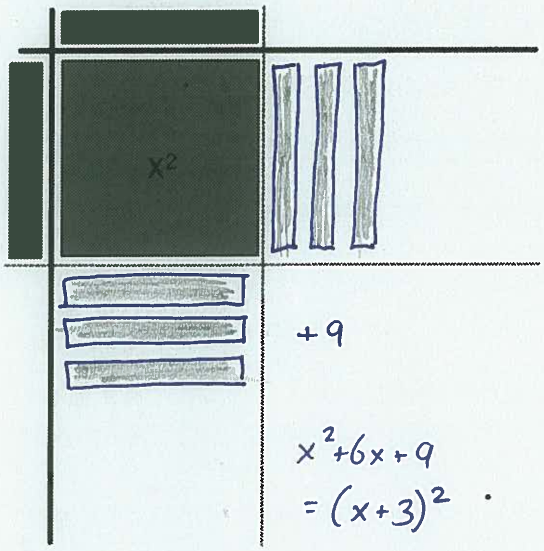
Math 20-1 KEY

L.xx – Worksheet – 3.3 Completing the Square

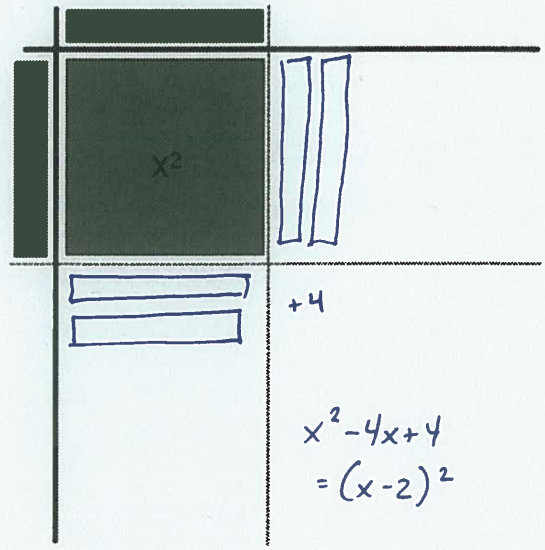
PART 1 – Textbook Questions

Pg 192 #1: Use a model to determine the value of *c* that makes each trinomial expression a perfect square. What is the equivalent binomial square expression for each?

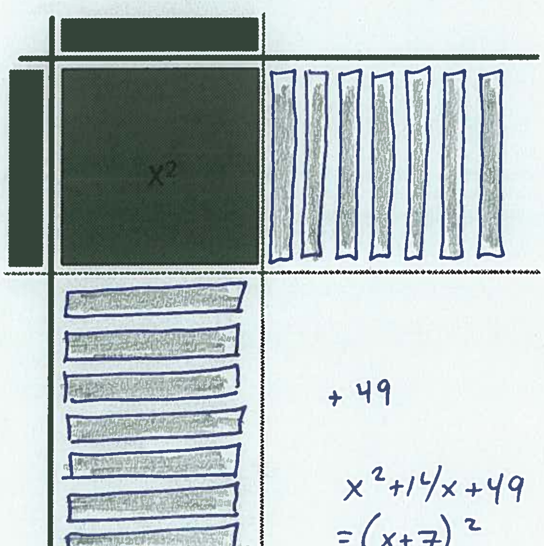
$x^2 + 6x + c \Rightarrow x^2 + 3x + 3x + c$



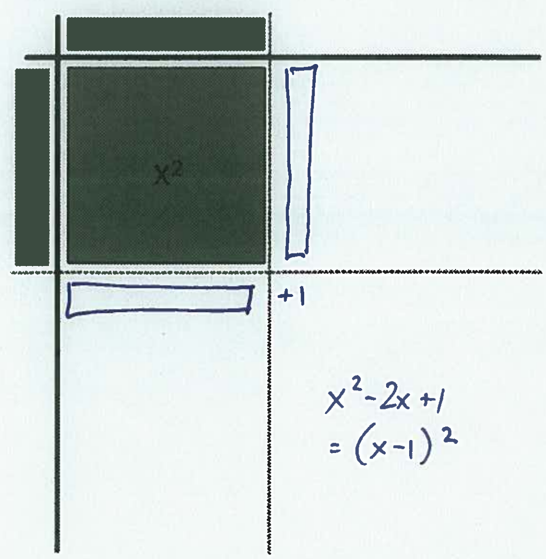
$x^2 - 4x + c \Rightarrow x^2 - 2x - 2x + c$



$x^2 + 14x + c \Rightarrow x^2 + 7x + 7x + c$



$x^2 - 2x + c \Rightarrow x^2 - 1x - 1x + c$



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Pg 192 #2: Write each function in vertex form by completing the square. Use your answer to identify the vertex of the function.

$y = x^2 + 8x \ + 0$

$(x+4)^2 = x^2 + 4x + 4x + 16$

$y = (x^2 + 4x + 4x) + 0$

$y = (x^2 + 4x + 4x + 16) + 0 - 16$

$y = (x+4)^2 - 16$

Vertex at $(-4, -16)$

$y = x^2 - 18x - 59$

$y = (x^2 - 18x) - 59$

$y = (x^2 - 9x - 9x) - 59$

$y = (x^2 - 9x - 9x + 81) - 59 - 81$

$y = (x-9)^2 - 140$

Vertex at $(9, -140)$

$y = x^2 - 10x + 31$

$y = (x^2 - 10x) + 31$

$y = (x^2 - 5x - 5x) + 31$

$y = (x^2 - 5x - 5x + 25) + 31 - 25$

$y = (x-5)^2 + 6$

Vertex at $(5, 6)$

$y = x^2 + 32x - 120$

$y = (x^2 + 32x) - 120$

$y = (x^2 + 16x + 16x) - 120$

$y = (x^2 + 16x + 16x + 256) - 120 - 256$

$y = (x+16)^2 - 376$

Vertex at $(-16, -376)$

Pg 192 #6ab: Determine the maximum or minimum value of each function and the value of x at which it occurs.

Positive means "opens up"

$y = x^2 + 6x - 2$

$y = (x^2 + 6x) - 2$

$y = (x^2 + 3x + 3x) - 2$

$y = (x^2 + 3x + 3x + 9) - 2 - 9$

$y = (x+3)^2 - 11$

Vertex at $(-3, -11)$

Min value of $y = -11$, which occurs at $x = -3$.

Positive means "opens up"

$y = 3x^2 - 12x + 1$

$y = (3x^2 - 12x) + 1$

$y = 3(x^2 - 4x) + 1$

$y = 3(x^2 - 2x - 2x) + 1$

$y = 3(x^2 - 2x - 2x + 4) + 1 - 12$

Added 3(4) so subtract

$y = 3(x-2)^2 - 11$

Vertex at $(2, -11)$

Min value at $y = -11$, which occurs at $x = 2$.

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Pg 192 #6cd: Determine the maximum or minimum value of each function and the value of x at which it occurs.

Negative means "open down"

$y = -x^2 - 10x \; +0$

$y = (-1x^2 - 10x) + 0$

$y = -1(x^2 + 10x) + 0$

$y = -1(x^2 + 5x + 5x) + 0$

$y = -1(x^2 + 5x + 5x + 25) + 0 \; + 25$

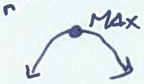
Added $-1(25)$ so add 25.

$y = -1(x+5)^2 + 25$

Vertex at $(-5, 25)$

Max at $y = 25$, when $x = -5$.

Negative means "opens down"



$y = -2x^2 + 8x - 3$

$y = (-2x^2 + 8x) - 3$

$y = -2(x^2 - 4x) - 3$

$y = -2(x^2 - 2x - 2x) - 3$

$y = -2(x^2 - 2x - 2x + 4) - 3 + 8$

Added $-2(4)$ so add $+8$.

$y = -2(x-2)^2 + 5$

Vertex at $(2, 5)$

Max at $y = 5$, when $x = 2$.

Pg 192 #8ac: Convert each function to vertex form.

$$y = x^2 + \frac{3}{2}x - 7$$

$y = (x^2 + \frac{3}{2}x) \; - 7$

$y = (x^2 + \frac{3}{4}x + \frac{3}{4}x) - 7$

$y = (x^2 + \frac{3}{4}x + \frac{3}{4}x + \frac{9}{16}) - 7 \; - \frac{9}{16}$

$y = (x + \frac{3}{4})^2 - \frac{121}{16}$

$$y = 2x^2 - \frac{5}{6}x + 1$$

$y = (2x^2 - \frac{5}{6}x) \; + 1$

$y = 2(x^2 - \frac{5}{12}x) + 1$

$y = 2(x^2 - \frac{5}{24}x - \frac{5}{24}x) + 1$

$y = 2(x^2 - \frac{5}{24}x - \frac{5}{24}x + \frac{25}{576}) + 1 \; - \frac{25}{288}$

$y = 2(x - \frac{5}{24})^2 + \frac{263}{288}$

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Pg 192 #15: Sandra is practicing at an archery club. The height, h, in feed, of the arrow on one of her shots can be modelled as a function of time, t, in seconds, since it was fired using the function $h(t) = -16t^2 + 10t + 4$.

a. What is the maximum height of the arrow, in feet, and when does it reach that height?
b. Verify your solution in two different ways.

(A) $h(t) = (-16t^2 + 10t) + 4$

$h(t) = -16(t^2 - \frac{5}{8}t) + 4$

$h(t) = -16(t^2 - \frac{5}{16}t - \frac{5}{16}t) + 4$

$h(t) = -16(t^2 - \frac{5}{16}t - \frac{5}{16}t + \frac{25}{256}) + 4 + \frac{25}{16}$ Added $(-16)(\frac{25}{256})$ so add $\frac{25}{16}$

$h(t) = -16(t - \frac{5}{16})^2 + \frac{89}{16}$

Vertex at $(\frac{5}{16}, \frac{89}{16})$.

So reaches height of $\frac{89}{16}$ feet (or 5.5625 ft) after $\frac{5}{16}$ seconds (or 0.3125 seconds).

(B) Verify using 2^{nd} CALC → MAXIMUM on Calculator.
Verify using Math 31 Calculus, where $0 = -32t + 10$
$t = \frac{5}{16}$.

Pg 192 #18ab: A concert promoter is planning the ticket price for an upcoming concert for a certain band. At the last concert, she charged \$70 per ticket and sold 2000 tickets. After conducting a survey, the promoter has determined that for every \$1 decrease in ticket price, she might expect to sell 50 more tickets.

a. What maximum revenue can the promoter expect.? What ticket price will give that revenue?
b. How many tickets can the promoter expect to sell at that price?

(A) Revenue = (Tickets sold)(Cost per ticket). Let x = Price Adjustment (dollars)

$= (2000 + 50x)(70 - 1x)$

$= 140{,}000 - 2000x + 3500x - 50x^2$

$= -50x^2 + 1500x + 140{,}000$

$= (-50x^2 + 1500x) + 140{,}000$

$= -50(x^2 - 30x) + 140{,}000$

$= -50(x^2 - 15x - 15x) + 140{,}000$

$= -50(x^2 - 15x - 15x + 225) + 140{,}000 + 11{,}250$ Added $-50(225)$ so add 11,250.

$= -50(x - 15)^2 + 151{,}250$

Vertex at (15, 151250).

So at a ticket price of $(70 - 15) = \$55$, the promoter can make \$151,250.00

(B) They will sell $(2000 + 50(15)) = 2750$ tickets.

KEY

Pg 192 #23: Use a quadratic function model to solve each problem.

a. Two numbers have a sum of 29 and a product that is a maximum. Determine the two numbers and the maximum product.

$x + y = 29 \qquad (x)(y) = \text{Maximum}$

So PRODUCT $= (x)(y)$ where $y = 29 - x$

$= (x)(29 - x)$

$= -x^2 + 29x + 0 \Rightarrow$ Opens down

$= (-x^2 + 29x) + 0$

$= -1\,(x^2 - 29x) + 0$

$= -1\,(x^2 - \frac{29}{2}x - \frac{29}{2}x) + 0$

$= -1\,(x^2 - \frac{29}{2}x - \frac{29}{2}x + \frac{841}{4}) + 0 + \frac{841}{4}$ Added $-1\,(\frac{841}{4})$ so add $+\frac{841}{4}$.

$= -1\,(x - \frac{29}{2})^2 + \frac{841}{4}$

Vertex at $(\frac{29}{2}, \frac{841}{4})$.

So when $x = \frac{29}{2}$ (and $y = \frac{29}{2}$) then product $= \frac{841}{4}$, which is max.

b. Two numbers have a difference of 13 and a product that is a minimum. Determine the two numbers and the minimum product.

$x - y = 13 \qquad (x)(y) = \text{Minimum}$

So PRODUCT $= (x)(y)$ where $y = x - 13$

$= (x)(x - 13)$

$= x^2 - 13x + 0 \Rightarrow$ Opens up

$= (x^2 - 13x) + 0$

$= (x^2 - \frac{13}{2}x - \frac{13}{2}x) + 0$

$= (x^2 - \frac{13}{2}x - \frac{13}{2}x + \frac{169}{4}) + 0 - \frac{169}{4}$

$= (x - \frac{13}{2})^2 - \frac{169}{4}$

Vertex at $(\frac{13}{2}, \frac{-169}{4})$

So when $x = \frac{13}{2}$ (and $y = \frac{-13}{2}$), then product $= \frac{-169}{4}$, which is Min.

KEY

PART 2 – Additional Practice (Easy)

Q1: Convert each function to vertex form.

$$f(x) = x^2 + 12x + 1000$$

$f(x) = (x^2+12x) + 1000$

$= (x^2+6x+6x) + 1000$

$= (x^2+6x+6x+36) + 1000 - 36$

$= (x+6)^2 + 964$

$$f(x) = x^2 - 6x + 1000$$

$f(x) = (x^2-6x) + 1000$

$= (x^2 - 3x - 3x) + 1000$

$= (x^2-3x-3x+9) + 1000 - 9$

$= (x-3)^2 + 991$

Q2: Convert each function to vertex form *(using decimals)*.

$$f(x) = x^2 + 5x + 1000$$

$f(x) = (x^2+5x) + 1000$

$= (x^2+2.5x+2.5x) + 1000$

$= (x^2+2.5x+2.5x+6.25) + 1000 - 6.25$

$= (x+2.5)^2 + 993.75$

$$f(x) = x^2 - 3x + 1000$$

$f(x) = (x^2-3x) + 1000$

$= (x^2-1.5x-1.5x) + 1000$

$= (x^2-1.5x-1.5x+2.25) + 1000 - 2.25$

$= (x-1.5)^2 + 997.75$

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Q3: Convert each function to vertex form *(using fractions)*. ⇒ Same questions as Q4!

$$f(x) = x^2 + 5x + 1000$$

$f(x) = (x^2 + 5x) + 1000$

$= (x^2 + \frac{5}{2}x + \frac{5}{2}x) + 1000$

$= (x^2 + \frac{5}{2}x + \frac{5}{2}x + \frac{25}{4}) + 1000 - \frac{25}{4}$

$= (x + \frac{5}{2})^2 + \frac{3975}{4}$

$$f(x) = x^2 - 3x + 1000$$

$f(x) = (x^2 - 3x) + 1000$

$= (x^2 - \frac{3}{2}x - \frac{3}{2}x) + 1000$

$= (x^2 - \frac{3}{2}x - \frac{3}{2}x + \frac{9}{4}) + 1000 - \frac{9}{4}$

$= (x - \frac{3}{2})^2 + \frac{3991}{4}$

Q4: Convert each function to vertex form *(using fractions)*.

$$f(x) = 2x^2 + 6x + 1000$$

$f(x) = (2x^2 + 6x) + 1000$

$= 2(x^2 + 3x) + 1000$

$= 2(x^2 + \frac{3}{2}x + \frac{3}{2}x + \frac{9}{4}) + 1000 - \frac{9}{2}$

$= 2(x + \frac{3}{2})^2 + \frac{1991}{2}$

$$f(x) = 3x^2 - 5x + 1000$$

$f(x) = (3x^2 - 5x) + 1000$

$= 3(x^2 - \frac{5}{3}x) + 1000$

$= 3(x^2 - \frac{5}{6}x - \frac{5}{6}x) + 1000$

$= 3(x^2 - \frac{5}{6}x - \frac{5}{6}x + \frac{25}{36}) + 1000 - \frac{25}{12}$

$= 3(x - \frac{5}{6})^2 + \frac{11975}{12}$

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Q5: Convert the function $f(x) = 3x^2 + 9x + 1000$ to vertex form.

Using Decimals

$f(x) = (3x^2+9x) + 1000$
$= 3(x^2+3x) + 1000$
$= 3(x^2+1.5x+1.5x) + 1000$
$= 3(x^2+1.5x+1.5x+2.25) + 1000 - 6.75$
$= 3(x+1.5)^2 + 993.25$

Using Fractions

$f(x) = (3x^2+9x) + 1000$
$= 3(x^2+3x) + 1000$
$= 3(x^2+\frac{3}{2}x+\frac{3}{2}x) + 1000$
$= 3(x^2+\frac{3}{2}x+\frac{3}{2}x+\frac{9}{4}) + 1000 - \frac{27}{4}$
$= 3(x+\frac{3}{2})^2 + \frac{3973}{4}$

Q6: Convert the function $f(x) = 2x^2 + 10x + 1000$ to vertex form.

Using Decimals

$f(x) = (2x^2+10x) + 1000$
$= 2(x^2+5x) + 1000$
$= 2(x^2+2.5x+2.5x) + 1000$
$= 2(x^2+2.5x+2.5x+6.25) + 1000 - 12.5$
$= 2(x+2.5)^2 + 987.5$

Using Fractions

$f(x) = (2x^2+10x) + 1000$
$= 2(x^2+5x) + 1000$
$= 2(x^2+\frac{5}{2}x+\frac{5}{2}x) + 1000$
$= 2(x^2+\frac{5}{2}x+\frac{5}{2}x+\frac{25}{4}) + 1000 - \frac{25}{2}$
$= 2(x+\frac{5}{2})^2 + \frac{1975}{2}$

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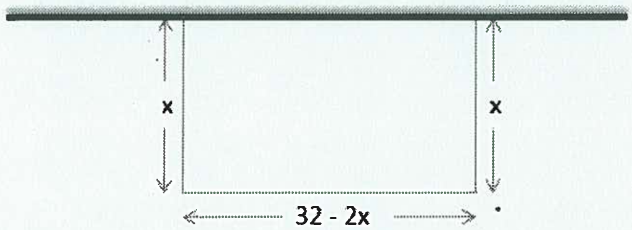
PART 3 – Additional Practice (Word Problems)

Q7: A farmer decides to build a fenced pasture on the side of a cliff. If the farmer has 32 yards of fencing, what are the dimensions of the pasture that give the maximum area?



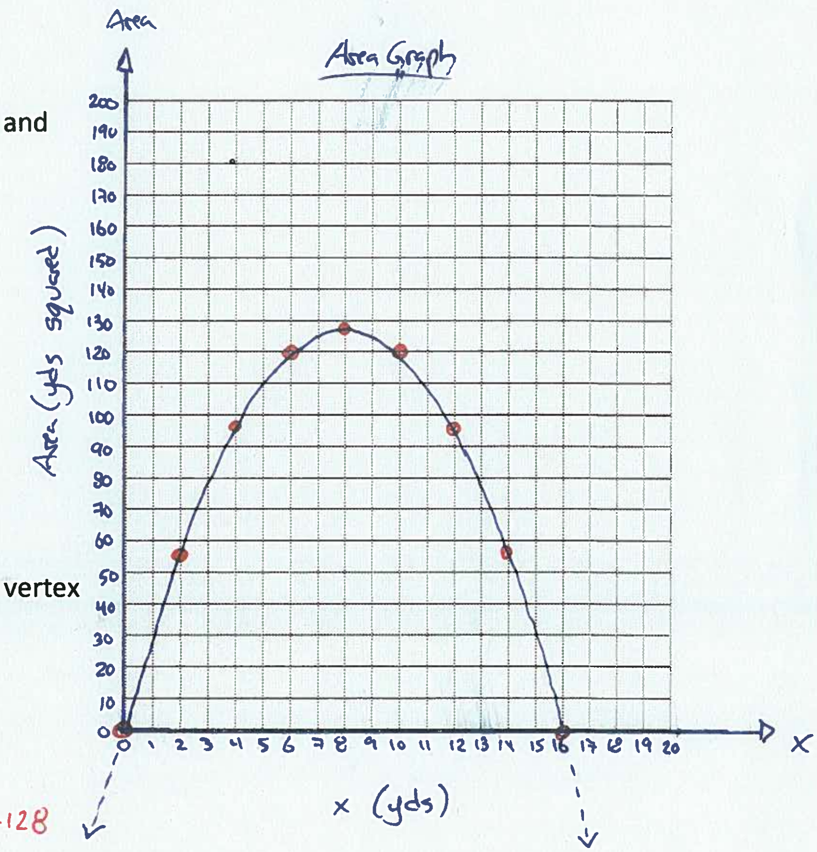
32 yards, and we've used up "x" on one side, "x" on a second side, leaving us with 32 - 2x for the third side.

a. Determine the area of the pasture as a quadratic function in *Standard Form*, $y = ax^2 + bx + c$.

$$\begin{aligned} \text{Area} &= (L)(w) \\ &= (x)(32-2x) \\ &= 32x - 2x^2 \\ &= -2x^2 + 32x + 0 \end{aligned}$$

b. Create a table of values for the function and graph it.

x	Area
0	0
2	56
4	96
6	120
8	128
10	120
12	96
14	56
16	0



c. Convert the equation to *Vertex Form*, $= a(x-p)^2 + q$, and explain what the vertex *(p,q)* represents.

$$\begin{aligned} \text{Area} &= (-2x^2 + 32x) + 0 \\ &= -2(x^2 - 16x) + 0 \\ &= -2(x^2 - 8x - 8x) + 0 \\ &= -2(x^2 - 8x - 8x + 64) + 0 + 128 \\ &= -2(x-8)^2 + 128 \end{aligned}$$

Vertex at (8, 128), or when $x = 8$, area has maximum value of 128.

d. Determine the dimensions necessary to maximize the area of the pasture.

The pasture is (x) by (32 - 2x)

or (8) by (32 - 16)

or 8 by 16 yards.