Math 20-1 KEY

L10 – 3.2 Standard Form

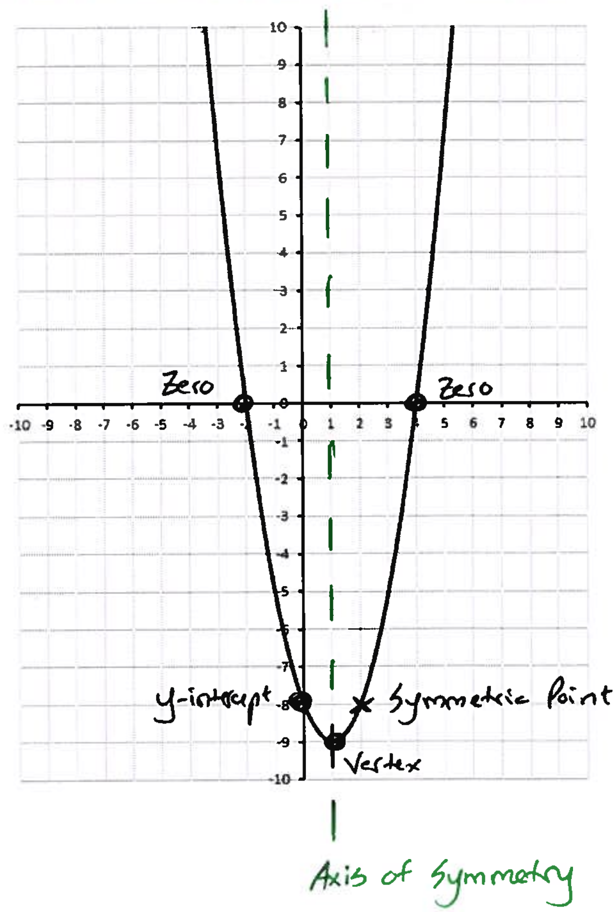
Key Ideas

Definitions:

- Quadratic Function: A function "f" whose value f(x) is given by a polynomial of degree two.
- Parabola: The symmetrical curve of the graph of a quadratic function.
- Vertex (of a Parabola): The lowest point of the graph when the graph opens upwards, or the highest point of the graph when the graph opens downwards.
- Minimum value of the function: The lowest value on the Range of a function. For a quadratic function that opens upwards, this would be the y-coordinate of the vertex.
- Maximum value of the function: The greatest value in the range of a function.
- Axis of Symmetry: A line through the vertex that divides the graph of a quadratic function into 2 congruent halves. The x-coordinate of the vertex defines the equation of the axis of symmetry.

Part 1 – Quadratic Functions Introduction

In the quadratic function $f(x) = x^2 - 2x - 8$, what are the important points that you would need if you were to sketch the graph?



Part 2 – Standard Form Transformations

In the equation $f(x) = ax^2 + bx + c$, what do adjusting ***a, b,*** and ***c*** actually do to the graph?

https://phet.colorado.edu/sims/html/graphing-quadratics/latest/graphing-quadratics_en.html

a – Changes shape.

b – Moves left/right. Doesn't change shape or y-intercept, so "rocking" motion

c – y-intercept

Part 3 – Identifying Zeroes, Vertex, y-Intercept, and Axis of Symmetry from a Graph

Zeroes: $x = -5, 1$

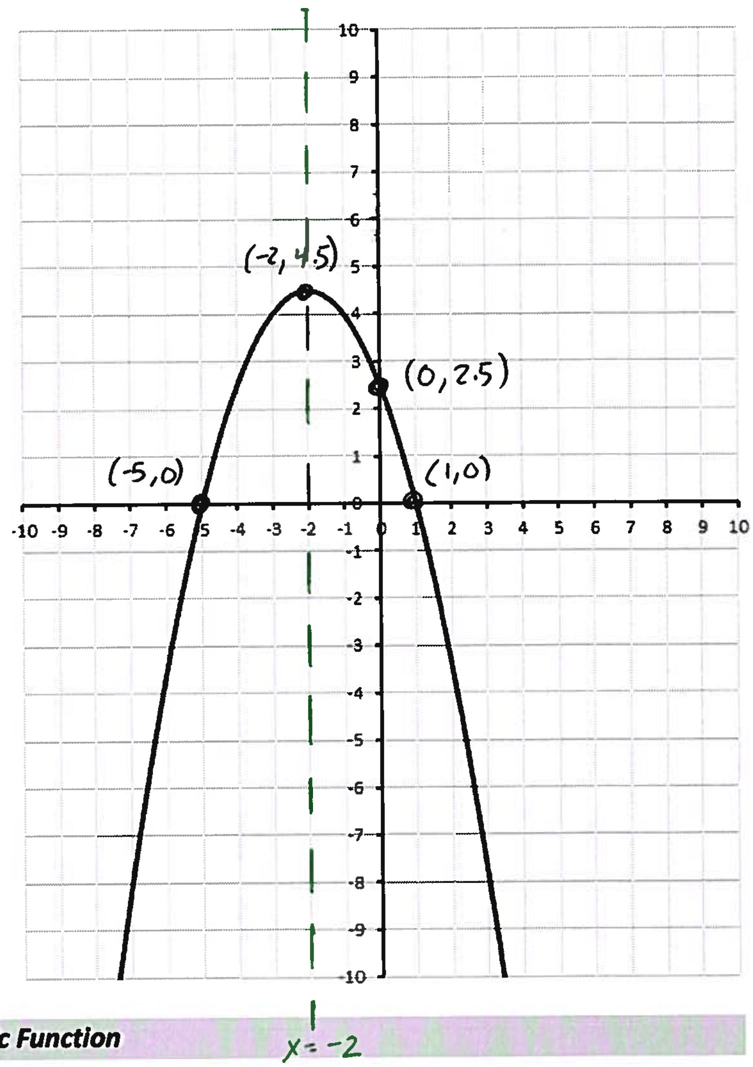
Vertex: $(-2, 4.5)$

y-Intercept: $(0, 2.5)$

Axis of Symmetry: $x = -2$

Maximum: 4.5

Occurs at vertex.



Part 4 – Domain and Range of a Quadratic Function

Domain:

$\{x \mid -\infty < x < \infty, x \in \mathbb{R}\}$

or

$\{x \in \mathbb{R}\}$ or $(-\infty, \infty)$

Range:

$\{y \mid -\infty < y \leq 4.5, y \in \mathbb{R}\}$

or

$\{y \mid y \leq 4.5, y \in \mathbb{R}\}$ or $(-\infty, 4.5]$

Math 20-1 KEY

Part 5 – Identifying Zeroes, Vertex, y-Intercept, and Axis of Symmetry from a Graph (your turn)

Zeroes: $x = -7, -1$

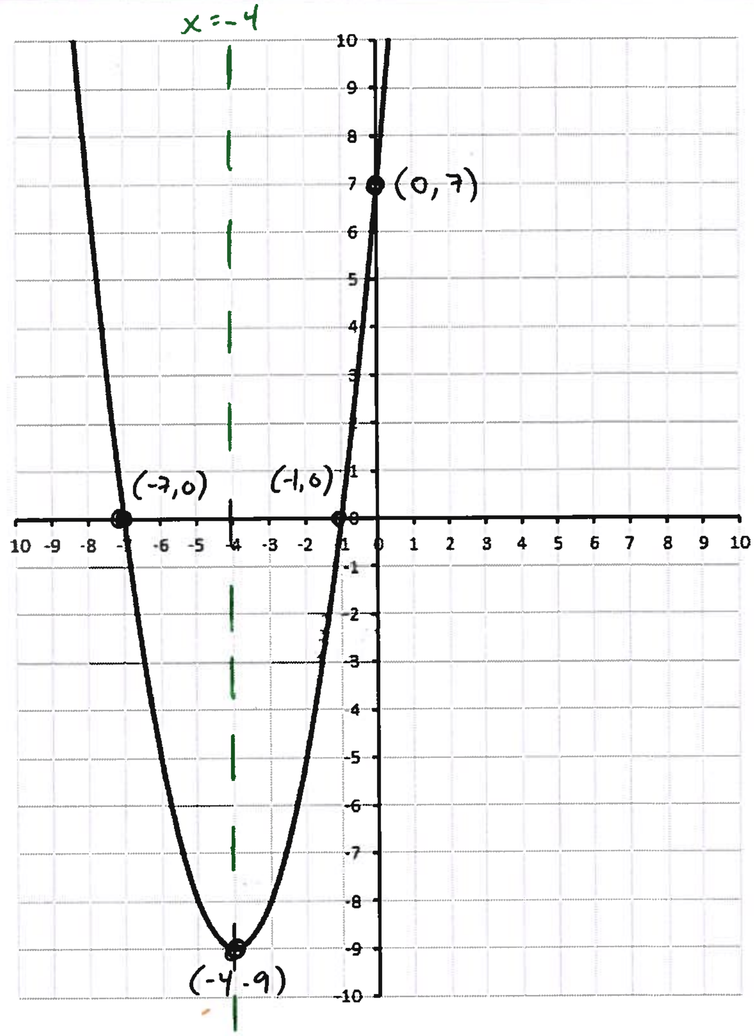
Vertex: $(-4, -9)$

y-Intercept: $(0, 7)$

Axis of Symmetry: $x = -4$

Minimum: -9

Occurs at vertex.



Part 6 – Domain and Range of a Quadratic Function (your turn)

Domain:

$\{x \mid -\infty < x < \infty, x \in \mathbb{R}\}$

or

$(-\infty, \infty)$

Range:

$\{y \mid -9 \leq y < \infty, y \in \mathbb{R}\}$

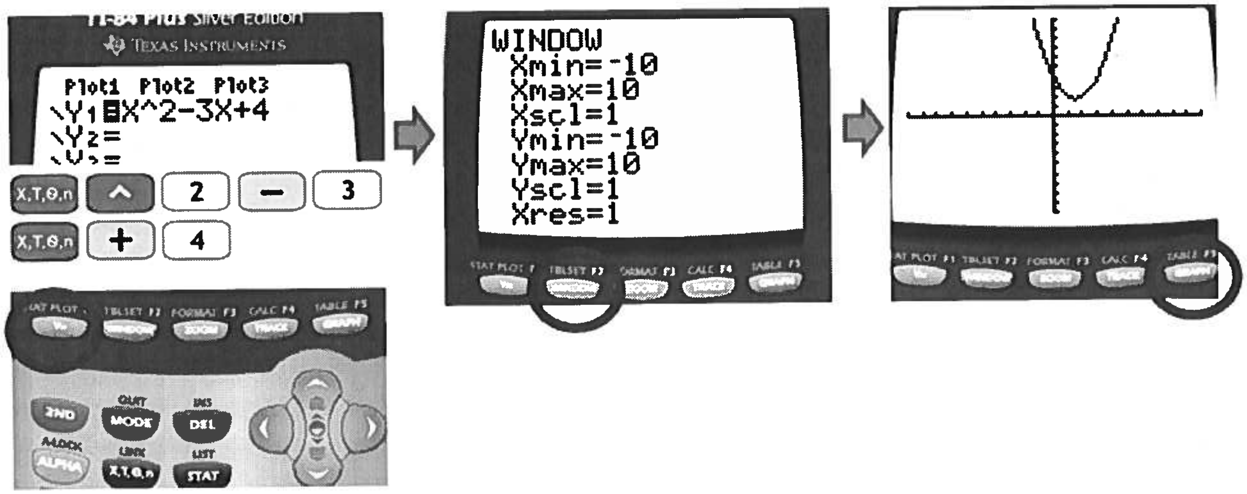
or

$[-9, \infty)$

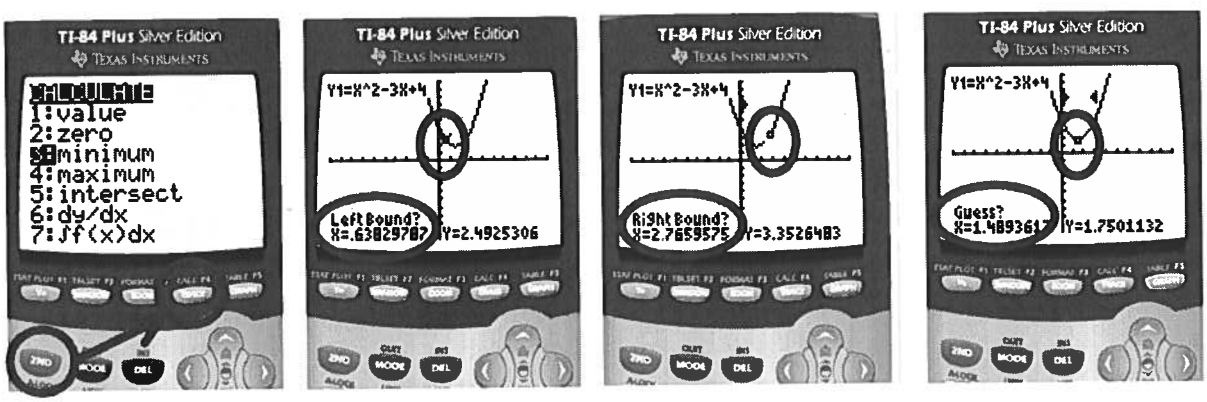
KEY

Part 7 – Using a T.I. Calculator to find Vertex and Zeroes

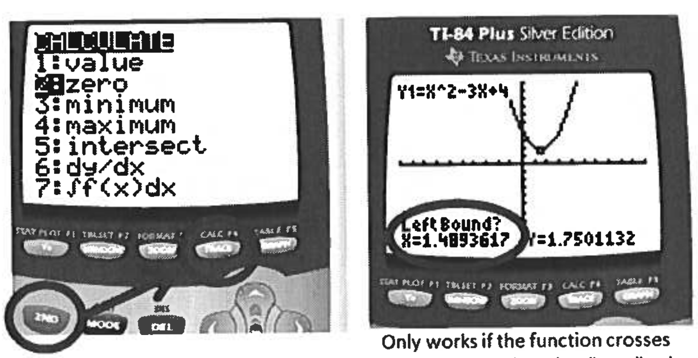
1. With your calculator, graph $f(x) = x^2 - 3x + 4$



2. Find the Max/Min/Vertex



3. Find the Zeroes



Only works if the function crosses the x-axis (i.e. there is a "zero" value of y)

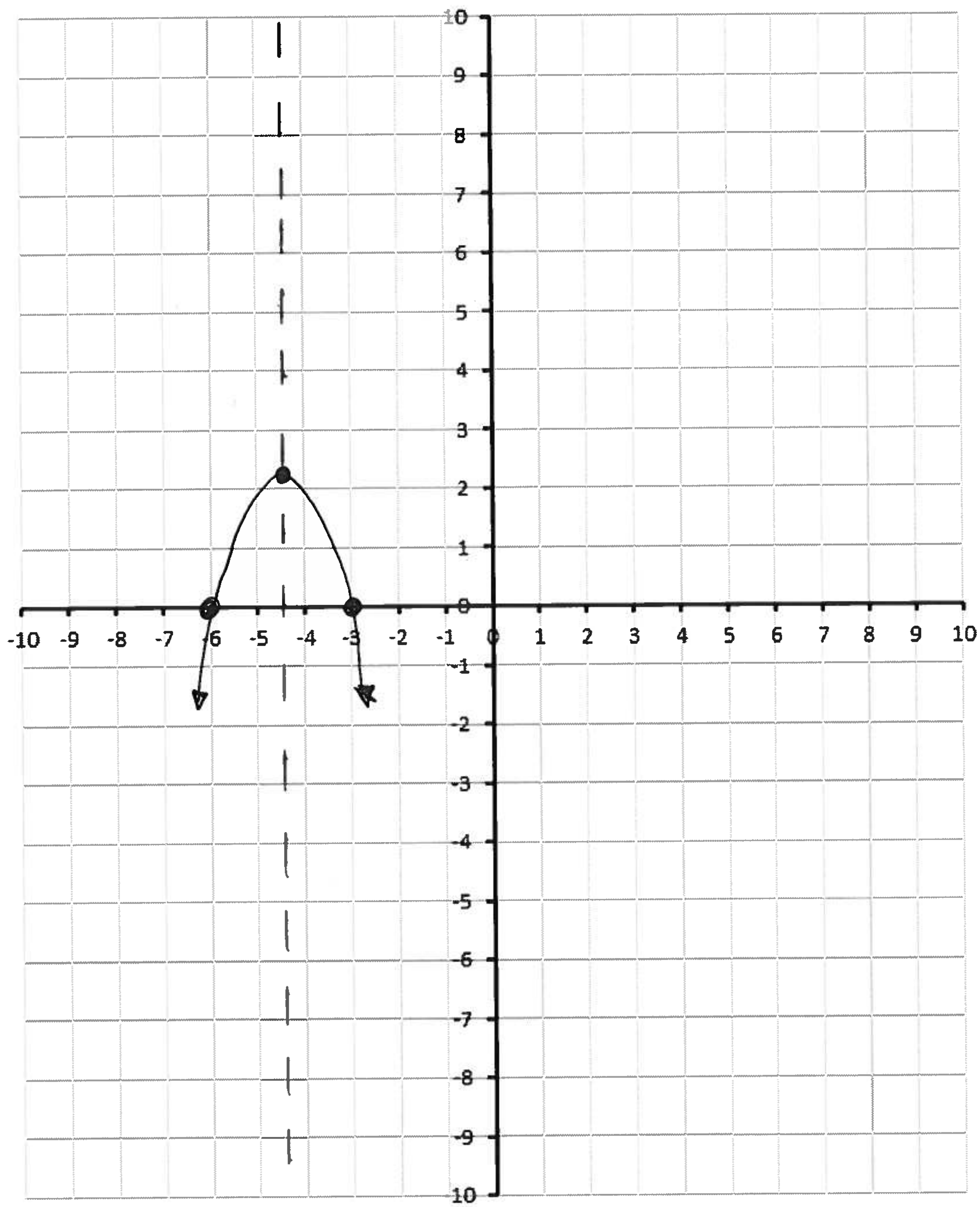
Key

Math 20-1

Using your T.I. Calculator, and given the function $f(x) = -x^2 - 9x - 18$, find

a. The Vertex $(-4.5, 2.25)$
b. The Maximum value of f(x) 2.25 (Occurs at Vertex)
c. The Zeroes $x = -6, -3$
d. The equation for the Axis of Symmetry $x = -4.5$

Sketch a quick graph below using these values.

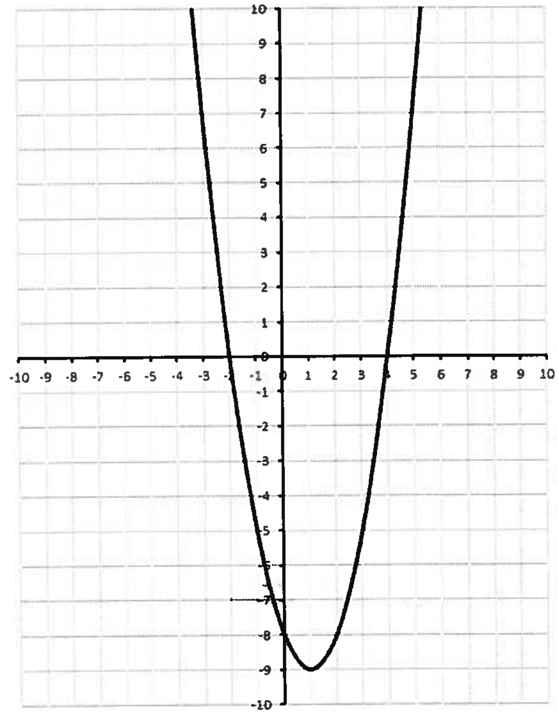


KEY

Math 20-1

Part 8 – Using Factoring to Find Zeroes

In the quadratic function $f(x) = x^2 - 2x - 8$, for what value of y are the "zeroes" found? Can we determine these values algebraically using factoring?



$y = x^2 - 2x - 8$

x-int when $y = 0$

$0 = x^2 - 2x - 8$

$+2 \quad -4$

$\square + \square = -2$

$\square \times \square = -8$

1, 8
2, 4

$0 = (x+2)(x-4)$

$x + 2 = 0 \qquad x - 4 = 0$

$x = -2 \qquad x = 4$

Zeroes are $x = -2, 4$

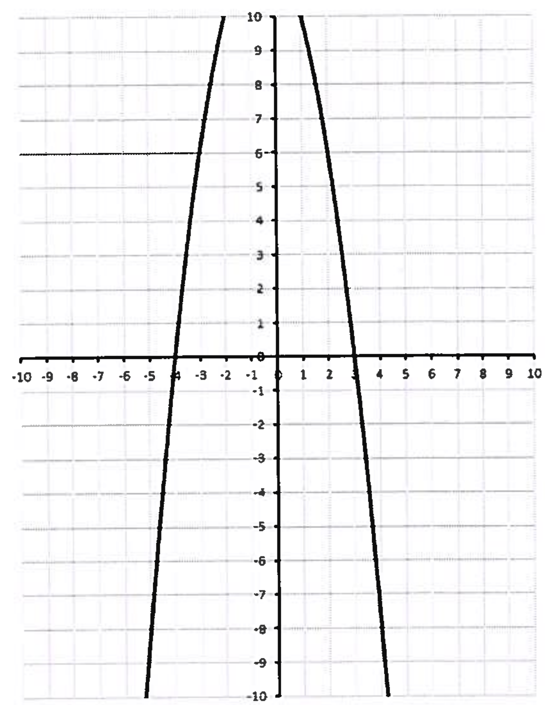
What is the equation for the axis of symmetry?

Avg distance between -2 and 4

$$\frac{(-2)+(4)}{2} = 1$$

$\boxed{x = 1}$

In the quadratic function $f(x) = -x^2 - x + 12$, for what value of y are the "zeroes" found? Can we determine these values algebraically using factoring?



$y = -x^2 - x + 12$

$0 = -x^2 - x + 12$

$0 = -1(x^2 + x - 12)$

$\div(-1) \qquad \div(-1)$

$-3 \quad +4$

$0 = x^2 + x - 12$

$\square + \square = 1$

$\square \times \square = -12$

$0 = (x-3)(x+4)$

1, 12
2, 6
3, 4

$x - 3 = 0 \qquad x + 4 = 0$

$x = 3 \qquad x = -4$

Zeroes are $x = -4, 3$

What is the equation for the axis of symmetry?

Avg distance between -4 and 3

$$\frac{(-4)+(3)}{2} = -0.5$$

$\boxed{x = -\frac{1}{2}}$

KEY

***Part 9 – Using Calculus to Find Vertex** (Optional – Will not be assessed)*

$y = ax^2 + bx + c$

$y = 2x^2 - 5x - 3$

Step #1: Take the derivative using Math 31 techniques.
The derivative of $ax^n + bx^{n-1} + \cdots + yx + z$
is $(n)ax^{n-1} + (n-1)bx^{n-2} + \cdots + y$

$y' = 4x - 5$

Step #2: This is our slope equation, where y' is our slope. Set the slope, y', equal to zero.

$0 = 4x - 5$

$x = \frac{5}{4}$

Step #3: We now know the x-value at which our tangent's slope is zero. This occurs at the vertex. So plug this value back into the original equation to find the y-coordinate of the vertex too.

$y = 2x^2 - 5x - 3$

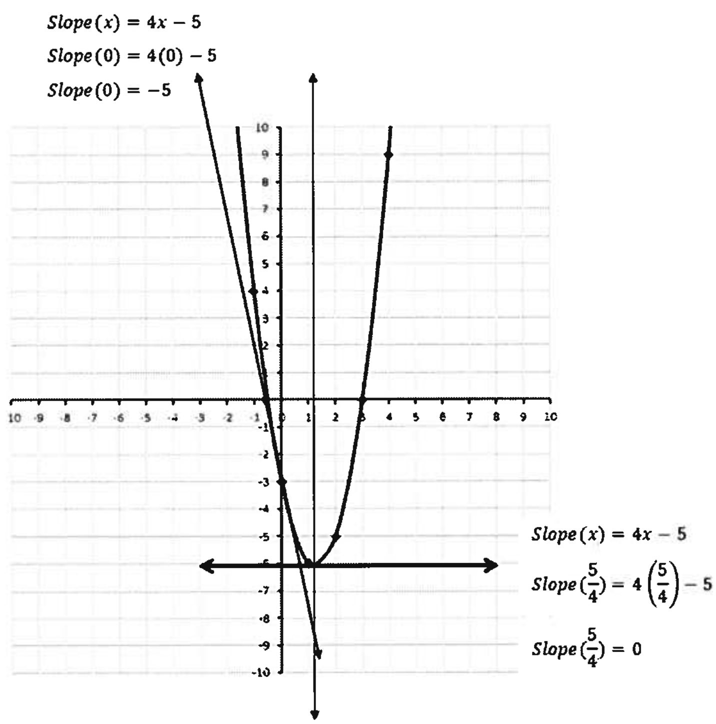
$y = 2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) - 3$

$y = 2\left(\frac{25}{16}\right) - 5\left(\frac{5}{4}\right) - 3$

$y = \frac{25}{8} - \frac{25}{4} - 3$

$y = -\frac{49}{8}$

So the vertex is along the axis of symmetry, located at $\left(\frac{5}{4}, -\frac{49}{8}\right)$, or approximately (1.25, -6.125).



KEY

Part 10 – Putting it all together

Given the equation $f(x) = x^2 - 4x - 5$,

a. Use factoring to determine the Zeroes.

x-int → set $y = 0$

$+1 \quad -5$

$\square + \square = -4$

$\square \times \square = -5$

$1, 5$

$0 = x^2 - 4x - 5$

$0 = (x + 1)(x - 5)$

$x + 1 = 0 \qquad x - 5 = 0$

$x = -1 \qquad x = 5$

Zeroes at $x = -1, 5$

b. Use the zeroes to determine the axis of symmetry.

Avg distance between

$\frac{(-1) + (5)}{2} = 2$ So $\boxed{x = 2}$

c. Use the method of your choice to determine the coordinates of the vertex.

i. Option #1: Use your T.I. Calculator

ii. Option #2: Use the x-coordinate (from the axis of symmetry) to find y.

iii. Option #3: Use calculus to find the x-value, then solve for the y-value.

Option #1: Calculator returns value of $(2, -9)$

Option #2: Vertex occurs on axis of symmetry.

$f(x) = x^2 - 4x - 5$

$f(2) = (2)^2 - 4(2) - 5$

$= 4 - 8 - 5$

$= -9$

So vertex is $(2, -9)$

Option #3: Calculus (for if we didn't yet know our zeroes or axis of symmetry).

THIS IS OPTIONAL AND WILL NOT BE ASSESSED!

$f(x) = x^2 - 4x^1 - 5$

$f'(x) = (2)x^1 - 4(1)x^0$

$\boxed{f'(x) = 2x - 4}$ Slope equation.

Slope is zero at vertex.

$0 = 2x - 4$

$+4 \qquad +4$

$4 = 2x$

$\div 2 \qquad \div 2$

$\boxed{2 = x}$ Vertex occurs at $x = 2$.

Plug back into original formula to get y-value.

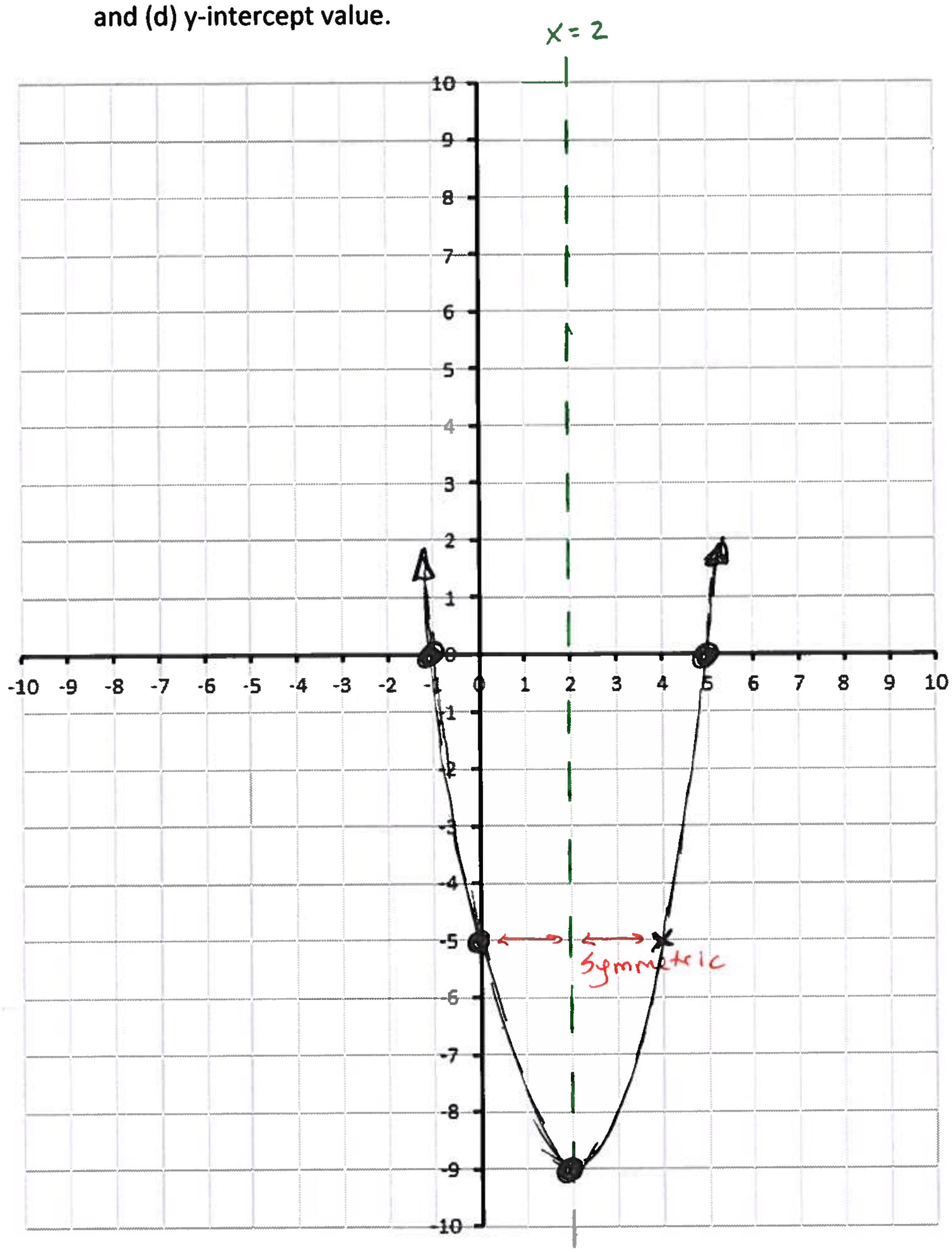
$f(2) = (2)^2 - 4(2) - 5$

$= -9$

So vertex is $(2, -9)$.

KEY

d. Sketch a graph of the function, with clearly labelled (i) vertex, (ii) zeroes, (iii) axis of symmetry, and (d) y-intercept value.



e. State the Domain and Range for the function.

Domain:

$\{x \mid -\infty < x < \infty, x \in \mathbb{R}\}$

or

$(-\infty, \infty)$

Range:

$\{y \mid -9 \le y < \infty, y \in \mathbb{R}\}$

or

$[-9, \infty)$