KEY

L11 – 3.1 Vertex Form

Key Ideas

Definitions:

- Vertex Form uses the Vertex (p,q) to quickly build the equation. Similar to "Slope-Point Form" in Math 10C
- Vertex Form $y = a(x - p)^2 + q$ is easier to sketch than Standard Form $y = ax^2 + bx + c$
- In Math 20-2, the vertex is labelled (h,k), making the equation $y = a(x - h)^2 + k$

Part 1 – Vertex Form Transformations

In the equation $f(x) = a(x - p)^2 + q$, what do adjusting ***a, p,*** and ***q*** actually do to the graph?

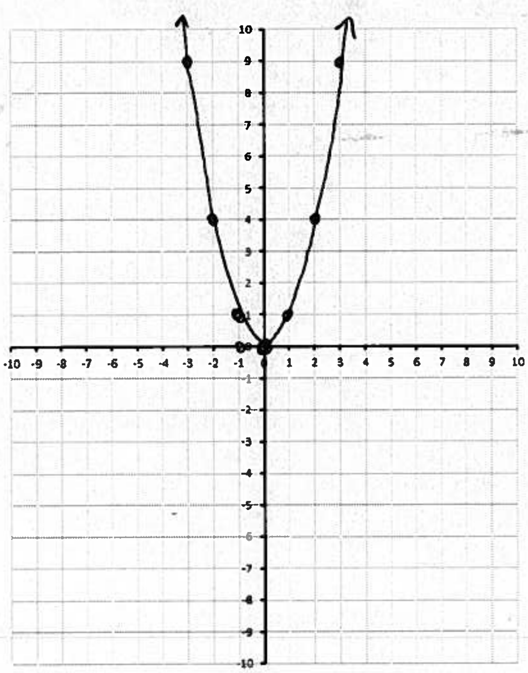
https://phet.colorado.edu/sims/html/graphing-quadratics/latest/graphing-quadratics_en.html

a – Vertical stretch → stretch larger or smaller.

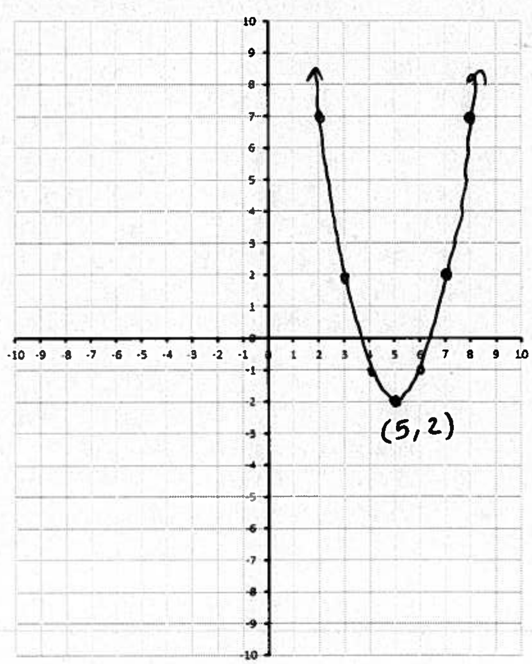
p – Horizontal translation left or right.

q – Vertical translation up or down.

$f(x) = 1(x - 0)^2 + 0$



$g(x) = 1(x - 5)^2 - 2$



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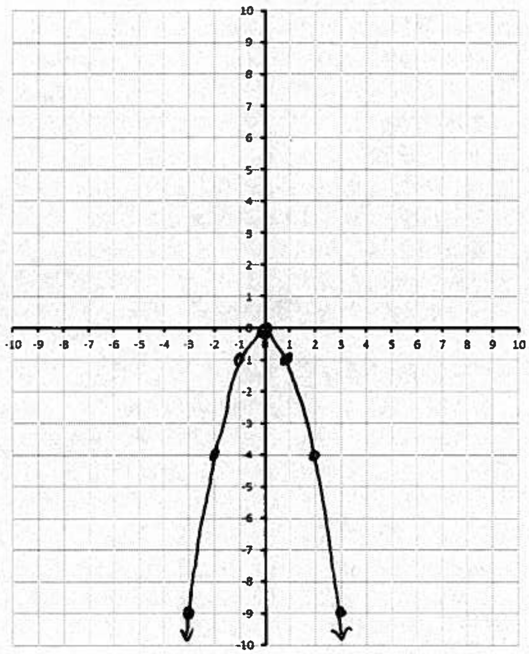
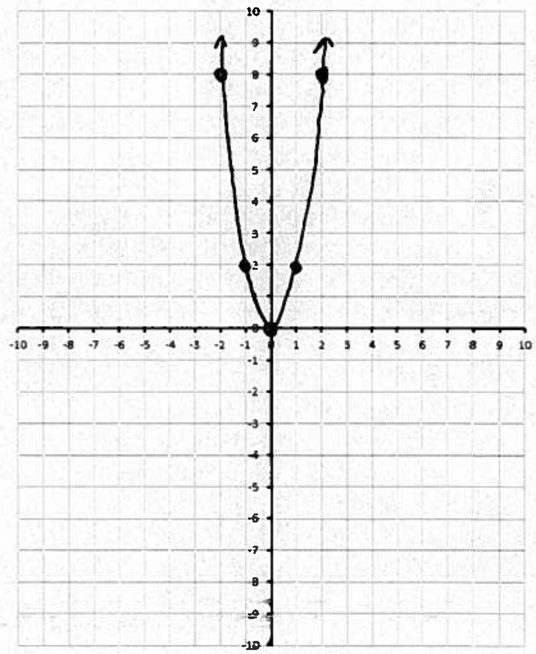
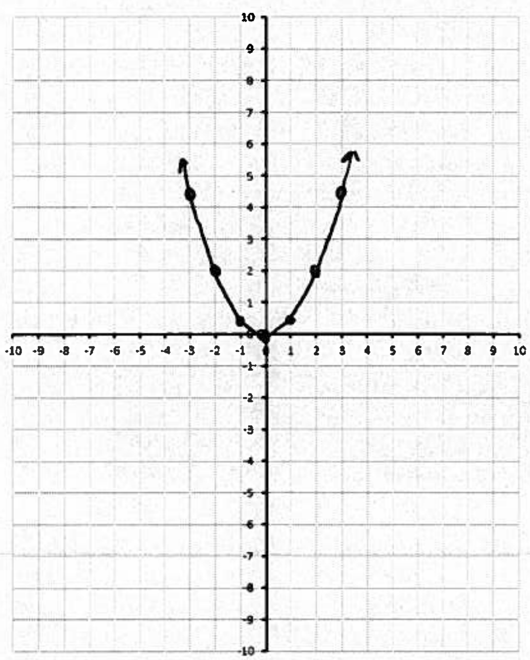
$h(x) = -1(x-0)^2 + 0$



$k(x) = 2(x-0)^2 + 0$

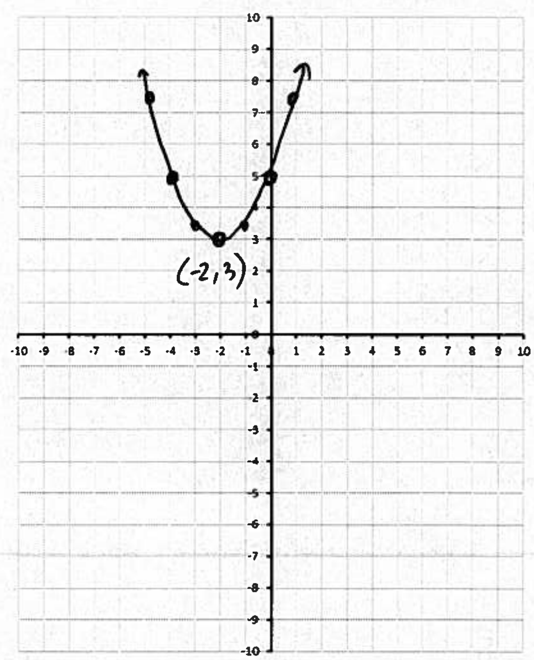


$m(x) = 0.5(x-0)^2 + 0$



$n(x) = 0.5(x - -2)^2 + 3$

$n(x) = 0.5(x+2)^2 + 3$



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Part 2 – Building an Equation using Vertex and a Point

Q1: A parabola has a vertex at (3,5) and passes through the point (1,9). Determine the equation of the line in Vertex Form.

$$y = a(x-p)^2 + q$$

$$y = a(x-3)^2 + 5 \qquad \text{Use } (1,9)$$

$$9 = a(\ \ -3)^2 + 5$$

$$9 = a(-2)^2 + 5$$

$$9 = a(4) + 5$$

$$-5 \qquad -5$$

$$4 = a(4)$$

$$\div 4 \qquad \div 4$$

$$1 = a$$

$$\boxed{y = 1(x-3)^2 + 5}$$

Q2: A parabola has a vertex at (-4,6) and passes throught the point (-2,2). Determine the equation of the line in Vertex Form.

$$y = a(x-p)^2 + q$$

$$y = a(x+4)^2 + 6 \qquad \text{Use } (-2,2)$$

$$2 = a(-2+4)^2 + 6$$

$$2 = a(2)^2 + 6$$

$$2 = a(4) + 6$$

$$-6 \qquad -6$$

$$-4 = a(4)$$

$$\div 4 \qquad \div 4$$

$$-1 = a$$

$$\boxed{y = -1(x+4)^2 + 6}$$

Q3: A parabola has a vertex at (0,6) and passes through the point (3,3). Determine the equation of the line in Vertex Form.

$$y = a(x-p)^2 + q$$

$$y = a(x-0)^2 + 6 \qquad \text{Use } (3,3)$$

$$3 = a(3-0)^2 + 6$$

$$3 = a(9) + 6$$

$$-6 \qquad -6$$

$$-3 = a(9)$$

$$\div 9 \qquad \div 9$$

$$-\frac{1}{3} = a$$

$$\boxed{y = -\frac{1}{3}(x-0)^2 + 6}$$

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Part 3 – Converting to Standard Form, $f(x) = ax^2 + bx + c$

Q4: Convert $y = 2(x - 3))^2 + 6$ into Standard Form.

$$y = 2(x-3)(x-3) + 6$$
$$y = 2(x^2 - 6x + 9) + 6$$
$$y = (2x^2 - 12x + 18) + 6$$
$$y = 2x^2 - 12x + 24$$

Q5: Convert $f(x) = \frac{1}{2}(x + 2)^2 - 8$ ~~into Standard Form~~.

$$f(x) = \tfrac{1}{2}(x+2)(x+2) - 8$$
$$= \tfrac{1}{2}(x^2 + 4x + 4) - 8$$
$$= (\tfrac{1}{2}x^2 + 2x + 2) - 8$$
$$= \tfrac{1}{2}x^2 + 2x - 6$$

Q6: Convert $g(x) = 3(x - 1)^2 + 12$ into Standard Form.

$$g(x) = 3(x-1)(x-1) + 12$$
$$= 3(x^2 - 2x + 1) + 12$$
$$= (3x^2 - 6x + 3) + 12$$
$$= 3x^2 - 6x + 15$$

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Part 4 – Finding the Vertex, Axis of Symmetry, and Zeroes using Vertex Form

Use the following information to answer Q7-Q10:

$$f(x) = 4\left(x+\frac{1}{4}\right)^2 - \frac{81}{4}$$

$f(x) = a(x - -1/4) + -81/4$

Q7: Determine the coordinate of the Vertex.

$\left(-\frac{1}{4}, -\frac{81}{4}\right)$

Q8: Determine the equation of the Axis of Symmetry.

$x = -\frac{1}{4}$

Q9: Convert to Standard Form to find the y-Intercept.

$f(x) = 4(x+1/4)(x+1/4) - \frac{81}{4}$

$= 4\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right) - \frac{81}{4}$

$= \left(4x^2 + 2x + \frac{1}{4}\right) - \frac{81}{4}$

$= 4x^2 + 2x - \frac{80}{4}$

$= 4x^2 + 2x - 20$

↓

y-intercept is -20

Q10: Determine the zeroes.

From Vertex Form	*From Standard Form*

From Vertex Form

$f(x) = 4(x+1/4)^2 - \frac{81}{4}$

x-int → set $y = 0$

$0 = 4(x+1/4)^2 - \frac{81}{4}$

$+\frac{81}{4}$ $\qquad +\frac{81}{4}$

$\frac{81}{4} = 4(x+1/4)^2$

$\div 4$ $\qquad \div 4$

$\frac{81}{16} = (x+1/4)^2$

$\sqrt{\frac{81}{16}} = (x+1/4)$

$+\frac{9}{4} = x + 1/4$

$-1/4 \qquad -1/4$

$\boxed{2 = x}$

$-\frac{9}{4} = x + 1/4$

$-1/4 \qquad -1/4$

$\boxed{-5/2 = x}$

From Standard Form

$f(x) = 4x^2 + 2x - 20$

x-int → set $y = 0$

$0 = 4x^2 + 2x - 20$

$\div 2 \quad \div 2 \quad \div 2 \quad \div 2$

$0 = 2x^2 + 1x - 10$

$-4 \quad +5$

$\square + \square = 1$

$\square \times \square = -20$

$0 = 2x^2 - 4x + 5x - 10$

$0 = (2x^2 - 4x) + (5x - 10)$

$0 = 2x(x-2) + 5(x-2)$

$0 = (x-2)(2x+5)$

$x - 2 = 0$

$\boxed{x = 2}$

$2x + 5 = 0$

$\boxed{x = -5/2}$

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Use the following information to answer Q11-Q15:

$g(x) = 1(x-1)^2 - 9$

$g(x) = a(x - +1)^2 + -9$

Q11: Determine the coordinate of the Vertex. $(1, -9)$

Q12: Determine the equation of the Axis of Symmetry.

$x = 1$

Q13: Convert to Standard Form to find the y-Intercept.

$$\begin{aligned} g(x) &= 1(x-1)(x-1) - 9 \\ &= 1(x^2 - 2x + 1) - 9 \\ &= (1x^2 - 2x + 1) - 9 \\ &= x^2 - 2x - 8 \end{aligned}$$

$\downarrow$

y-intercept is -8

Q14: Determine the zeroes.

From Vertex Form	*From Standard Form*
$g(x) = 1(x-1)^2 - 9$	$g(x) = x^2 - 2x - 8$
x-int → set $y = 0$	x-int → set $y = 0$
$0 = 1(x-1)^2 - 9$	$0 = x^2 - 2x - 8$
$+9$ $+9$	$+2$ -4; $\square + \square = -2$; $\square \times \square = -8$; 1,8; 2,4
$9 = (x-1)^2$	$0 = (x+2)(x-4)$
$\sqrt{9} = (x-1)$	$x+2 = 0$, $\boxed{x = -2}$; $x-4 = 0$, $\boxed{x = 4}$
$+3 = (x-1)$, $+1$ $+1$, $\boxed{4 = x}$; $-3 = (x-1)$, $+1$ $+1$, $\boxed{-2 = x}$	

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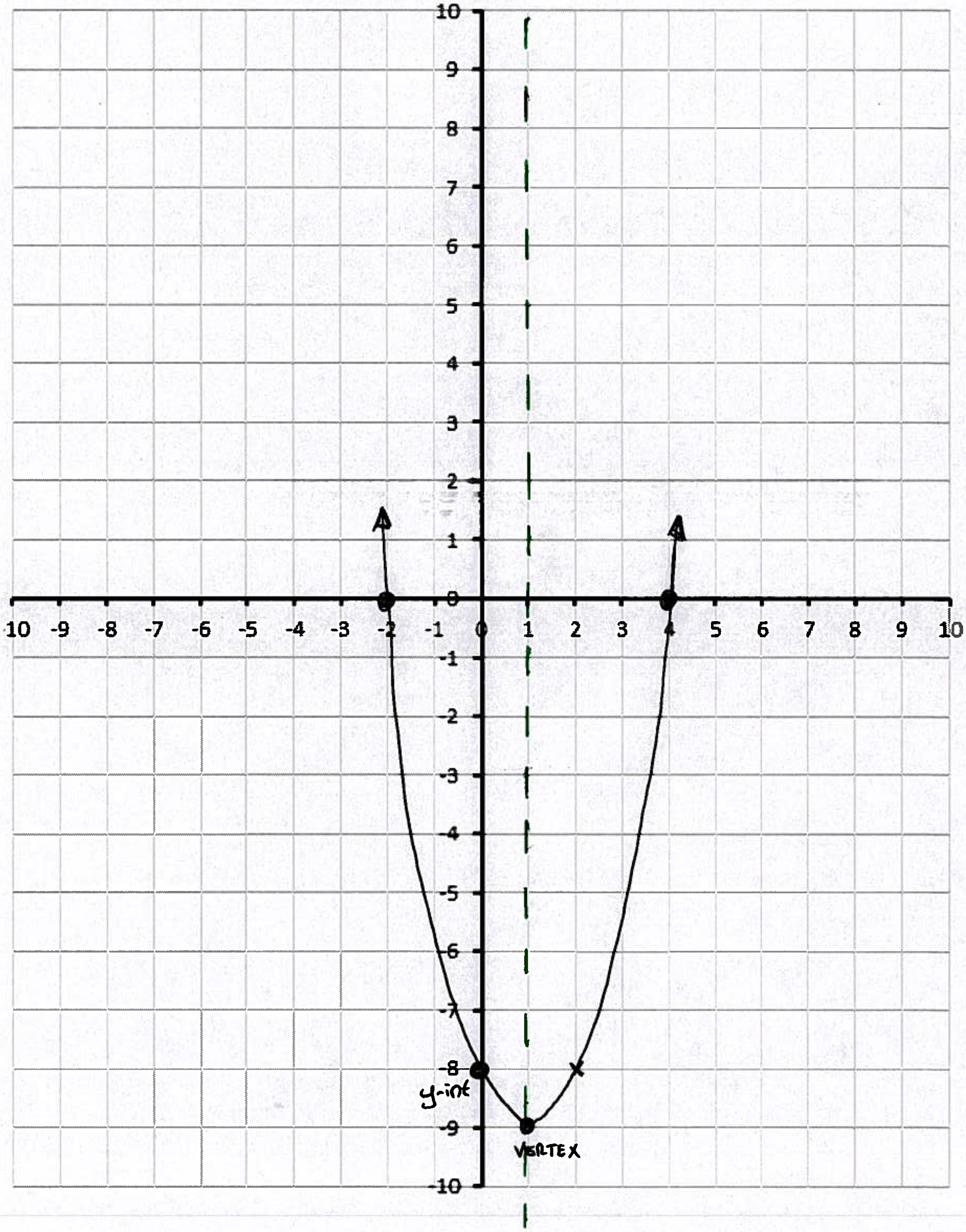
Q15: Sketch the function *g(x)* below.



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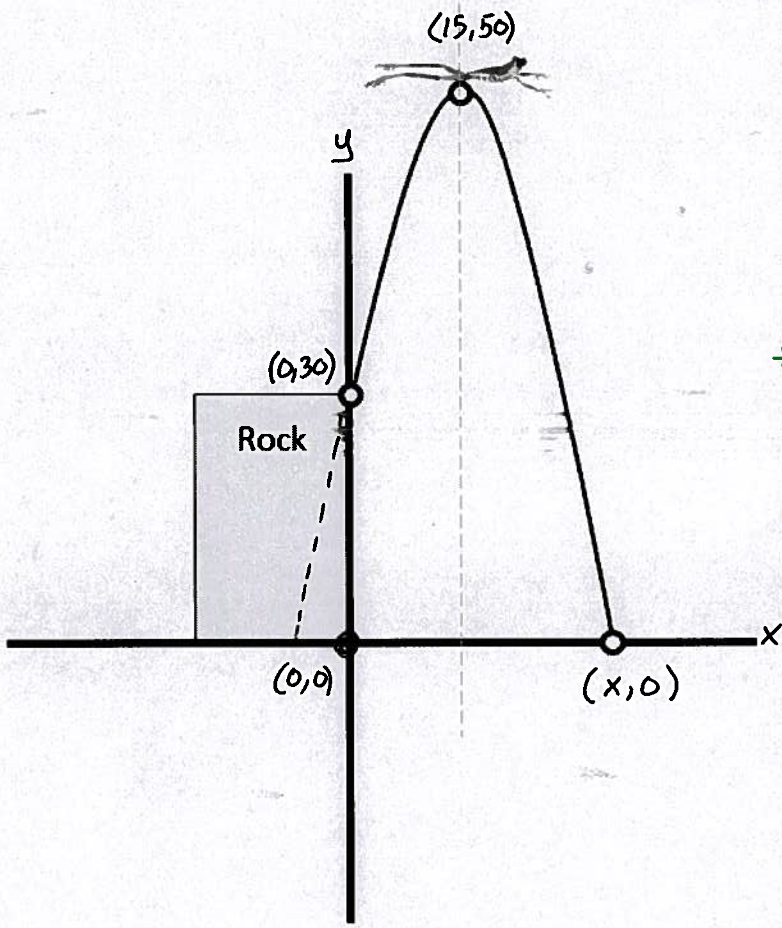
Part 5 – Projectile Motion using Vertex Form

Q16: A frog is standing on a 30cm tall rock. It jumps through the air, reaching a maximum height of 50cm when it is a horizontal distance of 15cm from the rock. How far from the base of the rock does the frog land?



$h(x) = a(x-p)^2 + q$

$h(x) = a(x-15)^2 + 50$

Use $(0,30)$

$30 = a(0-15)^2 + 50$

$30 = a(225) + 50$

$-50 \qquad -50$

$-20 = a(225)$

$\div 225 \qquad \div 225$

$\frac{-4}{45} = a$

$$\boxed{h(x) = -\frac{4}{45}(x-15)^2 + 50}$$

Hits bottom at $y = 0$

$0 = -\frac{4}{45}(x-15)^2 + 50$

$-50 \qquad -50$

$-50 = -\frac{4}{45}(x-15)^2$

$\cdot 45 \qquad \cdot 45$

$-2250 = -4(x-15)^2$

$\div(-4) \qquad \div(-4)$

$562.5 = (x-15)^2$

$\sqrt{562.5} = (x-15)$

$+23.72 = x - 15$

$+15 \qquad +15$

$\boxed{+38.72 = x}$

$-23.72 = x - 15$

$+15 \qquad +15$

$\boxed{-8.72 = x}$

Two zeroes for the graph, but which makes sense for the word problem?

$\boxed{x = 38.72 \text{ cm}}$

Q17: State the Domain and Range of the function.

Domain: $\{x \mid 0 \leq x \leq 38.72, x \in R\}$

Range: $\{y \mid 0 \leq y \leq 50, y \in R\}$