(x+2)}}{\cancel{(x+2)}(x+4)} \\
 &\quad \begin{array}{cc} \downarrow & \downarrow \\ x \neq -3 & x \neq -1 \end{array} \quad \begin{array}{cc} \downarrow & \downarrow \\ x \neq -2 & x \neq -4 \end{array} \\
 &= \frac{(x+2)}{(x+1)} + \frac{(x-5)}{(x+4)} = \frac{(x+2)}{(x+1)} \left( \frac{x+4}{x+4} \right) + \frac{(x-5)}{(x+4)} \left( \frac{x+1}{x+1} \right) \\
 &= \frac{x^2+6x+8}{(x+1)(x+4)} + \frac{x^2-4x-5}{(x+1)(x+4)} = \frac{(x^2+6x+8) + (x^2-4x-5)}{(x+1)(x+4)} \\
 &= \frac{2x^2+2x+3}{(x+1)(x+4)} \text{ where } x \neq -4, -3, -2, -1
 \end{aligned}$$

Q9: Simplify the following expression. State NPV's.

$$\begin{aligned}
 \frac{x^2-x-2}{x^2+4x+3} - \frac{x^2-x-12}{x^2-5x+4} &= \frac{(x-2)\cancel{(x+1)}}{(x+3)\cancel{(x+1)}} - \frac{\cancel{(x-4)}(x+3)}{\cancel{(x-4)}(x-1)} \\
 &\quad \begin{array}{cc} \downarrow & \downarrow \\ x \neq -3 & x \neq -1 \end{array} \quad \begin{array}{cc} \downarrow & \downarrow \\ x \neq 4 & x \neq 1 \end{array} \\
 &= \frac{(x-2)}{(x+3)} - \frac{(x+3)}{(x-1)} = \frac{(x-2)}{(x+3)} \left( \frac{x-1}{x-1} \right) - \frac{(x+3)}{(x-1)} \left( \frac{x+3}{x+3} \right) \\
 &= \frac{x^2-3x+2}{(x+3)(x-1)} - \frac{x^2+6x+9}{(x+3)(x-1)} = \frac{(x^2-3x+2) - (x^2+6x+9)}{(x+3)(x-1)} \\
 &= \frac{-9x-7}{(x+3)(x-1)} \text{ where } x \neq -3, -1, 1, 4
 \end{aligned}$$

## Part 4 – Adding and Subtracting Rational Expressions (Hardest)

Q10: Simplify the following expression. State NPV's.

$$\begin{aligned}
 \frac{x-1}{x^2+x-6} - \frac{x-2}{x^2+4x+3} &= \frac{(x-1)}{(x+3)(x-2)} - \frac{(x-2)}{(x+3)(x+1)} \\
 &\quad \begin{array}{l} \swarrow x \neq -3 \quad \searrow x \neq 2 \\ \swarrow x \neq -3 \quad \searrow x \neq -1 \end{array} \\
 &= \frac{(x-1)}{(x+3)(x-2)} \left( \frac{x+1}{x+1} \right) - \frac{(x-2)}{(x+3)(x+1)} \left( \frac{x-2}{x-2} \right) \\
 &= \frac{x^2-1}{(x+3)(x-2)(x+1)} - \frac{x^2-4x+4}{(x+3)(x-2)(x+1)} \\
 &= \frac{(x^2-1) - (x^2-4x+4)}{(x+3)(x-2)(x+1)} = \frac{4x-5}{(x+3)(x-2)(x+1)} \quad \text{where } x \neq -3, -1, 2
 \end{aligned}$$

Q11: Simplify the expressions. State NPV's.

$$\begin{aligned}
 \frac{2-\frac{4}{y}}{y-\frac{4}{y}} &= \left(2-\frac{4}{y}\right) \div \left(y-\frac{4}{y}\right) & \frac{1+\frac{1}{x}}{x-\frac{1}{x}} &= \left(1+\frac{1}{x}\right) \div \left(x-\frac{1}{x}\right) \\
 & & & \\
 &= \left[ \frac{2}{1} \left( \frac{y}{y} \right) - \frac{4}{y} \right] \div \left[ \frac{y}{1} \left( \frac{y}{y} \right) - \frac{4}{y} \right] & &= \left[ \frac{1}{1} \left( \frac{x}{x} \right) + \frac{1}{x} \right] \div \left[ \frac{x}{1} \left( \frac{x}{x} \right) - \frac{1}{x} \right] \\
 &= \left[ \frac{2y-4}{y} \right] \div \left[ \frac{y^2-4}{y} \right] \quad \begin{array}{l} y^2-4 \neq 0 \\ y^2 \neq 4 \\ y \neq -2, +2 \end{array} & &= \left[ \frac{x+1}{x} \right] \div \left[ \frac{x^2-1}{x} \right] \quad \begin{array}{l} x^2-1 \neq 0 \\ x^2 \neq 1 \\ x \neq -1, 1 \end{array} \\
 &= \frac{2y-4}{y} \cdot \frac{y}{y^2-4} = \frac{2(y-2)}{y} \cdot \frac{y}{(y+2)(y-2)} & &= \frac{x+1}{x} \cdot \frac{x}{x^2-1} \\
 &= \frac{2}{y+2} \quad \text{where } y \neq -2, 0, 2 & &= \frac{(x+1)}{x} \cdot \frac{x}{(x+1)(x-1)} \\
 & & &= \frac{1}{x-1} \quad \text{where } x \neq -1, 0, 1
 \end{aligned}$$