First Name:_____   Last Name:_____

L'09 – Worksheet – Review for Exponents and Radicals SQ   /36 marks

Part 1 – Exponents   ____/22 marks

Q1: Which of the following is a rational number?

⇘ Can be written as a fraction.

    a. π

    b. $\sqrt{5}$

    (c.) $3.\overline{3}$
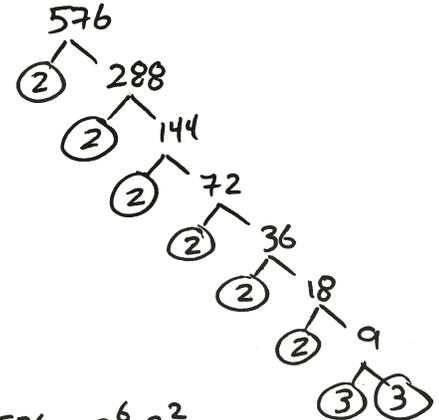
    d. $\sqrt[3]{25}$

Use the following information to answer Q2-Q3:

> The number **576** can be prime factored to determine if it is a perfect square, a perfect cube, both, or neither.

Q2: The number 576 can be prime factored to the form $a^b c^d$, where **a, b, c,** and **d** are ___, ___, ___, and ___.

*(Record your **four digit** answer in the Numerical Response boxes below)*

| 2 | 6 | 3 | 2 | or 3 2 2 6
|---|---|---|---|

576

②　288

②　144

②　72

②　36

②　18

②　9

③ ③

$576 = 2^6 \cdot 3^2$

Q3: The number 576 is

    (a.) A perfect square

    b. A perfect cube

    c. Both

    d. Neither

Q4: The expression $\dfrac{9x^6 y^{-4} z^3}{27x^3 z^{-2}}$ can be simplified to the form $\dfrac{x^a z^c}{3y^b}$, where **a, b,** and **c** are ___, ___, and ___.

*(Record your **three digit** answer in the Numerical Response boxes below)*

3	4	5	

$$\frac{9x^6 y^{-4} z^3}{27x^3 z^{-2}} = \frac{9x^6 z^3 \cdot z^2}{27x^3 y^4}$$

$$= \frac{x^3 z^5}{3y^4} \quad as \quad \frac{x^a z^c}{3y^b}$$

so $a = 3$

     $b = 4$

     $c = 5$

Use the following information to answer Q5-Q7:

$$\frac{[(x^5y^3z)(x^8y\ z^3)]^2}{(xy^2z^2)^4}$$

Q5: The numerator simplifies to $x^a y^b z^c{}^d$, where $a, b, c,$ and d are —, —, —, and —.

(Record your　　digit answer in the Numerical Response boxes below)

2	6	8	8

$$\left[x^{13}y^4z^4\right]^2 = x^{26}y^8z^8$$

$a = 2, b = 6$
$c = 8$
$d = 8$

Q6: The denominator simplifies to $x^a y^b z^c$, where *a, b,* and *c* are ___, ___, and ___.

*(Record your **three digit** answer in the Numerical Response boxes below)*

4	8	8	

$$x^4 y^8 z^8$$

$a = 4$
$b = 8$
$c = 8$

Q7: *(Long Answer)* Simplify the expression $\dfrac{[(x^5y^3z)(x^8y\ z^3)]^2}{(xy^2z^2)^4}$ using positive exponents only. *(2 marks)*

$$\frac{x^{26}y^8z^8}{x^4y^8z^8} = \boxed{x^{22}}$$

Q8: *(Long Answer)* Simplify the expression $(9)(3^3)(27)$ using a common base. *(3 marks)*

$$(3^2)(3^3)(3^3)$$

$$3^{2+3+3}$$

$$\boxed{3^8}$$

Use the following information to answer Q9-Q10:

> There are 2000 beetles wondering Wm. E. Hay's hallways at the beginning of the day. As students file into the school, they stomp the beetles, killing half of them every 30 minutes.

Q9: *(Long Answer)* Write an equation to represent this experiment. Let **N** be the number of beetles and let **t** be the time in minutes. *(1 mark)*

$$N = 2000(0.5)^{t/30}$$

Q10: *(Long Answer)* How many beetles are there after 80 minutes? *(1 mark)*

$$N = 2000(0.5)^{80/30}$$
$$= 2000(0.5)^{2.\overline{6}}$$
$$= 2000(0.15749)$$
$$= 314.98$$
$$N \approx 315 \text{ beetles.}$$

Use the following information to answer Q11:

Unsimplified Expressions		
Equation 1	$(x^2)^3$	$= x^6$
Equation 2	$x^2 x^3$	$= x^5$
Equation 3	$\dfrac{x^{12}}{x^4}$	$= x^8$
Equation 4	$\left(\dfrac{x^5}{x^7}\right)^3$	$= x^{-6}$

Q11: The expressions, ordered from the expression with the largest simplified exponent to the one with the smallest simplified exponent, are _3_, _1_, _2_, and _4_.

*(Record your **four digit** answer in the Numerical Response boxes below)*

3	1	2	4

Also accept 865-6, even though you can't put a negative into Numerical Response boxes.

Q12: *(Long Answer)* Simplify the expression $\dfrac{\left[x^5\left(y^{\frac{3}{2}}\right)^3\right]^{\frac{1}{4}}}{(xy^2)^3}$ using only positive exponents *(3 marks)*

$$\frac{\left[x^5\left(y^{3/2}\right)^3\right]^{1/4}}{(xy^2)^3} = \frac{\left[x^5 y^{9/2}\right]^{1/4}}{x^3 y^6} = \frac{x^{5/4}\, y^{9/8}}{x^3 y^6}$$

$$= x^{(5/4 - 3)}\, y^{(9/8 - 6)}$$

$$= x^{-7/4}\, y^{-39/8}$$

$$= \frac{1}{x^{7/4}\, y^{39/8}}$$

Use the following information to answer Q13-Q14:

Billy simplifies the expression $\dfrac{x^{\frac{2}{3}}(x^3)^{\frac{1}{4}}}{x^{\frac{1}{6}}}$ as shown below:

Step 1	$\dfrac{x^{\frac{2}{3}}x^{\frac{3}{4}}}{x^{\frac{1}{6}}}$
Step 2	$\dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{6}}}$
Step 3	$x^{\frac{1}{3}}$

Q13: In which step did Billy make an error when simplifying the expression?

    a. Step 1 only
    (b) Step 2 only
    c. Step 3 only
    d. Step 1 and Step 3

Note: I did Q14 first, then did Q13.

Q14: *(Long Answer)* Simplify the original expression. *(2 marks)*

$$\frac{x^{\frac{2}{3}} \cdot (x^3)^{\frac{1}{4}}}{x^{\frac{1}{6}}} = \frac{x^{\frac{2}{3}} \cdot x^{\frac{3}{4}}}{x^{\frac{1}{6}}} = \frac{x^{\frac{2}{3}+\frac{3}{4}}}{x^{\frac{1}{6}}} = \frac{x^{\frac{17}{12}}}{x^{\frac{1}{6}}}$$

$$= x^{(\frac{17}{12} - \frac{1}{6})} = \boxed{x^{\frac{5}{4}}}$$

Use the following information to answer Q15:

Carbon-14 is used in radioactive carbon dating, and has a half-life of 5730 years. The time it takes

Carbon-14 to radioactively decay is modelled using $y = a(0.5)^{\frac{x}{5730}}$, where *a* is the starting

amount of Carbon-14, *x* is the age of the item being dated, and *y* is the current amount of Carbon-

Q15: *(Long Answer)* If a sample is 8000 years old, what percentage of the original Carbon-14 would be remaining? *(2 marks)*

$$y = 100\% \, (0.5)^{8000 \div 5730}$$
$$y = 100 \, (0.5)^{1.39616}$$
$$= 100 \, (0.3799)$$
$$y = 37.99\% \text{ left}$$

Part 2 – Radicals ___/14 marks

Q16: The entire radical $\sqrt{98x^3y}$ can be written as a mixed radical in the form $ax^b\sqrt{cx^dy}$, where **a, b, c,** and **d** are ___, ___, ___, and ___.
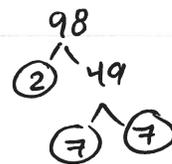
*(Record your **four digit** answer in the Numerical Response boxes below)*

7	1	2	1

$$\sqrt{2\cdot 7^2 \cdot x^2 \cdot x \cdot y}$$

$$7x\sqrt{2xy} \quad \text{or} \quad 7x^1\sqrt{2x^1y}$$

So $a=7$, $b=1$
$c=2$, $d=1$

98
② 49
⑦ ⑦

Q17: The entire radical $\sqrt[3]{32x^5y^2z^6}$ can be written as a mixed radical in the form $2xz^a\sqrt[3]{4x^by^c}$, where **a, b,** and **c** are ___, ___, and ___.

*(Record your **three digit** answer in the Numerical Response boxes below)*

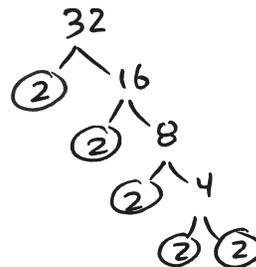| 2	2	2	

$$\sqrt[3]{2^3\cdot 2^2 \cdot x^3 \cdot x^2 \cdot y^2 \cdot z^3 \cdot z^3}$$

$$2x\,z\,z\,\sqrt[3]{2^2x^2y^2}$$

$$2xz^2\sqrt[3]{2^2x^2y^2}$$

$a=2$ $b=2$, $c=2$

32
② 16
② 8
② 4
② ②

Q18: The mixed radical $2x^3\sqrt{15x}$ can be written as the following entire radical:

a. $\sqrt{30x^6}$
b. $\sqrt{60x^6}$
c. $\sqrt{30x^7}$
d. $\sqrt{60x^7}$ ← (circled)

$$\sqrt{(2x^3)^2}\cdot\sqrt{15x}$$

$$\sqrt{4x^6}\cdot\sqrt{15x}$$

$$\sqrt{60x^7}$$

Q19: *(Long Answer)* Express $3^{\frac{2}{3}} x^3 \sqrt{3x^5}$ as a simplified power. *(3 marks)*

$$3^{2/3} \cdot x^3 \left(3x^5\right)^{1/2}$$

$$3^{2/3} \cdot x^3 \cdot 3^{1/2} \cdot x^{5/2}$$

$$3^{(2/3 + 1/2)} \cdot x^{(3 + 5/2)}$$

$$\boxed{3^{7/6} \cdot x^{11/2}}$$

Q20: The expression $5^{\frac{1}{2}} x^2 \sqrt[3]{5x^2}$ can be written as a simplified power in the form $5^{\frac{a}{b}} x^{\frac{c}{d}}$, where *a, b, c,* and *d* are ___, ___, ___, and ___.

*(Record your **four digit** answer in the Numerical Response boxes below)*

5	6	8	3

$$5^{1/2} \cdot x^2 \left(5x^2\right)^{1/3}$$

$$5^{1/2} \cdot x^2 \cdot 5^{1/3} \cdot x^{2/3}$$

$$5^{(1/2 + 1/3)} \cdot x^{(2 + 2/3)}$$

$$5^{5/6} \cdot x^{8/3}$$

so $a = 5, b = 6$
$c = 8, d = 3$

*Use the following information to answer Q21:*

A clock in a town square has a moving pendulum arm that starts on the far right, swings to the left, and returns to the right. The time it takes the arm to make this full pendulum swing can be approximately modelled using the equation:
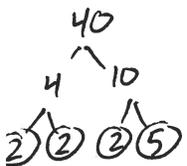
$$T = 6\sqrt{\dfrac{L}{10}}$$

Where **T** is the time for a full swing, in seconds, and **L** is the length of the clock arm, measured in meters.

Q21: *(Long Answer)* If the arm of the clock is 5 meters long, what is the time for a full swing, written as a mixed radical? *(3 marks)*

$$T = 6\sqrt{\frac{5}{10}} \qquad = \sqrt{(6)^2} \cdot \sqrt{\frac{5}{10}}$$

$$= \sqrt{36} \cdot \sqrt{\frac{5}{10}}$$

$$= \sqrt{\frac{180}{10}}$$

$$= \sqrt{18}$$

$$= \sqrt{2 \cdot 3^2}$$

$$\boxed{T = 3\sqrt{2}}$$

18
② 9
③ ③
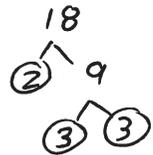
Q22: *(Long Answer)* Express the following as simplified mixed radicals *(4 marks)*

40
4 10
② ② ② ⑤

$$\sqrt[3]{40x^4y^5}$$

$$\sqrt[3]{2^3 \cdot 5 \cdot x^3 \cdot x \cdot y^3 \cdot y^2}$$

$$2xy\sqrt[3]{5xy^2}$$

$$\left(x^{\frac{1}{2}}\right)^{\frac{8}{3}}$$

$$x^{8/6} = x^{4/3}$$

$$= \left(x^4\right)^{1/3}$$

$$= \sqrt[3]{x^4}$$

$$= \sqrt[3]{x^3 \cdot x}$$

$$= x\sqrt[3]{x}$$