Math 20-1 KEY

L12 – 3.3 Completing the Square

Key Ideas

Definitions:

- Used for converting between Standard Form and Vertex Form
- Steps:
  1. Group first two terms ($ax^2 + bx$)
  2. Factor a-value out of the first two terms (ax^2 becomes $1x^2$)
  3. Divide the x-term by two and then square it. Write this value as a third term in the brackets.
  4. Multiply "a" by this third term, then subtract that amount from your constant term.
  5. Factor the term in the brackets and simplify the constant terms.

Part 1 – Completing the Square Practice

Q1: Convert the following into Vertex Form (a=1):

$f(x) = x^2 + 8x + 7$

$= (x^2+8) + 7$

$= (x^2+4+4x) + 7$ → $(x+4)(x+4) = x^2+4x+4x+16$ Lets add this

$= (x^2+4x+4x+16) + 7 - 16$

This cancels out the prev +16, so we have not changed the overall function.

$f(x) = (x+4)^2 - 9$

$g(x) = x^2 + 2x - 3$

$= (x^2+2x) - 3$

$= (x^2+1x+1x) - 3$ → $(x+1)(x+1) = x^2+1x+1x+1$

$= (x^2+1x+1x+1) - 3 - 1$

$g(x) = (x+1)^2 - 4$

Math 20-1

KEY

$h(x) = x^2 + 8x + 15$

$= (x^2+8x) + 15$

$= (x^2+4x+4x) + 15$ $\longrightarrow$ $(x+4)(x+4) = x^2+4x+4x+16$

$= (x^2+4x+4x+16) + 15 - 16$

$h(x) = (x+4)^2 - 1$

$k(x) = x^2 + 12x + 32$

$= (x^2+12x) + 32$

$= (x^2+6x+6x) + 32$ $\longrightarrow$ $(x+6)(x+6) = x^2+6x+6x+36$

$= (x^2+6x+6x+36) + 32 - 36$

$= (x+6)^2 - 4$

Math 20-1

KEY

Q2: Convert the following into Vertex Form (a>1, No Fractions):

$f(x) = 2x^2 - 4x - 6$

$= (2x^2 - 4x) - 6$

$= 2(x^2 - 2x) - 6$

$= 2(x^2 - 1x - 1x) - 6$ → $(x-1)(x-1) = x^2 - 1x - 1x + 1$

$= 2(x^2 - 1x - 1x + 1) - 6 - 2$

we added 2(1), so we need to subtract 2.

$f(x) = 2(x-1)^2 - 8$

$g(x) = 3x^2 + 36x + 105$

$= (3x^2 + 36x) + 105$

$= 3(x^2 + 12x) + 105$

$= 3(x^2 + 6x + 6x) + 105$ → $(x+6)(x+6) = x^2 + 6x + 6x + 36$

$= 3(x^2 + 6x + 6x + 36) + 105 - 108$

we added 3(36), so we need to subtract 108.

$g(x) = 3(x+6)^2 - 3$

Math 20-1 KEY

$h(x) = 4x^2 + 72x + 320$

$= (4x^2 + 72x) + 320$

$= 4(x^2 + 18x) + 320$

$= 4(x^2 + 9x + 9x) + 320$ → $(x+9)(x+9) = x^2 + 9x + 9x + 81$

$= 4(x^2 + 9x + 9x + 81) + 320 - 324$

Added $4(81)$ so need to subtract 324.

$h(x) = 4(x+9)^2 - 4$

$k(x) = \frac{1}{2}x^2 + 4x + 8$

$= \left(\frac{1}{2}x^2 + 4x\right) + 8$

$= \frac{1}{2}(x^2 + 8x) + 8$

$= \frac{1}{2}(x^2 + 4x + 4x) + 8$ → $(x+4)(x+4) = x^2 + 4x + 4x + 16$

$= \frac{1}{2}(x^2 + 4x + 4x + 16) + 8 - 8$

Added $\frac{1}{2}(16)$ so need to subtract 8.

$k(x) = \frac{1}{2}(x+4)^2 + 0$

Math 20-1

KEY

Q3: Convert the following into Vertex Form (with Fractions):

$f(x) = x^2 + 3x + 1000$

$= (x^2 + 3x) + 1000$

$= (x^2 + \frac{3}{2}x + \frac{3}{2}x) + 1000$ $\longrightarrow$ $(x + \frac{3}{2})(x + \frac{3}{2}) = x^2 + \frac{3}{2}x + \frac{3}{2}x + \frac{9}{4}$

$= (x^2 + \frac{3}{2}x + \frac{3}{2}x + \frac{9}{4}) + 1000 - \frac{9}{4}$

$\boxed{f(x) = (x + \frac{3}{2})^2 + \frac{3991}{4}}$

$g(x) = x^2 - 5x + 1000$

$= (x^2 - 5x) + 1000$

$= (x^2 - \frac{5}{2}x - \frac{5}{2}x) + 1000$ $\longrightarrow$ $(x - \frac{5}{2})(x - \frac{5}{2}) = x^2 - \frac{5}{2}x - \frac{5}{2}x + \frac{25}{4}$

$= (x - 5/2 \quad -\frac{5}{2}x + \frac{25}{4}) + 1000 - \frac{25}{4}$

$= (x - 5/2)^2 + \frac{3975}{4}$

KEY

Math 20-1

$h(x) = 2x^2 + 9x + 1000$

$= (2x^2 + 9x) + 1000$

$= 2(x^2 + \frac{9}{2}x) + 1000$

$= 2\left(x^2 + \frac{9}{4}x + \frac{9}{4}x\right) + 1000$ $\longrightarrow$ $(x + \frac{9}{4})(x + \frac{9}{4}) = x^2 + \frac{9}{4}x + \frac{9}{4}x + \frac{81}{16}$

$= 2\left(x^2 + \frac{9}{4}x + \frac{9}{4}x + \frac{81}{16}\right) + 1000 - \frac{81}{8}$

= Added $2(81/16)$, so subtract $81/8$.

$= 2\left(x + \frac{9}{4}x\right)^2 + \frac{7919}{8}$

$k(x) = 3x^2 + 7x + 1000$

$= (3x^2 + 7x) + 1000$

$= 3(x^2 + \frac{7}{3}x) + 1000$

$= 3\left(x^2 + \frac{7}{6}x + \frac{7}{6}x\right) + 1000$ $\longrightarrow$ $(x + \frac{7}{6})(x + \frac{7}{6}) = x^2 + \frac{7}{6}x + \frac{7}{6}x + \frac{49}{36}$

$= 3\left(x^2 + \frac{7}{6}x + \frac{7}{6}x + \frac{49}{36}\right) + 1000 - \frac{49}{12}$

= Added $3(\frac{49}{36})$ so subtract $\frac{49}{12}$.

$= 3\left(x + \frac{7}{6}\right)^2 + \frac{11951}{12}$

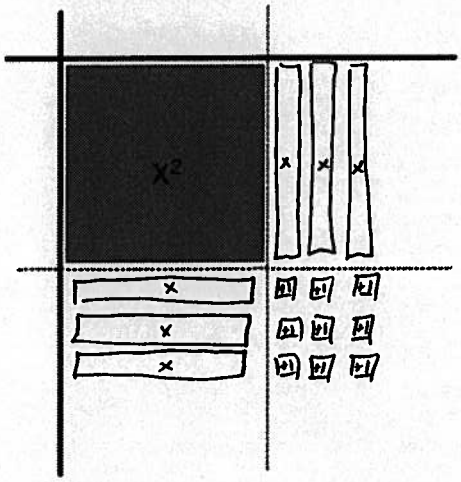
Math 20-1

KEY

Part 2 – Modelling with Algebra Tiles

Q4: Complete the square for the following using Algebra Tiles:

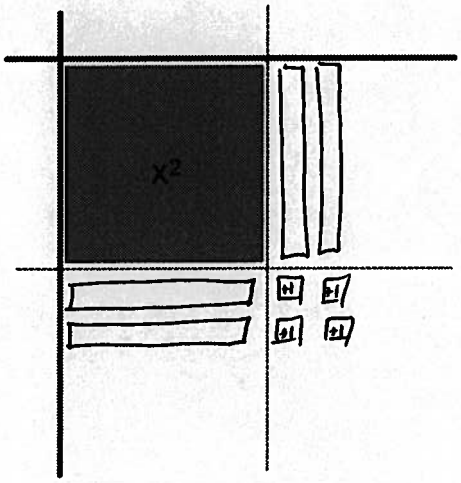
$x^2 + 6x + 10$



$(x^2 + 3x + 3x + 9) + 1$

$(x+3)^2 + 1$

$x^2 + 4x - 5$



$(x^2 + 2x + 2x + 4) - 9$

$(x+2)^2 - 9$

Math 20-1 KEY

Part 3 – Back and Forth

Q5: Convert $f(x) = -x^2 + 8x - 12$ to Vertex Form.

$f(x) = (-1x^2 + 8x) - 12$

$= -1(x^2 - 8x) - 12$

$= -1(x^2 - 4x - 4x) - 12$

$= -1(x^2 - 4x - 4x + 16) - 12$ $+16$

Added $-1(16)$ so add 16.

$f(x) = -1(x-4)^2 + 4$

Q6: Convert $f(x) = -1(x-4)^2 + 4$ to Standard Form.

$f(x) = -1(x-4)(x-4) + 4$

$= -1(x^2 - 4x - 4x + 16) + 4$

$f(x) = -1x^2 + 8x - 12$

Q7: For the function $f(x) = -x^2 + 8x - 12$, identify:

a. Coordinates of the Vertex
b. Y-Intercept
c. Equation for Axis of Symmetry
d. Zeroes

(A) $f(x) = -1(x-4)^2 + 4$, Vertex is $(4, 4)$

(B) $f(x) = -x^2 + 8x - 12$, y-int is -12

(C) $x = 4$

(D) $0 = -x^2 + 8x - 12$

$0 = x^2 - 8x + 12$

$0 = (x-2)(x-6)$

$x - 2 = 0$ $\quad$ $x - 6 = 0$

$x = 2$ $\quad$ $x = 6$

Math 20-1

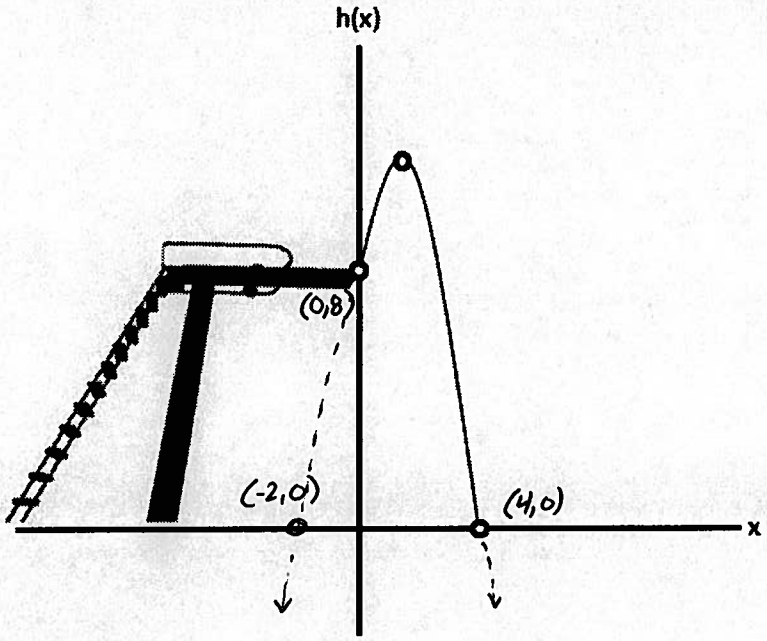
KEY

Part 4 – Working with Standard Form and Vertex Form

Use the following information to answer Q8-Q10:

A diver jumps off of a 8m tall diving board. The equation that models their height, h, as a function of horizontal distance, x, is given by:

$$h(x) = -x^2 + 2x + 8$$



Q8: Working with Standard Form...

a. Factor to find the zeroes. Which one is significant, and why?
b. Determine the y-intercept. What is the significance?
c. Use the zeroes to find the axis of symmetry.
d. Use the axis of symmetry to find the Vertex. What is the significance?

(A) $0 = -x^2 + 2x + 8$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$x-4=0$ → $x=4$ Landing position.

$x+2=0$ → $x=-2$ Gibberish.

(B) y-int is 8. Height of diving board.

(C) $\frac{(4)+(-2)}{2} = 1$ $\boxed{x=1}$

(D) $h(1) = -(1)^2 + 2(1) + 8$

$= -1 + 2 + 8$

$= 9$

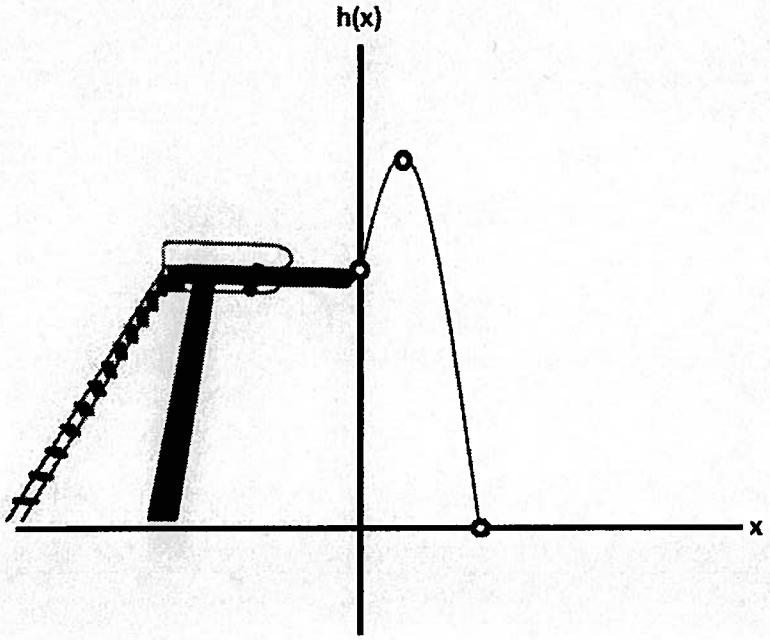
Vertex (1,9). Max height.

Math 20-1 KEY

Use the following information to answer Q8-Q10:

A diver jumps off of a 8m tall diving board. The equation that models their height, *h*, as a function of horizontal distance, *x*, is given by:

$$h(x) = -x^2 + 2x + 8$$



Q9: Working with Vertex Form...

a. Convert the function to Vertex Form.
b. Find the zeroes. Which one is significant, and why?
c. Find the Vertex. What is the significance?

(A) $h(x) = -1x^2 + 2x + 8$
$= (-1x^2 + 2x) + 8$
$= -1(x^2 - 2x) + 8$
$= -1(x - 1x - 1x) + 8$
$= -1(x - 1x - 1x + 1) + 8 + 1$
$h(x) = -1(x-1)^2 + 9$

(B) $0 = -1(x-1)^2 + 9$
$-9 \quad\quad -9$
$-9 = -1(x-1)^2$
$\div(-1) \quad \div(-1)$
$9 = (x-1)^2$
$\sqrt{9} = (x-1)$

$+3 = x - 1$
$+1 \quad +1$
$4 = x$ Landing position

$-3 = x - 1$
$+1 \quad +1$
$-2 = x$ Gibberish

(C) Vertex at (1, 9). Max height.

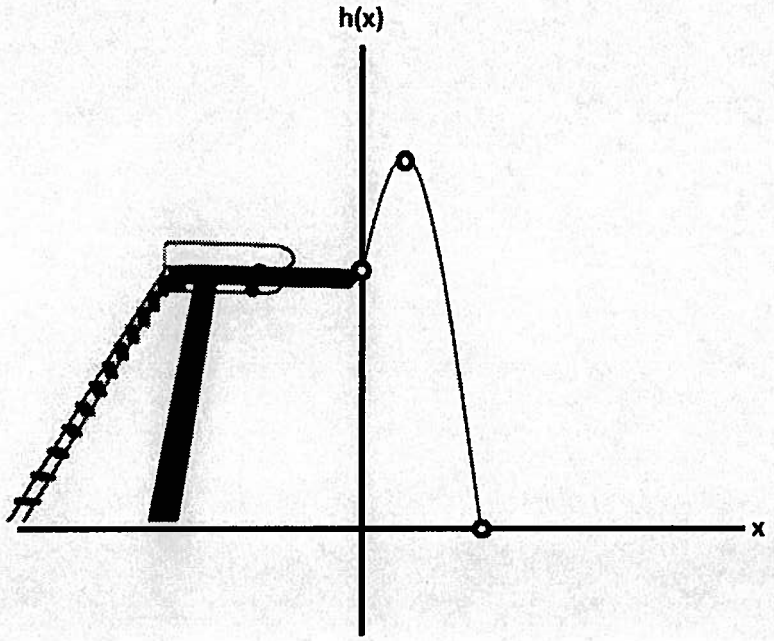
Math 20-1

Key

Use the following information to answer Q8-Q10:

A diver jumps off of a 5m tall diving board. The equation that models their height, *h*, as a function of horizontal distance, *x*, is given by:

$$h(x) = -x^2 + 2x + 8$$



Q10: Working with your Calculator...

a. Find the zeroes. Which one is significant, and why?
b. Find the Vertex. What is the significance?

(A) Input $y = -x^2 + 2x + 8$

2^{nd} Calc → Zero

$x = -2, 4$

(B) 2^{nd} Calc → Maximum

(1, 9)