KEY

Math 20-1

L08 – Word Problems

Key Ideas

Adding or Subtracting Radicals:

- Make sure you have a common denominator. Add or subtract the numerator.
- Include your Non-Permissible Values.

Multiplying or Dividing Radicals:

- Factor first.
- To divide, multiply by the reciprocal. Don't forget the extra Non-Permissible Values!

Solving Rational Equations

- Make sure you have a common denominator.
- Use the numerators to solve the equation.
- Include your Non-Permissible Values.

Part 1 – Word Problems involving Rates

Q1: Bob can paint 1 room in 20 minutes. Sam can paint 1 room in 25 minutes. If they work together, how much time does it take them to paint 4 rooms?

BOB
Paints 1 room in 20 min.
Paints $\frac{1}{20}^{th}$ of a room per min.

SAM
Paints 1 room in 25 min.
Paints $\frac{1}{25}^{th}$ of a room per minute.

$\left(\frac{1}{20}\text{ room per min}\right)(n\text{ min}) + \left(\frac{1}{25}\text{ room per min}\right)(n\text{ min}) = 4\text{ rooms}$

$\left(\frac{1}{20}\right)(n) + \left(\frac{1}{25}\right)(n) = 4$

$\frac{n}{20} + \frac{n}{25} = 4$

$\frac{n}{20}\left(\frac{5}{5}\right) + \frac{n}{25}\left(\frac{4}{4}\right) = \frac{4}{1}\left(\frac{100}{100}\right)$

$\frac{5n}{100} + \frac{4n}{100} = \frac{400}{100}$

Look at numerators.

$(5n) + (4n) = 400$

$9n = 400$

$n = 44.\overline{4}\text{ min}$

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Part 2 – Word Problems involving Relative Motion

Q2: A river current is moving at 2 km/h. The kayaker can paddle km upstream in the same amount of time it takes him to travel 26 km downstream.

$$\text{RATE/SPEED} = \frac{\text{DISTANCE}}{\text{TIME}} \Rightarrow \text{TIME} = \frac{\text{DISTANCE}}{\text{RATE}}$$

Let v = rate of kayaker in still water.

GOING UPSTREAM

$$\text{TIME} = \frac{\text{DISTANCE}}{\text{RATE}}$$

$$\text{TIME} = \frac{10}{v-2}$$

$v \neq 2$

GOING DOWNSTREAM

$$\text{TIME} = \frac{\text{DISTANCE}}{\text{RATE}}$$

$$\text{TIME} = \frac{26}{v+2}$$

$v \neq -2$

$$\text{TIME} = \text{TIME}$$

$$\frac{10}{v-2} = \frac{26}{v+2}$$

$$\frac{10}{v-2}\left(\frac{v+2}{v+2}\right) = \frac{26}{v+2}\left(\frac{v-2}{v-2}\right)$$

$$\frac{10v+20}{(v-2)(v+2)} = \frac{26v-52}{(v+2)(v-2)}$$

Look at numerators.

$$10v + 20 = 26v - 52$$

$-10v \quad -10v$

$$20 = 16v - 52$$

$+52 \quad +52$

$$72 = 16v$$

$v = 4.5$ km/h in still water.

KEY

Part 3 – Word Problems involving Sums

Q3: The sum of two consecutive numbers is 27. What are the numbers?

Let first number = n
Next number = $n+1$

$$(\text{First Number}) + (\text{Second Number}) = 27$$
$$(n) + (n+1) = 27$$
$$2n+1 = 27$$
$$2n = 26$$
$$n = 13 \quad (\text{So } n+1 = 14)$$

The numbers are 13 and 14.

Q4: If the sum of two numbers is 13 and the sum of their reciprocals is 0.325, what are the two numbers?

Let first number = m
Let second number = n

$$m + n = 13$$
$$m = 13 - n$$

$$\frac{1}{m} + \frac{1}{n} = 0.325$$
$$\frac{1}{13-n} + \frac{1}{n} = \frac{0.325}{1} \quad \rightarrow \quad n \neq 0, 13$$
$$\frac{1}{13-n}\left(\frac{n}{n}\right) + \frac{1}{n}\left(\frac{13-n}{13-n}\right) = \frac{0.325}{1}\left(\frac{n}{n}\right)\left(\frac{13-n}{13-n}\right)$$
$$\frac{n}{(n)(13-n)} + \frac{13-n}{(n)(13-n)} = \frac{4.225n - 0.325n^2}{(n)(13-n)}$$

Look at numerators.

$$(n) + (13-n) = 4.225n - 0.325n^2$$
$$13 = 4.225n - 0.325n^2$$
$$0.325n^2 - 4.225n + 13 = 0$$
$$\div 0.325 \quad \div 0.325 \quad \div 0.325 \quad \div 0.325$$
$$n^2 - 13n + 40 = 0$$
$$(n-5)(n-8) = 0$$
$$n = 5 \quad n = 8$$

If $n = 5$ then $m = 8$.
If $n = 8$ then $m = 5$.

Our numbers are 5 and 8.