

Multiplying Rational Expressions

Simplify each expression.

$$1) \frac{59n}{99} \cdot \frac{80}{33n} = \frac{4720n}{3267n}$$

$$= \frac{4720}{3267}$$

$$2) \frac{53}{43} \cdot \frac{46n^2}{31} = \frac{2438n^2}{1333}$$

$$5) \frac{96}{38n} \cdot \frac{25}{45} = \frac{\cancel{2}(48)}{\cancel{2}(19)(n)} \cdot \frac{\cancel{5}(5)}{\cancel{5}(9)}$$

$$= \frac{(48)}{\cancel{19}(n)} \cdot \frac{(5)}{(9)} = \frac{\cancel{3}(16)}{\cancel{19}(n)} \cdot \frac{(5)}{\cancel{3}(3)}$$

$$= \frac{80}{57n} \quad \text{where } n \neq 0$$

$$6) \frac{84}{3} \cdot \frac{48x}{95} = \frac{\cancel{12}(7)}{(3)} \cdot \frac{\cancel{12}(4)x}{\cancel{5}(19)}$$

$$= \frac{\cancel{3}(4)(7)}{(3)} \cdot \frac{\cancel{3}(4)(4)x}{\cancel{5}(19)}$$

$$= \frac{1344x}{95}$$

$$9) \frac{2(p+6)}{4} \cdot \frac{p-3}{2(p-3)}$$

$$= \frac{\cancel{2}(p+6)}{\cancel{2}(2)} \cdot \frac{\cancel{(p-3)}}{\cancel{2}(p-3)} = \frac{p+6}{2} \cdot \frac{1}{2}$$

$\hookrightarrow p \neq 3$

$$= \frac{p+6}{4} \quad \text{where } p \neq 3$$

$$10) \frac{9(r+4)}{r+4} \cdot \frac{9r}{9(r-5)} = \frac{9\cancel{(r+4)}}{\cancel{(r+4)}} \cdot \frac{\cancel{9}(r)}{\cancel{9}(r-5)}$$

$\hookrightarrow r \neq -4 \quad \hookrightarrow r \neq 5$

$$= \frac{9r}{r-5} \quad \text{where } r \neq -4, 5$$

$$13) \frac{1}{v+10} \cdot \frac{10v+30}{v+3} = \frac{1}{(v+10)} \cdot \frac{10\cancel{(v+3)}}{\cancel{(v+3)}} = \frac{10}{v+10}$$

$\downarrow \quad \downarrow$
 $v \neq -10 \quad v \neq -3$

$$15) \frac{x+7}{7x+35} \cdot \frac{x^2-3x-40}{x-8}$$

$$= \frac{(x+7)}{(7)(x+5)} \cdot \frac{(x-8)(x+5)}{(x-8)}$$

\downarrow $x \neq -5$ \downarrow $x \neq 8$

$$= \frac{x+7}{7} \text{ where } x \neq -5, 8$$

$$16) \frac{20a^2-100a}{a-1} \cdot \frac{1}{16a^3-80a^2} = \frac{20a(a-5)}{(a-1)} \cdot \frac{1}{16a^2(a-5)}$$

\downarrow $a \neq 1$ \downarrow $a \neq 0$ \downarrow $a \neq 5$

$$= \frac{20a}{16a^2(a-1)} = \frac{5}{4a(a-1)} \text{ where } a \neq 0, 1, 5$$

$$19) \frac{21x^2-21x}{18x^2-18x} \cdot \frac{6x}{6x^2} = \frac{21x(x-1)}{18x(x-1)} \cdot \frac{(6x)}{(6x)(x)}$$

\downarrow $x \neq 0$ \downarrow $x \neq 1$ \downarrow $x \neq 0$ \downarrow $x \neq 0$

$$= \frac{(21x)}{(18x)(x)} = \frac{(3)(7)(x)}{(3)(6)(x)(x)}$$

$$= \frac{7}{6x} \text{ where } x \neq 0, 1$$

$$20) \frac{1}{p-9} \cdot \frac{p^2+6p-27}{p+9} = \frac{1}{(p-9)} \cdot \frac{(p+9)(p-3)}{(p+9)}$$

\downarrow $p \neq 9$ \downarrow $p \neq -9$

$$= \frac{p-3}{p-9} \text{ where } p \neq -9, 9$$

$$23) \frac{x^2-10x+25}{10x-100} \cdot \frac{x-10}{45-9x}$$

$$= \frac{(x-5)(x-5)}{10(x-10)} \cdot \frac{(x-10)}{-9(x-5)}$$

\downarrow $x \neq 10$ \downarrow $x \neq 5$

$$= \frac{x-5}{-90} \text{ where } x \neq 5, 10$$

$$24) \frac{45x^2}{x-9} \cdot \frac{x^2-5x-36}{3x^3+12x^2} = \frac{(3)(15)x^2}{(x-9)} \cdot \frac{(x-9)(x+4)}{3x^2(x+4)}$$

\downarrow $x \neq 9$ \downarrow $x \neq 0$ \downarrow $x \neq -4$

$$= 15, \text{ where } x \neq -4, 0, 9$$

$$27) \frac{m+1}{3m-15} \cdot \frac{8m-80}{m^2-9m-10} = \frac{(m+1)}{3(m-5)} \cdot \frac{8(m-10)}{(m-10)(m+1)} = \frac{8}{3(m-5)}$$

\downarrow $m \neq 5$ \downarrow $m \neq 10$ \downarrow $m \neq -1$

$$\text{where } m \neq -1, 5, 10$$

Dividing Rational Expressions

Simplify each expression.

$$1) \frac{10n}{9} \div \frac{13n^2}{16} = \frac{10\cancel{n}}{9} \cdot \frac{16}{13\cancel{n}^2}$$

$$= \frac{160}{117n}$$

$$2) \frac{16n}{17} \div \frac{8n}{6} = \frac{16\cancel{n}}{17} \cdot \frac{6}{8\cancel{n}}$$

$$= \frac{(2)(8\cancel{n})}{(17)} \cdot \frac{(6)}{(8\cancel{n})}$$

$$= \frac{12}{17} \text{ where } n \neq 0$$

$$5) \frac{7}{18} \div \frac{6}{9a} = \frac{7}{(2)(9)} \cdot \frac{(9)(a)}{(6)}$$

$$= \frac{7a}{12} \text{ where } a \neq 0$$

$$6) \frac{5}{20} \div \frac{5x}{3} = \frac{(5)}{(4)(5)} \cdot \frac{3}{(5x)}$$

$$= \frac{3}{20x} \text{ where } x \neq 0$$

$$9) \frac{7a^2}{7a^3 + 56a^2} \div \frac{2}{a^2 + 7a - 8} = \frac{(7a^2)}{(7a^2)(a+8)} \div \frac{(2)}{(a+8)(a-1)}$$

$$= \frac{(7\cancel{a}^2)}{(7\cancel{a}^2)(a+8)} \cdot \frac{(a+8)(a-1)}{(2)} = \frac{a-1}{2} \text{ where } a \neq -8, 0, 1$$

$$\begin{aligned}
 11) \quad & \frac{x^2 + 10x + 16}{x^2 + 6x + 8} \div \frac{1}{x+4} \\
 & = \frac{(x+8)(x+2)}{(x+2)(x+4)} \div \frac{1}{(x+4)} \\
 & \quad \downarrow \quad \downarrow \quad \downarrow \\
 & \quad x \neq -2 \quad x \neq -4 \quad x \neq -4 \\
 & = \frac{(x+8)(x+2)}{(x+2)(x+4)} \cdot \frac{(x+4)}{1} \\
 & = x+8, \text{ where } x \neq -4, -2
 \end{aligned}$$

$$\begin{aligned}
 12) \quad & \frac{49x+21}{6x} \div \frac{42x+18}{6} \\
 & = \frac{7(7x+3)}{6x} \div \frac{6(7x+3)}{6} \quad x \neq -3/7 \\
 & \quad \downarrow \\
 & \quad x \neq 0 \\
 & = \frac{7(7x+3)}{6x} \cdot \frac{6}{6(7x+3)} \\
 & = \frac{7}{6x} \text{ where } x \neq -3/7, 0
 \end{aligned}$$

$$\begin{aligned}
 15) \quad & \frac{8}{4n^2 - 16n} \div \frac{1}{n-4} \\
 & = \frac{(2)(4)}{(4n)(n-4)} \div \frac{1}{(n-4)} \\
 & \quad \downarrow \quad \downarrow \quad \downarrow \\
 & \quad n \neq 0 \quad n \neq 4 \quad n \neq 4 \\
 & = \frac{(2)(4)}{(4)(n)(n-4)} \cdot \frac{(n-4)}{1} \\
 & = \frac{2}{n} \text{ where } n \neq 0, 4
 \end{aligned}$$

$$\begin{aligned}
 16) \quad & \frac{a-4}{a^2 - 2a - 8} \div \frac{1}{a-5} \\
 & = \frac{(a-4)}{(a-4)(a+2)} \div \frac{1}{(a-5)} \\
 & \quad \downarrow \quad \downarrow \quad \downarrow \\
 & \quad a \neq 4 \quad a \neq -2 \quad a \neq 5 \\
 & = \frac{1}{(a+2)} \cdot \frac{(a-5)}{1} \\
 & = \frac{a-5}{a+2} \text{ where } a \neq -2, 4, 5
 \end{aligned}$$

$$\begin{aligned}
 19) \quad & \frac{16x-56}{8} \div \frac{8x-28}{4} \quad x \neq 7/2 \\
 & = \frac{8(2x-7)}{8} \div \frac{4(2x-7)}{4} \\
 & = \frac{(2x-7)}{1} \cdot \frac{1}{(2x-7)} \\
 & = 1, \text{ where } x \neq 7/2
 \end{aligned}$$

$$\begin{aligned}
 20) \quad & \frac{10x^2 - 28x + 16}{2x-4} \div \frac{25x^2 - 25x + 4}{5x^2 - 41x + 8} \\
 & = \frac{2(5x^2 - 14x + 8)}{2(x-2)} \div \frac{(5x-1)(5x-4)}{(5x-1)(x-8)} \\
 & \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 & \quad x \neq 2 \quad x \neq 1/5 \quad x \neq 4/5 \\
 & = \frac{2(5x-4)(x-2)}{2(x-2)} \cdot \frac{(5x-1)(x-8)}{(5x-1)(5x-4)} \\
 & = x-8, \text{ where } x \neq 1/5, 4/5, 2, 8
 \end{aligned}$$

Adding + Subtracting Rational Expressions

Simplify each expression.

$$\begin{aligned}
 1) \quad & \frac{u+5v}{8v^2u^2} - \frac{u-6v}{8v^2u^2} \\
 &= \frac{(u+5v) - (u-6v)}{8v^2u^2} \\
 &= \frac{11}{8v^2u^2} \quad \text{where } v \neq 0, u \neq 0
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \frac{5n}{30m} + \frac{2m+4n}{30m} \\
 & \quad \downarrow \quad \quad \downarrow \\
 & \quad m \neq 0 \quad m \neq 0 \\
 &= \frac{(5n) + (2m+4n)}{30m} \\
 &= \frac{2m+9n}{30m} \quad \text{where } m \neq 0
 \end{aligned}$$

$$\begin{aligned}
 5) \quad & \frac{4a-5}{6a^2+30a} + \frac{a-1}{6a^2+30a} \\
 &= \frac{(4a-5)}{6a(a+5)} + \frac{(a-1)}{6a(a+5)} \\
 & \quad \downarrow \quad \downarrow \quad \downarrow \quad \rightarrow a \neq -5 \\
 & \quad a \neq 0 \quad a \neq -5 \quad a \neq 0 \\
 &= \frac{(4a-5) + (a-1)}{6a(a+5)} = \frac{5a-6}{6a(a+5)} \\
 & \quad \text{where } a \neq -5, 0
 \end{aligned}$$

$$\begin{aligned}
 6) \quad & \frac{5x-4}{9x^3+27x^2} - \frac{x+6}{9x^3+27x^2} \\
 &= \frac{(5x-4)}{9x^2(x+3)} - \frac{(x+6)}{9x^2(x+3)} \\
 &= \frac{(5x-4) - (x+6)}{9x^2(x+3)} \\
 & \quad \downarrow \quad \rightarrow x \neq 0 \quad \rightarrow x \neq -3 \\
 &= \frac{4x-10}{9x^2(x+3)} \quad \text{or} \quad \frac{2(2x-5)}{9x^2(x+3)} \\
 & \quad \text{where } x \neq -3, 0
 \end{aligned}$$

$$\begin{aligned}
 9) \quad & \frac{7x}{2x} - \frac{x-2}{20x+16} = \frac{7x}{2x} - \frac{(x-2)}{4(5x+4)} \\
 & \quad \downarrow \quad \quad \downarrow \\
 & \quad x \neq 0 \quad \quad x \neq -4/5 \\
 &= \frac{14(5x+4)}{4(5x+4)} - \frac{(x-2)}{4(5x+4)} = \frac{69x+58}{4(5x+4)} \quad \text{where } x \neq -4/5, 0
 \end{aligned}$$

$$11) \frac{7v}{8} - \frac{8v-4}{(5v-2)}$$

$v \neq \frac{2}{5}$

$$= \frac{7v}{8} \left(\frac{5v-2}{5v-2} \right) - \frac{8v-4}{5v-2} \left(\frac{8}{8} \right)$$

$$= \frac{35v^2 - 14v}{8(5v-2)} - \frac{64v-32}{8(5v-2)} = \frac{(35v^2 - 14v) - (64v - 32)}{8(5v-2)}$$

$$= \frac{35v^2 - 78v + 32}{8(5v-2)} \text{ where } v \neq \frac{2}{5}$$

$$12) \frac{4}{n+7} - \frac{7}{n-2}$$

$n \neq -7, 2$

$$= \frac{4}{n+7} \left(\frac{n-2}{n-2} \right) - \frac{7}{n-2} \left(\frac{n+7}{n+7} \right)$$

$$= \frac{4n-8}{(n+7)(n-2)} - \frac{7n+49}{(n+7)(n-2)}$$

$$= \frac{(4n-8) - (7n+49)}{(n+7)(n-2)}$$

$$= \frac{-3n-57}{(n+7)(n-2)} \text{ where } n \neq -7, 2$$

$$15) \frac{6x}{3} + \frac{7}{15x+3}$$

$x \neq -\frac{1}{5}$

$$= \frac{3(2x)}{3} + \frac{7}{3(5x+1)}$$

$$= \frac{3(2x)}{3} \left(\frac{5x+1}{5x+1} \right) + \frac{7}{3(5x+1)}$$

$$= \frac{30x^2 + 6x}{3(5x+1)} + \frac{7}{3(5x+1)}$$

$$= \frac{30x^2 + 6x + 7}{3(5x+1)} \text{ where } x \neq -\frac{1}{5}$$

$$16) \frac{5v}{v-3} + \frac{5}{v+6}$$

$v \neq 3, -6$

$$= \frac{5v}{v-3} \left(\frac{v+6}{v+6} \right) + \frac{5}{v+6} \left(\frac{v-3}{v-3} \right)$$

$$= \frac{5v^2 + 30v}{(v-3)(v+6)} + \frac{5v-15}{(v-3)(v+6)}$$

$$= \frac{(5v^2 + 30v) + (5v-15)}{(v-3)(v+6)}$$

$$= \frac{5v^2 + 35v - 15}{(v-3)(v+6)} = \frac{5(v^2 + 7v - 3)}{(v-3)(v+6)}$$

where $v \neq -6, 3$

$$19) \frac{4x}{x+3} - \frac{4x}{x+6}$$

$x \neq -3, -6$

$$= \frac{4x}{x+3} \left(\frac{x+6}{x+6} \right) - \frac{4x}{x+6} \left(\frac{x+3}{x+3} \right)$$

$$= \frac{4x^2 + 24x}{(x+3)(x+6)} - \frac{4x^2 + 12x}{(x+6)(x+3)}$$

$$= \frac{(4x^2 + 24x) - (4x^2 + 12x)}{(x+3)(x+6)}$$

$$= \frac{12x}{(x+3)(x+6)}, \text{ where } x \neq -6, -3$$

$$20) \frac{2x}{3x+3} - \frac{2}{x+5} = \frac{2x}{3(x+1)} - \frac{2}{(x+5)}$$

$x \neq -1, -5$

$$= \frac{2x}{3(x+1)} \left(\frac{x+5}{x+5} \right) - \frac{2}{(x+5)} \left(\frac{3}{3} \right) \left(\frac{x+1}{x+1} \right)$$

$$= \frac{2x^2 + 10x}{3(x+1)(x+5)} - \frac{6x+6}{3(x+1)(x+5)}$$

$$= \frac{(2x^2 + 10x) - (6x+6)}{3(x+1)(x+5)}$$

$$= \frac{2x^2 + 4x - 6}{3(x+1)(x+5)} = \frac{2(x^2 + 2x - 3)}{3(x+1)(x+5)}$$

$$= \frac{2(x+3)(x-1)}{3(x+1)(x+5)} \text{ where } x \neq -5, -1$$