■ KEY ■

L05 – Worksheet – 6.3 Adding and Subtracting Rational Expressions

Pg 336 #3ab: Simplify. Identify all non-permissible values.

$$\frac{1}{(x-3)(x+1)} - \frac{4}{(x+1)}$$

$x \neq 3 \qquad x \neq -1 \qquad x \neq -1$

$$= \frac{1}{(x-3)(x+1)} - \frac{4}{(x+1)}\left(\frac{x-3}{x-3}\right)$$

$$= \frac{1}{(x-3)(x+1)} - \frac{4x-12}{(x+1)(x-3)}$$

$$= \frac{(1) - (4x-12)}{(x-3)(x+1)}$$

$$= \frac{-4x+13}{(x-3)(x+1)} \quad \text{where } x \neq -1, 3$$

$$\frac{x-5}{x^2+8x-20} + \frac{2x+1}{x^2-4}$$

$$= \frac{(x-5)}{(x+10)(x-2)} + \frac{(2x+1)}{(x+2)(x-2)}$$

$x \neq -10 \quad x \neq 2 \qquad x \neq -2 \quad x \neq 2$

$$= \frac{(x-5)}{(x+10)(x-2)}\left(\frac{x+2}{x+2}\right) + \frac{(2x+1)}{(x+2)(x-2)}\left(\frac{x+10}{x+10}\right)$$

$$= \frac{x^2-3x-10}{(x+10)(x-2)(x+2)} + \frac{2x^2+21x+10}{(x+2)(x-2)(x+10)}$$

$$= \frac{3x^2+18x}{(x+10)(x-2)(x+2)} \quad \text{or} \quad \frac{3x(x+6)}{(x+10)(x-2)(x+2)}$$

$$\text{where } x \neq -2, -10, 2$$

Pg 336 #5ef: Add or subtract. Give answers in simplest form. Identify all non-permissible values.

$$\frac{2s}{5t^2} + \frac{1}{10t} - \frac{6}{15t^3}$$

$t \neq 0 \qquad t \neq 0 \qquad t \neq 0$

$$= \frac{2s}{5t^2}\left(\frac{6t}{6t}\right) + \frac{1}{10t}\left(\frac{3t^2}{3t^2}\right) - \frac{6}{15t^3}\left(\frac{2}{2}\right)$$

$$= \frac{12st}{30t^3} + \frac{3t^2}{30t^3} - \frac{12}{30t^3}$$

$$= \frac{(12st) + (3t^2) + (-12)}{30t^3}$$

$$= \frac{12st + 3t^2 - 12}{30t^3}$$

$$= \frac{3(t^2 + 4st - 4)}{30t^3}$$

$$= \frac{t^2 + 4st - 4}{10t^3} \quad \text{where } t \neq 0.$$

$$\frac{6xy}{a^2b} - \frac{2x}{ab^2y} + 1$$

$a \neq 0$
$b \neq 0$ $\qquad$ $a \neq 0$
$\qquad\qquad$ $b \neq 0$
$\qquad\qquad$ $y \neq 0$

$$= \frac{6xy}{a^2b}\left(\frac{by}{by}\right) - \frac{2x}{ab^2y}\left(\frac{a}{a}\right) + \frac{1}{1}\left(\frac{a^2b^2y}{a^2b^2y}\right)$$

$$= \frac{6bxy^2}{a^2b^2y} - \frac{2ax}{a^2b^2y} + \frac{a^2b^2y}{a^2b^2y}$$

$$= \frac{(6bxy^2 - 2ax + a^2b^2y)}{a^2b^2y}$$

$$\text{where } a \neq 0, b \neq 0, y \neq 0$$

Pg 336 #7ad: Simplify each rational expression, and then add or subtract. Express answers in simplest form. Identify all non-permissible values.

$$\frac{3x+15}{x^2-25} + \frac{4x^2-1}{2x^2+9x-5}$$

$$= \frac{3(x+5)}{(x+5)(x-5)} + \frac{(2x+1)(2x-1)}{(2x-1)(x+5)}$$

$x \neq -5 \quad x \neq 5 \quad x \neq \frac{1}{2} \quad x \neq -5$

$$= \frac{3}{(x-5)}\left(\frac{x+5}{x+5}\right) + \frac{2x+1}{x+5}\left(\frac{x-5}{x-5}\right)$$

$$= \frac{3x+15}{(x-5)(x+5)} + \frac{2x^2-9x-5}{(x+5)(x-5)}$$

$$= \frac{(3x+15) + (2x^2-9x-5)}{(x-5)(x+5)}$$

$$= \frac{2x^2-6x+10}{(x-5)(x+5)} = \frac{2(x^2-3x+5)}{(x-5)(x+5)}, \; x \neq -5, \tfrac{1}{2}, 5$$

Pg 337 #10ad: Simplify. State any non-permissible values.

$$\frac{2-\frac{6}{x}}{1-\frac{9}{x^2}} = \left(2-\frac{6}{x}\right) \div \left(1-\frac{9}{x^2}\right)$$

$$= \left[2\left(\frac{x}{x}\right) - \frac{6}{x}\right] \div \left[1\left(\frac{x^2}{x^2}\right) - \frac{9}{x^2}\right]$$

$$= \left[\frac{2x}{x} - \frac{6}{x}\right] \div \left[\frac{x^2}{x^2} - \frac{9}{x^2}\right]$$

$$= \frac{2x-6}{x} \div \frac{x^2-9}{x^2}$$

$$= \frac{2(x-3)}{x} \div \frac{(x+3)(x-3)}{x^2}$$

$x \neq -3 \quad x \neq 3$

$x \neq 0 \quad x \neq 0$

$$= \frac{2(x-3)}{x} \cdot \frac{x^2}{(x+3)(x-3)}$$

$$= \frac{2x}{x+3} \text{ where } x \neq -3, 0, 3$$

$$\frac{2w}{w^2+5w+6} - \frac{w-6}{w^2+6w+8}$$

$$= \frac{2w}{(w+2)(w+3)} - \frac{(w-6)}{(w+2)(w+4)}$$

$w \neq -2 \quad w \neq -3 \qquad w \neq -2 \quad w \neq -4$

$$= \frac{2w}{(w+2)(w+3)}\left(\frac{w+4}{w+4}\right) - \frac{(w-6)}{(w+2)(w+4)}\left(\frac{w+3}{w+3}\right)$$

$$= \frac{2w^2+8w}{(w+2)(w+3)(w+4)} - \frac{w^2-3w-18}{(w+2)(w+3)(w+4)}$$

$$= \frac{(2w^2+8w) - (w^2-3w-18)}{(w+2)(w+3)(w+4)}$$

$$= \frac{w^2+11w+18}{(w+2)(w+3)(w+4)} = \frac{(w+2)(w+9)}{(w+2)(w+3)(w+4)}$$

$$= \frac{(w+9)}{(w+3)(w+4)} \text{ where } w \neq -4, -3, -2$$

$$\frac{\frac{1}{x+4}+\frac{1}{x-4}}{\frac{x}{x^2-16}+\frac{1}{x+4}} = \left[\frac{1}{x+4} + \frac{1}{x-4}\right] \div \left[\frac{x}{x^2-16} + \frac{1}{x+4}\right]$$

$$= \left[\frac{1}{x+4}\left(\frac{x-4}{x-4}\right) + \frac{1}{x-4}\left(\frac{x+4}{x+4}\right)\right] \div \left[\frac{x}{(x-4)(x+4)} + \frac{1}{(x+4)}\left(\frac{x-4}{x-4}\right)\right]$$

$$= \left[\frac{(x-4)+(x+4)}{(x+4)(x-4)}\right] \div \left[\frac{(x)+(x-4)}{(x+4)(x-4)}\right]$$

$$= \left[\frac{2x}{(x+4)(x-4)}\right] \div \left[\frac{2x-4}{(x+4)(x-4)}\right] \qquad x \neq 2$$

$x \neq -4 \quad x \neq 4 \qquad x \neq -4 \quad x \neq 4$

$$= \frac{2x}{(x+4)(x-4)} \cdot \frac{(x+4)(x-4)}{2(x-2)}$$

$$= \frac{x}{x-2} \text{ where } x \neq -4, 2, 4$$

Pg 337 #14abcd: Suppose you can type an average of *n* words per minute.

   a. What is an expression for the number of minutes it would take to type an assignment with 200 words?

   b. Write a sum of rational expressions to represent the time it would take you to type three assignments of 200, 500, and 1000 words, respectively.

   c. Simplify the sum in part **b.** What does the simplified rational expression tell you?

   d. Suppose your typing speed decreases by 5 words per minute for each new assignment. Write a rational expression to represent how much longer it would take to type the three assignments. Express your answer in simplest form.

Ⓐ   WORDS PER MIN × MINUTES = TOTAL WORDS

$$\text{MINUTES} = \frac{\text{TOTAL WORDS}}{\text{WORDS PER MIN}}$$

$$= \frac{200}{n}$$

Ⓑ   MINUTES FOR ASSIGN 1 + MINUTES FOR ASSIGN 2 + MINUTES FOR ASSIGN 3

$$= \frac{200}{n} + \frac{500}{n} + \frac{1000}{n}$$

Ⓒ
$$= \frac{1700}{n}$$
where $n \neq 0$

This is the time it would take you to type up all three assignments.

Ⓓ   FIRST ASSIGNMENT RATE IS "n"
SECOND ASSIGNMENT RATE IS "$n-5$"
THIRD ASSIGNMENT RATE IS "$n-10$"

$$\text{MINUTES} = \frac{200}{n} + \frac{500}{n-5} + \frac{1000}{n-10} \quad \text{where } n \neq 0, 5, 10$$

$$= \frac{200}{n}\left(\frac{n-5}{n-5}\right)\left(\frac{n-10}{n-10}\right) + \frac{500}{n-5}\left(\frac{n}{n}\right)\left(\frac{n-10}{n-10}\right) + \frac{1000}{n-10}\left(\frac{n}{n}\right)\left(\frac{n-5}{n-5}\right)$$

$$= \frac{200(n^2-15n+50)}{n(n-5)(n-10)} + \frac{500(n^2-10n)}{n(n-5)(n-10)} + \frac{1000(n^2-5n)}{n(n-5)(n-10)}$$

$$= \frac{(200n^2 - 3000n + 10,000) + (500n^2 - 5000n) + (1000n^2 - 5000n)}{n(n-5)(n-10)}$$

$$= \frac{1700n^2 - 13,000n + 10,000}{n(n-5)(n-10)}$$

ADDITIONAL TIME $= \dfrac{1700n^2 - 13,000n + 10,000}{n(n-5)(n-10)} - \dfrac{1700}{n}\left(\dfrac{n-5}{n-5}\right)\left(\dfrac{n-10}{n-10}\right)$

$$= \frac{1700n^2 - 13,000n + 10,000}{n(n-5)(n-10)} - \frac{1700n^2 - 25,500n + 85,000}{n(n-5)(n-10)}$$

$$= \frac{(1700n^2 - 13,000n + 10,000) - (1700n^2 - 25,500n + 85,000)}{n(n-5)(n-10)}$$

$$= \frac{12500n - 75000}{n(n-5)(n-10)} = \frac{12500(n-6)}{n(n-5)(n-10)} \quad \text{where } n \neq 0, 5, 10$$

Math 20-1

Pg 338 #15ad: Simplify. Identify all non-permissible values.

$$\frac{x-2}{x+5} + \frac{x^2-2x-3}{x^2-x-6} * \frac{x^2+2x}{x^2-4x} \qquad = \frac{(x-2)}{(x+5)} + \frac{\cancel{(x-3)}(x+1)}{\cancel{(x-3)}(x+2)} \cdot \frac{\cancel{(x)}(x+2)}{\cancel{(x)}(x-4)}$$

$$ x \neq -5 \qquad x \neq 3 \quad x \neq -2 \qquad x \neq 0 \quad \to x \neq 4$$

$$= \frac{(x-2)}{(x+5)} + \frac{(x+1)}{(x-4)}$$

$$= \frac{(x-2)}{(x+5)}\left(\frac{x-4}{x-4}\right) + \frac{(x+1)}{(x-4)}\left(\frac{x+5}{x+5}\right)$$

$$= \frac{x^2-6x+8}{(x+5)(x-4)} + \frac{x^2+6x+5}{(x-4)(x+5)}$$

$$= \frac{2x^2+13}{(x+5)(x-4)} \qquad \text{where } x \neq -5, -2, 0, 3, 4$$

$$ x \neq -\tfrac{1}{2} \qquad\qquad x \neq -3$$

$$\frac{x+1}{x+6} - \frac{x^2-4}{x^2+2x} \div \frac{2x^2+7x+3}{2x^2+x} \qquad = \frac{(x+1)}{(x+6)} - \frac{(x+2)(x-2)}{(x)(x+2)} \div \frac{(2x+1)(x+3)}{(x)(2x+1)}$$

$$ x \neq -6 \qquad x \neq 0 \quad x \neq -2 \qquad x \neq 0 \quad x \neq -\tfrac{1}{2}$$

$$= \frac{(x+1)}{(x+6)} - \frac{(x-2)}{\cancel{(x)}} \cdot \frac{\cancel{(x)}}{(x+3)} \qquad = \frac{(x+1)}{(x+6)} - \frac{(x-2)}{(x+3)}$$

$$= \frac{(x+1)}{(x+6)}\left(\frac{x+3}{x+3}\right) - \frac{(x-2)}{(x+3)}\left(\frac{x+6}{x+6}\right) \qquad = \frac{x^2+4x+3}{(x+6)(x+3)} - \frac{x^2+4x-12}{(x+6)(x+3)}$$

$$= \frac{(x^2+4x+3)-(x^2+4x-12)}{(x+6)(x+3)} \qquad = \frac{15}{(x+6)(x+3)} \quad \text{where } x \neq -6, -3, -2, -\tfrac{1}{2}, 0$$