

1.1x - Worksheet - 3.1 Investigating Quadratic Functions in Vertex Form

Pg 157 #2ab: Describe how the graphs of the functions in each pair are related. Then, sketch the graph of the second function in each pair, and determine the vertex, the equation of the axis of symmetry, the domain and range, and any intercepts.

Equations	Graph of Second Function	Vertex	Equation of the Axis of Symmetry	Domain and Range	Intercepts																		
$y = x^2$ $y = x^2 + 1$ <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><th>x</th><th>y</th></tr> <tr><td>-3</td><td>10</td></tr> <tr><td>-2</td><td>5</td></tr> <tr><td>-1</td><td>2</td></tr> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>2</td></tr> <tr><td>2</td><td>5</td></tr> <tr><td>3</td><td>10</td></tr> </table> $y = (x-0)^2 + 1$ Shifted up 1 unit.	x	y	-3	10	-2	5	-1	2	0	1	1	2	2	5	3	10		(0, 1)	$x = 0$	Domain: $\{x \mid -\infty < x < \infty, x \in \mathbb{R}\}$ or $\{x \in \mathbb{R}\}$ Range: $\{y \mid 1 \leq y < \infty, y \in \mathbb{R}\}$ or $\{y \mid y \geq 1, y \in \mathbb{R}\}$	None.		
x	y																						
-3	10																						
-2	5																						
-1	2																						
0	1																						
1	2																						
2	5																						
3	10																						
$y = x^2$ $y = (x-2)^2$ <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><th>x</th><th>y</th></tr> <tr><td>-3</td><td>25</td></tr> <tr><td>-2</td><td>16</td></tr> <tr><td>-1</td><td>9</td></tr> <tr><td>0</td><td>4</td></tr> <tr><td>1</td><td>1</td></tr> <tr><td>2</td><td>0</td></tr> <tr><td>3</td><td>1</td></tr> <tr><td>4</td><td>4</td></tr> </table> $y = (x-2)^2 + 0$ Shifted right 2 units.	x	y	-3	25	-2	16	-1	9	0	4	1	1	2	0	3	1	4	4		(2, 0)	$x = 2$	Domain: $\{x \mid -\infty < x < \infty, x \in \mathbb{R}\}$ or $\{x \in \mathbb{R}\}$ Range: $\{y \mid 0 \leq y < \infty, y \in \mathbb{R}\}$ or $\{y \mid y \geq 0, y \in \mathbb{R}\}$	$x = 2$ $\frac{1}{2}$ (2, 0)
x	y																						
-3	25																						
-2	16																						
-1	9																						
0	4																						
1	1																						
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4	4																						

Pg 157 #2cd: Describe how the graphs of the functions in each pair are related. Then, sketch the graph of the second function in each pair, and determine the vertex, the equation of the axis of symmetry, the domain and range, and any intercepts.

Equations	Graph of Second Function	Vertex	Equation of the Axis of Symmetry	Domain and Range	Intercepts																
$y = x^2$ $y = x^2 - 4$ <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><th>x</th><th>y</th></tr> <tr><td>-3</td><td>5</td></tr> <tr><td>-2</td><td>0</td></tr> <tr><td>-1</td><td>-3</td></tr> <tr><td>0</td><td>-4</td></tr> <tr><td>1</td><td>-3</td></tr> <tr><td>2</td><td>0</td></tr> <tr><td>3</td><td>5</td></tr> </table> $y = (x-0)^2 - 4$ Shifted down 4 units.	x	y	-3	5	-2	0	-1	-3	0	-4	1	-3	2	0	3	5		$(0, -4)$	$x = 0$	Domain: $\{x \mid -\infty < x < \infty, x \in \mathbb{R}\}$ or $\{x \in \mathbb{R}\}$ Range: $\{y \mid -4 \leq y < \infty, y \in \mathbb{R}\}$ or $\{y \mid y \geq -4, y \in \mathbb{R}\}$	$x = -2, +2$ or $(-2, 0)$ $(2, 0)$
x	y																				
-3	5																				
-2	0																				
-1	-3																				
0	-4																				
1	-3																				
2	0																				
3	5																				
$y = x^2$ $y = (x+3)^2$ <table border="1" style="display: inline-table; vertical-align: middle;"> <tr><th>x</th><th>y</th></tr> <tr><td>-3</td><td>0</td></tr> <tr><td>-2</td><td>1</td></tr> <tr><td>-1</td><td>4</td></tr> <tr><td>0</td><td>9</td></tr> <tr><td>1</td><td>16</td></tr> <tr><td>2</td><td>25</td></tr> <tr><td>3</td><td>36</td></tr> </table> $y = (x+3)^2 + 0$ or $y = (x-(-3))^2 + 0$ Shifted left 3 units.	x	y	-3	0	-2	1	-1	4	0	9	1	16	2	25	3	36		$(-3, 0)$	$x = -3$	Domain: $\{x \mid -\infty < x < \infty, x \in \mathbb{R}\}$ or $\{x \in \mathbb{R}\}$ Range: $\{y \mid 0 \leq y < \infty, y \in \mathbb{R}\}$ or $\{y \mid y \geq 0, y \in \mathbb{R}\}$	$x = -3$ or $(-3, 0)$
x	y																				
-3	0																				
-2	1																				
-1	4																				
0	9																				
1	16																				
2	25																				
3	36																				

Pg 157 #4ab: Sketch the graph of each function. Identify the vertex, the axis of symmetry, the direction of opening, the maximum or minimum value, the domain and range, and any intercepts.

a) $y = -(x - 3)^2 + 9$

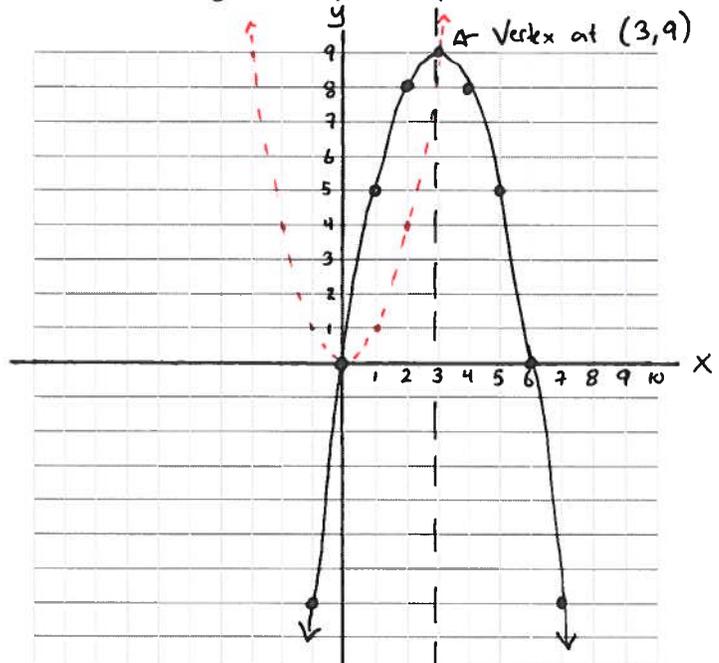
Opens down shifted 3 units right. shifted 9 units up.

x	y
-1	-7
0	0
1	5
2	8
3	9
4	8
5	5
6	0
7	-7

Max value at vertex.
Max value of $y = 9$.
No min value.

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \mid y \leq 9, y \in \mathbb{R}\}$

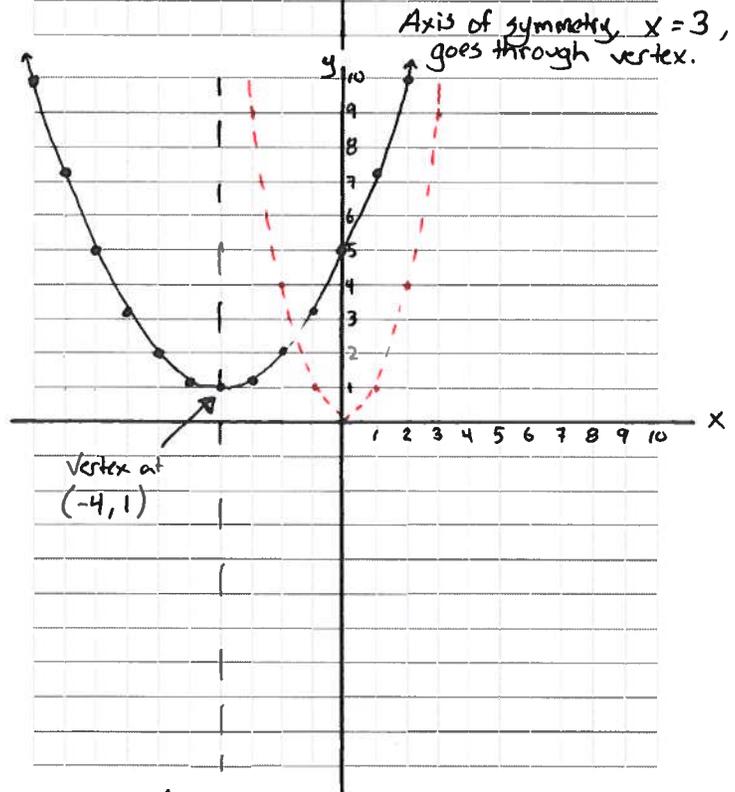


b) $y = 0.25(x + 4)^2 + 1$

Positive, so opens up. less than 1, so "stretched" by $\frac{1}{4}$ (compressed). $(x + 4)^2$ so shifted left 4 units. shifted up 1 unit.

x	y
-10	10
-9	7.25
-8	5
-7	3.25
-6	2
-5	1.25
-4	1
-3	1.25
-2	2
-1	3.25
0	5
1	7.25
2	10

Min value at vertex.
Min value of $y = 1$.
No max value.



Axis of symmetry, $x = -4$, goes through vertex.

Pg 157 #4cd: Sketch the graph of each function. Identify the vertex, the axis of symmetry, the direction of opening, the maximum or minimum value, the domain and range, and any intercepts.

a) $y = -3(x - 1)^2 + 12$

Negative means flipped upside down.
 3 means stretched by a factor of 3.
 Shifted by 1 unit right.
 Shifted by 12 units up.

x	y
-3	-36
-2	-15
-1	0
0	9
1	12
2	9
3	0
4	-15
5	-36

Max value at vertex.
 Max value at $y = 12$.
 No min value.

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \mid y \leq 12, y \in \mathbb{R}\}$

b) $y = \frac{1}{2}(x - 2)^2 - 2$

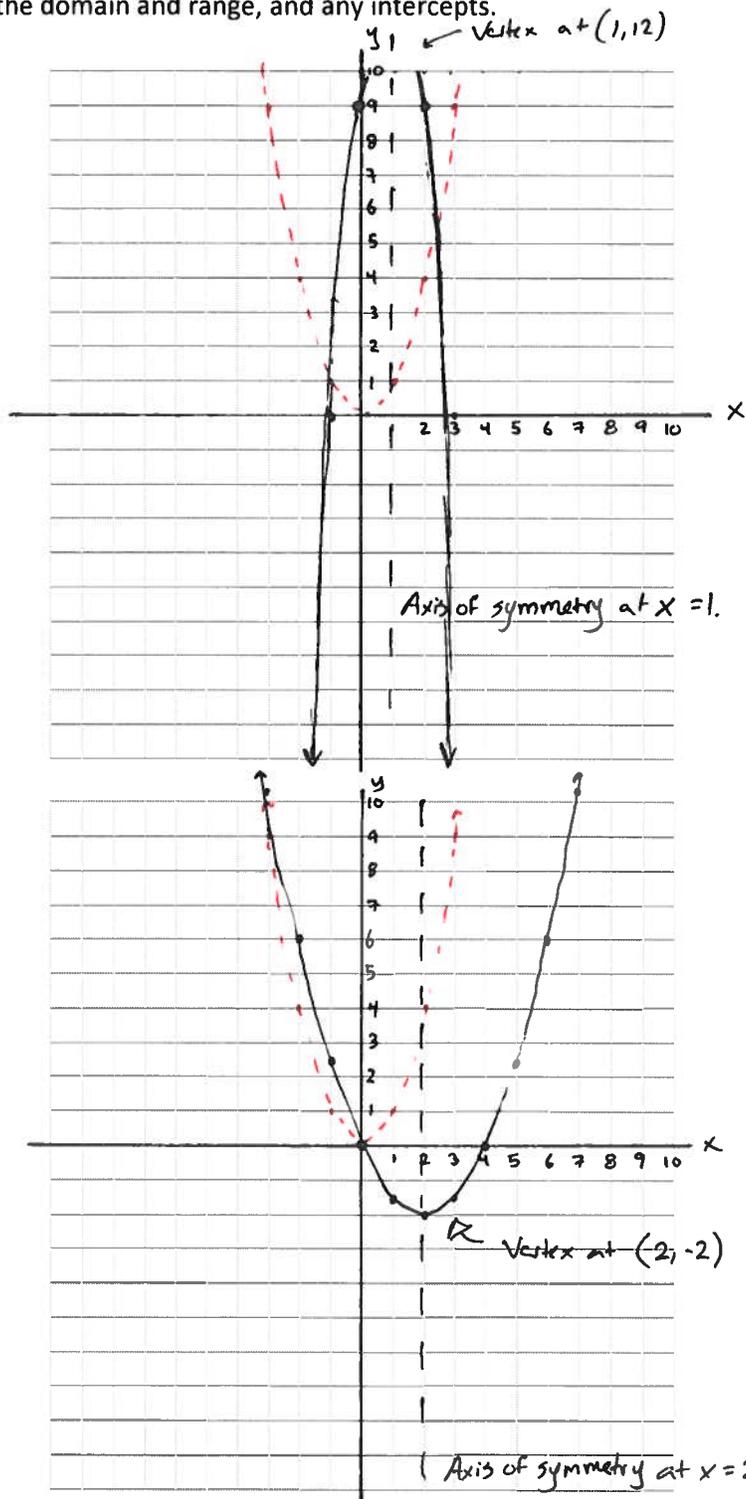
Positive means open up.
 $\frac{1}{2}$ means "stretched" by a factor of $\frac{1}{2}$. (Compressed)
 Shifted by 2 units right.
 Shifted down by 2 units.

x	y
-3	10.5
-2	6
-1	2.5
0	0
1	-1.5
2	-2
3	-1.5
4	0
5	2.5
6	6
7	10.5

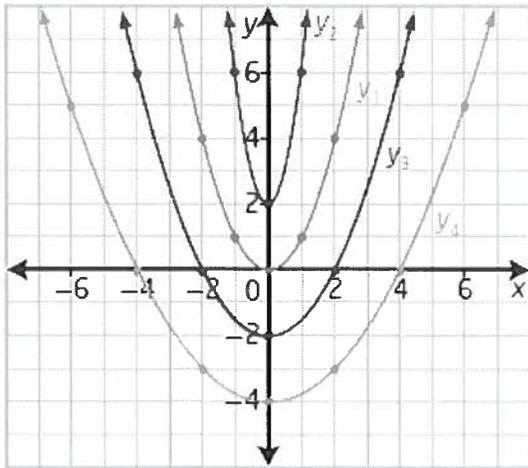
Min value at vertex.
 Min value at $y = -2$.
 No max value.

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \mid y \geq -2, y \in \mathbb{R}\}$



Pg 157 #5: Given each parabola in the graph below:



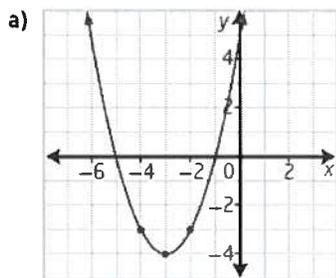
Vertex form: $y = a(x-p)^2 + q$

Vertex is at (p, q) .

Use another point to find a .

	a) Write a quadratic function in vertex form for each parabola in the graph.	b) Suppose four new parabolas <u>open downward</u> instead of upward but have the same shape and vertex as each parabola in the graph. Write a quadratic function in vertex form for each new parabola.	c) Write the quadratic functions in vertex form of four parabolas that are identical to the four in the graph but <u>translated 4 units to the left</u> .	d) Suppose the four parabolas in the graph are <u>translated 2 units down</u> . Write a quadratic function in vertex form for each new parabola.
y_1	$y = a(x-0)^2 + 0$ Use $(x, y) = (1, 1)$ $1 = a(1-0)^2 + 0$ $a = 1$ $y = 1(x-0)^2 + 0$ $y = x^2$	$y = -x^2$	$y = (x-4)^2$ $y = (x+4)^2$	$y = x^2 - 2$ $y = x^2 - 2$
y_2	$y = a(x-0)^2 + 2$ Use $(x, y) = (1, 6)$ $6 = a(1-0)^2 + 2$ $a = 4$ $y = 4(x-0)^2 + 2$ $y = 4x^2 + 2$	$y = -4x^2 + 2$	$y = 4(x-4)^2 + 2$ $y = 4(x+4)^2 + 2$	$y = 4x^2 + 2 - 2$ $y = 4x^2$
y_3	$y = a(x-0)^2 - 2$ Use $(x, y) = (2, 0)$ $0 = a(2-0)^2 - 2$ $a = \frac{1}{2}$ $y = \frac{1}{2}(x-0)^2 - 2$ $y = \frac{1}{2}x^2 - 2$	$y = -\frac{1}{2}x^2 - 2$	$y = \frac{1}{2}(x-4)^2 - 2$ $y = \frac{1}{2}(x+4)^2 - 2$	$y = \frac{1}{2}x^2 - 2 - 2$ $y = \frac{1}{2}x^2 - 4$
y_4	$y = a(x-0)^2 - 4$ Use $(x, y) = (2, -3)$ $-3 = a(2-0)^2 - 4$ $a = \frac{1}{4}$ $y = \frac{1}{4}(x-0)^2 - 4$ $y = \frac{1}{4}x^2 - 4$	$y = -\frac{1}{4}x^2 - 4$	$y = \frac{1}{4}(x-4)^2 - 4$ $y = \frac{1}{4}(x+4)^2 - 4$	$y = \frac{1}{4}x^2 - 4 - 2$ $y = \frac{1}{4}x^2 - 6$

Pg 157 #8: Determine the quadratic function in vertex form for each parabola.

Vertex at $(-3, -4)$

$$y = a(x-p)^2 + q$$

$$y = a(x - (-3))^2 + (-4)$$

$$y = a(x+3)^2 - 4$$

Use point $(-2, -3)$

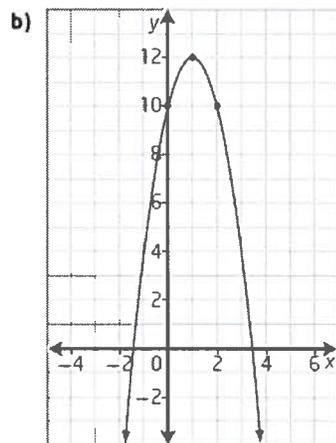
$$-3 = a(-2+3)^2 - 4$$

$$1 = a(1)^2$$

$$a = 1$$

$$\text{so } y = 1(x+3)^2 - 4$$

$$\boxed{y = (x+3)^2 - 4}$$

Vertex at $(1, 12)$

$$y = a(x-p)^2 + q$$

$$y = a(x-1)^2 + 12$$

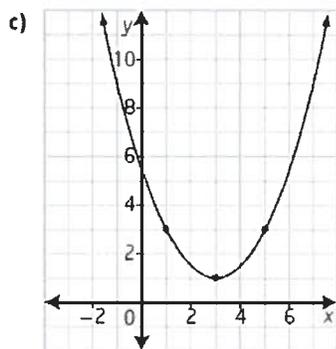
Use point $(2, 10)$

$$10 = a(2-1)^2 + 12$$

$$-2 = a(1)^2$$

$$a = -2$$

$$\text{so } \boxed{y = -2(x-1)^2 + 12}$$

Vertex at $(3, 1)$

$$y = a(x-3)^2 + 1$$

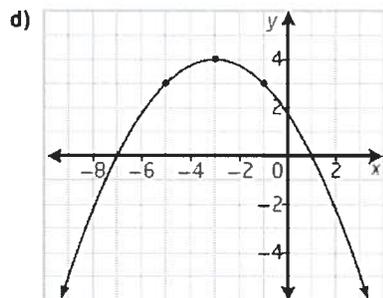
Use point $(5, 3)$

$$3 = a(5-3)^2 + 1$$

$$2 = a(2)^2$$

$$a = \frac{1}{2}$$

$$\text{so } \boxed{y = \frac{1}{2}(x-3)^2 + 1}$$

Vertex at $(-3, 4)$

$$y = a(x - (-3))^2 + 4$$

$$y = a(x+3)^2 + 4$$

Use point $(-1, 3)$

$$3 = a(-1+3)^2 + 4$$

$$-1 = a(2)^2$$

$$a = -\frac{1}{4}$$

$$\text{so } \boxed{y = -\frac{1}{4}(x+3)^2 + 4}$$

Pg 157 #9abc: Determine a quadratic function in vertex form that has the given characteristics.

Vertex at $(0,0)$, passing through the point $(6,-9)$

$$y = a(x-p)^2 + q$$

$$y = a(x-0)^2 + 0$$

$$y = ax^2$$

Use point $(6,-9)$

$$-9 = a(6)^2$$

$$\frac{-9}{36} = a$$

$$a = -\frac{1}{4}$$

$$y = -\frac{1}{4}x^2$$

Vertex at $(0,-6)$, passing through the point $(3,21)$

$$y = a(x-p)^2 + q$$

$$y = a(x-0)^2 + -6$$

$$y = ax^2 - 6$$

Use point $(3,21)$

$$21 = a(3)^2 - 6$$

$$27 = a(9)$$

$$a = 3$$

$$y = 3x^2 - 6$$

Vertex at $(2,5)$, passing through the point $(4,-11)$

$$y = a(x-p)^2 + q$$

$$y = a(x-2)^2 + 5$$

Use point $(4,-11)$

$$-11 = a(4-2)^2 + 5$$

$$-16 = a(2)^2$$

$$-4 = a$$

$$y = -4(x-2)^2 + 5$$

Pg 157 #10: The point $(4,16)$ is on the graph of $f(x) = x^2$. Describe what happens to the point when each of the following sets of transformations is performed in the order listed. Identify the corresponding point on the transformed graph.

- a) A horizontal translation of 5 units to the left and then a vertical translation of 8 units up.

$$(4,16) \longrightarrow (-1,16) \longrightarrow (-1,24)$$

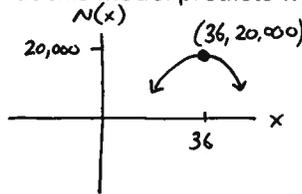
- c) A reflection in the x-axis and then a horizontal translation of 10 units to the right.

$$(4,16) \longrightarrow (4,-16) \longrightarrow (14,-16)$$

Pg 157 #14: The finance team at an advertising company is using the quadratic function $N(x) = -2.5(x - 36)^2 + 20,000$ to predict the effectiveness of a TV commercial for a certain product, where N is the predicted number of people who buy the product if the commercial is aired x times per week.

- Explain how you could sketch the graph of the function, and identify its characteristics.
- According to this model, what is the optimal number of times the commercial should be aired?
- What is the maximum number of people that this model predicts will buy the product?

(A) $N(x) = -2.5(x - 36)^2 + 20,000$
 Vertex is at $(36, 20,000)$
 Parabola opens down.



- (B) Function is at a max when $x = 36$.
 So commercial should be aired 36 times.
- (C) With 36 airings, 20,000 people should buy this product.

Pg 157 #16: See next page.

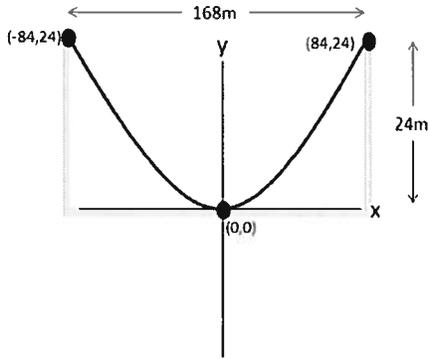
Pg 157 #19: The function $y = x^2 + 4$ represents a translation of 4 units up, which is the positive direction. The function $y = (x + 4)^2$ represents a translation of 4 units to the left, which is the negative direction. How can you explain this difference?

$y = a(x - p)^2 + q$ is the general form.

With $y = (x + 4)^2$, it is actually $y = (x - (-4))^2$, so it is shifted -4 units.

Pg 157 #16ab: The main section of the suspension bridge in Parc de la Gorge de Coaticook, Quebec, has cables in the shape of a parabola. Suppose that the points on the tops of the towers where the cables are attached are 168m apart and 24m vertically above the minimum height of the cables.

- a) Determine the quadratic function in vertex form that represents the shape of the cables. Identify the origin you used.
- b) Choose two other locations for the origin. Write the corresponding quadratic function for the shape of the cables for each.



Vertex at $(0,0)$.

$$y = a(x-0)^2 + 0$$

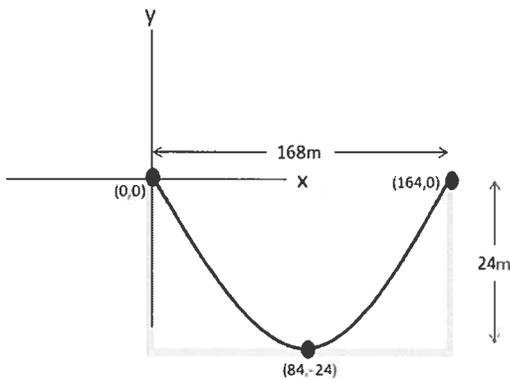
Use point $(84, 24)$

$$24 = a(84)^2$$

$$a = 0.0034$$

$$y = 0.0034(x-0)^2 + 0$$

$$y = 0.0034x^2$$



Vertex at $(84, -24)$

$$y = a(x-84)^2 - 24$$

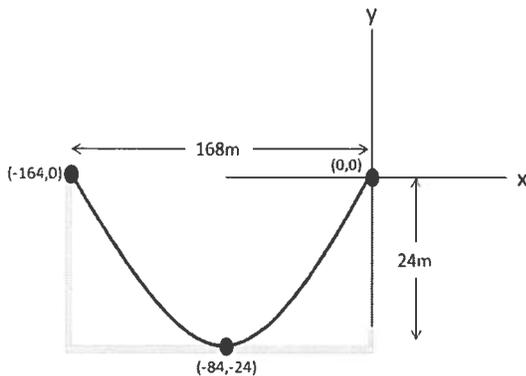
Use point $(164, 0)$

$$0 = a(164-84)^2 - 24$$

$$24 = a(84)^2$$

$$a = 0.0034$$

$$y = 0.0034(x-84)^2 - 24$$



Vertex at $(-84, -24)$

$$y = a(x-(-84))^2 - 24$$

$$y = a(x+84)^2 - 24$$

Use point $(0,0)$

$$0 = a(0+84)^2 - 24$$

$$24 = a(84)^2$$

$$a = 0.0034$$

$$y = 0.0034(x+84)^2 - 24$$