Math 10C

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L08 – 4.5 Exponents and Problem Solving

Part 1 – Investigating Exponential Growth

Use the following information to answer Q1-Q4:

> A bacterial culture in a lab has 500 cells. The number of cells doubles every hour.

Q1: Complete the following table:

Time (hours)	Number of Bacteria Cells
0	500
1	1000
2	2000
3	4000
4	8000
5	16000
6	32000

Q2: Create an equation, where N is the number of cells and t is the number of hours, for this scenario.

$$N = (500)(2)^t$$

Q3: How many cells are there in 20 hours?

$$N = (500)(2)^{20}$$
$$= (500)(1,048,576)$$
$$= 524,288,000 \text{ cells}$$

Q4: How many cells where there 4 hours before the initial count?

$$N = (500)(2)^{-4}$$
$$= (500)(0.0625)$$
$$= 31.25 \text{ cells} \Rightarrow \text{Doesn't make sense.}$$
$$N = 31 \text{ cells (nice whole number!)}$$

Q5: When will there be 250 000 cells?

Guess and check.

$N = (500)(2)^5 = 16,000$

$N = (500)(2)^{10} = 512,000$

$N = (500)(2)^9 = 256,000 \rightarrow$ so approximately after 9 hours.

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Part 2 – The Exponential Growth and Decay Equation

$$y = ab^{\frac{x}{t}}$$

a – *represents the starting amount*
b – *represents the rate of change (doubling, tripling, half ...)*
t – *represents the time or how often the rate of change occurs*

Double? $y = (\text{starting}) \times (2)^{x/t}$

Halfing? $y = (\text{starting}) \times (0.5)^{x/t}$

Part 3 – Bacteria Growth

Use the following information to answer Q6-Q9:

The growth of 5000 bacterium cells in a lab can be modelled using the expression $N = 5000(1.5)^{\frac{h}{40}}$, where *N* is the number of bacteria after *h* hours.

Q6: What does the value 1.5 in the expression tell you?

That it "one and a halves" every time increment.

Q7: How many bacteria are there after 40 h?

That it "one and a halves" every 40 hours.

Q8: How many more bacteria are there after 3 h?

$N = 5000(1.5)^{3/40}$
$= 5000(1.5)^{0.075}$
$N = 5000(1.03087\ldots)$
$N = 5154.3849\ldots$
So an extra 154 bacteria.

Q9: What does *h* = 0 indicate?

Starting amount.

$N = 5000(1.5)^{0/40} = 5000(1.5)^{0} = 5000$

Starting amount

Starting amount

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Use the following information to answer Q10-Q12:

A Petri dish originally has 20 bacteria in it at the stroke of midnight. Every 5 hours the number of bacteria triples.

Q10: How many bacteria are present at noon?

*(Record your **three digit** answer in the Numerical Response boxes below)*

$y = a\,b^{x/t}$

$y = 20(3)^{x/5}$

2	7	9	

Noon is 12 hours later.

$y = 20(3)^{12/5}$

$= 20(3)^{2.4}$

$= 20(13.9666)$

$= 279.333$

So 279 bacteria (whole number)

Q11: How many bacteria were present 2 hours before midnight?

$y = 20(3)^{-2/5}$

$= 20(3)^{-0.4}$

$= 20(0.64439...)$

$= 12.887...$

≈ 13 bacteria (whole number)

Q12: At what time would there be 180 bacteria present?

$20 \times 3 = 60 \times 3 = 180$

$t = 0 \qquad t = 5 \qquad t = 10$

10 hours after midnight, or 10am

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Part 4 – Radioactive Decay

Use the following information to answer Q13:

Household smoke detectors use a sample of 3.0×10^{-7} grams of Americium-241. Americium-241 has a half-life of 432.6 years.

Q13: How many grams of Americium-241 are present in a smoke detector after 10 years?

$y = ab^{x/t}$

$y = (3\times10^{-7})(0.5)^{10/432.6}$

$y = (3\times10^{-7})(0.5)^{0.023116}$

$= (3\times10^{-7})(0.9841)$

$= 2.95\times10^{-7}$ grams

Use the following information to answer Q14:

Carbon-14 radioactive dating uses an isotope of Carbon with a half-life of 5730 years.

Q14: What percentage of the original Carbon-14 in a living mammal would be present 8000 years after it has died?

$y = ab^{x/t}$

$y = 100(0.5)^{8000/5730}$

$y = 100(0.5)^{1.39616}$

$= 100(0.379938\ldots)$

$= 37.99\%$ left.

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Part 5 – Depreciation

Use the following information to answer Q15:

Unfortunately, over the years vehicles depreciate in price. You bought a used Ford F-150 for $15,000. It depreciates 13% per year

Q15: Write an equation to represent this scenario.

$y = ab^{x/t}$

$y = (15{,}000)(0.87)^{x/1}$

Q16: What will the Ford F-150 be worth 5 years after you purchase it?

$y = (15{,}000)(0.87)^5$

$= (15{,}000)(0.49842)$

$= 7{,}476.31$

So approximately $\$7{,}476.31$

Part 6 – Practice

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