KEY

L07 – 6.4 Rational Equations

Key Ideas

To solve a Rational Equation:

- Simplify to solve for the variable.
- When the equation is in the form $ax^2 + bx + c = 0$, factor to the form $(x + a)(x + b) = 0$ to find the solutions.
- The goal of solving these equations is to either (a) eliminate the denominator, or (b) have a least common denominator.

Special Note: Some equations in the form $ax^2 + bx + c = 0$ are not factorable, and will need to be solved using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. This will be taught in ***Lesson 20*** later this unit.

Part 1 – Solving Equations with Fractions

Q1: Solve for x.

$$\frac{x}{(2)(3)} + \frac{2x}{(3)(5)} = \frac{1}{(2)}$$

$\frac{x}{(2)(3)}\left(\frac{5}{5}\right) + \frac{2x}{(3)(5)}\left(\frac{2}{2}\right) = \frac{1}{(2)}\left(\frac{3}{3}\right)\left(\frac{5}{5}\right)$

$\frac{5x}{30} + \frac{4x}{30} = \frac{15}{30}$ Now look at numerator.

$5x + 4x = 15$

$9x = 15$

$x = 15/9$

$$\frac{x+2}{(3)} - \frac{x}{(5)} = x$$

$\frac{(x+2)}{3}\left(\frac{5}{5}\right) - \frac{x}{5}\left(\frac{3}{3}\right) = \frac{x}{1}\left(\frac{3}{3}\right)\left(\frac{5}{5}\right)$

$\frac{5x+10}{15} - \frac{3x}{15} = \frac{15x}{15}$ Now look at numerator.

$5x + 10 - 3x = 15x$

$10 = 13x$

$x = 10/13$

Q2: Solve for x.

$$\frac{x}{20} + \frac{3}{15} = \frac{x}{10}$$

$\frac{x}{20}\left(\frac{3}{3}\right) + \frac{3}{15}\left(\frac{4}{4}\right) = \frac{x}{10}\left(\frac{6}{6}\right)$

$\frac{3x}{60} + \frac{12}{60} = \frac{6x}{60}$ Now look at numerator.

$3x + 12 = 6x$

$12 = 3x$

$x = 4$

$$\frac{x+5}{4} - 3 = x$$

$\frac{x+5}{4} - \frac{3}{1}\left(\frac{4}{4}\right) = \frac{x}{1}\left(\frac{4}{4}\right)$

$\frac{x+5}{4} - \frac{12}{4} = \frac{4x}{4}$ Now look at numerator.

$(x+5) - (12) = 4x$

$x - 7 = 4x$

$-7 = 3x$

$x = -7/3$

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Part 2 – Solving Equations with Fractions

Q3: Solve for x.

$$\frac{x}{4} - \frac{7}{x} = 3 \quad \rightarrow x \neq 0$$

$\frac{x}{4}\left(\frac{x}{x}\right) - \frac{7}{x}\left(\frac{4}{4}\right) = \frac{3}{1}\left(\frac{4x}{4x}\right)$

$\frac{x^2}{4x} - \frac{28}{4x} = \frac{12x}{4x}$ Now look at numerators.

$x^2 - 28 = 12x$

$x^2 - 12x - 28 = 0$

$(x-14)(x+2) = 0$

$x = 14 \qquad x = -2$

So $x = -2, 14$.

$$\frac{4}{n^2} = \frac{5}{n} - \frac{1}{n^2} \quad \rightarrow n \neq 0$$

$\frac{4}{n^2} = \frac{5}{n}\left(\frac{n}{n}\right) - \frac{1}{n^2}$

$\frac{4}{n^2} = \frac{5n}{n^2} - \frac{1}{n^2}$ Now look at numerators.

$4 = 5n - 1$

$5 = 5n$

$n = 1$

Q4: Solve for x.

$$\frac{1}{3x^2} = \frac{x+3}{2x^2} - \frac{1}{6x^2} \quad \rightarrow x \neq 0$$

$\frac{1}{3x^2}\left(\frac{2}{2}\right) = \frac{(x+3)}{2x^2}\left(\frac{3}{3}\right) - \frac{1}{6x^2}$

$\frac{2}{6x^2} = \frac{3x+9}{6x^2} - \frac{1}{6x^2}$ Now look at numerator.

$2 = (3x+9) - (1)$

$2 = 3x + 8$

$-6 = 3x$

$x = -2$

$$\frac{2}{x^2-4} + \frac{10}{6x+12} = \frac{1}{x-2}$$

$\frac{2}{(x+2)(x-2)} + \frac{10}{6(x+2)} = \frac{1}{(x-2)} \rightarrow x \neq -2, 2$

$\frac{2}{(x+2)(x-2)}\left(\frac{6}{6}\right) + \frac{10}{6(x+2)}\left(\frac{x-2}{x-2}\right) = \frac{1}{(x-2)}\left(\frac{x+2}{x+2}\right)\left(\frac{6}{6}\right)$

$\frac{12}{6(x+2)(x-2)} + \frac{10x-20}{6(x+2)(x-2)} = \frac{6x+12}{6(x+2)(x-2)}$

$(12) + (10x - 20) = (6x + 12)$

$10x - 8 = 6x + 12$

$4x = 20$

$x = 5$

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Q5: Solve for x.

$$\frac{x-6}{x} = \frac{x+4}{x} + 1 \quad \rightarrow x \neq 0$$

$\frac{x-6}{x} = \frac{x+4}{x} + \frac{1}{1}\left(\frac{x}{x}\right)$ Now look at numerator

$x-6 = (x+4) + (x)$

$x-6 = 2x+4$

$-10 = x$

$$\frac{1}{2n} + \frac{1}{4n^2} = \frac{1}{4n} \quad \rightarrow n \neq 0$$

$\frac{1}{2n}\left(\frac{2n}{2n}\right) + \frac{1}{4n^2} = \frac{1}{4n}\left(\frac{n}{n}\right)$

$\frac{2n}{4n^2} + \frac{1}{4n^2} = \frac{n}{4n^2}$ Now look at numerator.

$2n+1 = n$

$n = -1$

Q6: Solve for x.

$$\frac{1}{2x} - \frac{x-1}{2x^2} = \frac{3}{x} \quad \rightarrow x \neq 0$$

$\frac{1}{2x}\left(\frac{x}{x}\right) - \frac{(x-1)}{2x^2} = \frac{3}{x}\left(\frac{2x}{2x}\right)$

$\frac{x}{2x^2} - \frac{(x-1)}{2x^2} = \frac{6x}{2x^2}$ Now look at numerator

$(x) - (x-1) = 6x$

$x - x + 1 = 6x$

$1 = 6x$

$x = 1/6$

$\frac{n^2-n-6}{n^2} - \frac{2n+12}{n} = \frac{n-6}{2n} \rightarrow n \neq 0$

$\frac{n^2-n-6}{n^2}\left(\frac{2}{2}\right) - \frac{2n+12}{n}\left(\frac{2n}{2n}\right) = \frac{n-6}{2n}\left(\frac{n}{n}\right)$

$\frac{2n^2-2n-12}{2n^2} - \frac{4n^2+24n}{2n^2} = \frac{n^2-6n}{2n^2}$

$(2n^2-2n-12) - (4n^2+24n) = n^2-6n$

$-2n^2-26n-12 = n^2-6n$

$0 = 3n^2+20n+12$

+2 +18

□+□ = 20

□×□ = 36

1, 36
2, 18
4, 9
6, 6

$0 = 3n^2+2n+18n+12$

$0 = (3n^2+2n) + (18n+12)$

$0 = n(3n+2) + 6(3n+2)$

$0 = (3n+2)(n+6)$

$n = -2/3$ $\quad$ $n = -6$

So $n = -6$ or $-2/3$

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Q7: Solve for the variable.

$$\frac{x+2}{x} = \frac{x-1}{x} - \frac{4x+2}{x^2-3x}$$

$\frac{x+2}{x} = \frac{x-1}{x} - \frac{2(2x+1)}{x(x-3)}$ → $x \neq 0, 3$

$\frac{x+2}{x}\left(\frac{x-3}{x-3}\right) = \frac{x-1}{x}\left(\frac{x-3}{x-3}\right) - \frac{4x+2}{(x)(x-3)}$

$\frac{x^2-x-6}{(x)(x-3)} = \frac{x^2-4x+3}{(x)(x-3)} - \frac{4x+2}{(x)(x-3)}$

$(x^2-x-6) = (x^2-4x+3) - (4x+2)$

$x^2-x-6 = x^2-8x+1$

$7x = 7$

$x = 1$

$$\frac{1}{2} = \frac{x^2-7x+10}{4x} - \frac{1}{2x}$$

$\frac{1}{2}\left(\quad\right) = \frac{x^2-7x+10}{4x} - \frac{1}{2x}\left(\frac{2}{2}\right)$

$\frac{2x}{4x} = \frac{x^2-7x+10}{4x} - \frac{2}{4x}$

$2x = (x^2-7x+10) - (2)$

$2x = x^2-7x+8$

$0 = x^2-9x+8$

$0 = (x-8)(x-1)$

$x = 8$ $\quad$ $x = 1$

So $x = 1$ or 8.