KEY

L14 – Word Problems

Key Ideas

Most word problems involve the following steps:

1. Read and interpret the question (and possibly diagram it). Build the basic equation.
2. Convert the equation to Vertex Form.
3. Solve for the Max/Min (Vertex), y-Intercept (starting amount), or zeroes.

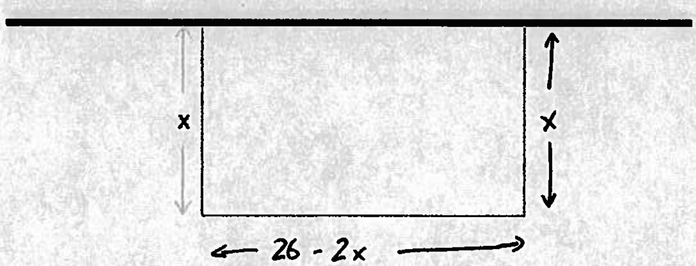
Most word problems fall under one of three types:

- Maximizing area (solve for vertex)
- Maximizing profit (solve for vertex)
- Projectile motion (solve for maximum/vertex or solve for zeroes/landing position)

We will model how to set up each type of problem.

Part 1 – Maximizing Area

Q1: A house owner is building a fence against their barn for chickens to run around. They have 26m of fence. What is the maximum area they can build for their chicken coup?



Area = LW

$A(x) = (x)(26-2x)$

$= 26x - 2x^2$

$\boxed{A(x) = -2x^2 + 26x + 0}$

Standard Form

$A(x) = -2x^2 + 26x + 0$

$= (-2x^2 + 26x) + 0$

$= -2(x^2 - 13x) + 0$

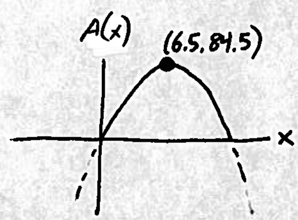
$= -2\left(x^2 - \frac{13}{2}x - \frac{13}{2}x\right) + 0$

$= -2\left(x^2 - \frac{13}{2}x - \frac{13}{2}x + \frac{169}{4}\right) + 0 + \frac{169}{2}$

$\boxed{A(x) = -2\left(x - \frac{13}{2}\right)^2 + \frac{169}{2}}$

Vertex Form

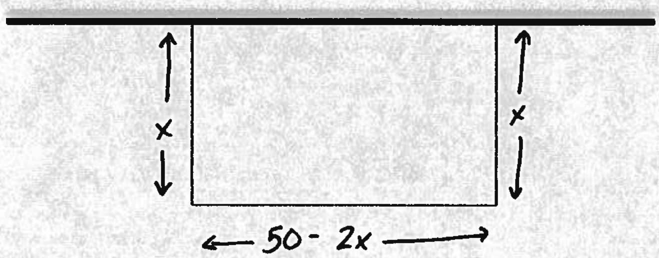
Vertex at $\left(\frac{13}{2}, \frac{169}{2}\right)$ or $(6.5, 84.5)$



$\boxed{\text{Maximum area is } 84.5\,m^2}$

KEY

Q2: On the other side of the barn, the same house owners are building a duck coup for their ducks to run around. This time they have 50m of fence, because ducks taste better than chickens and deserve more area as a result. What is the maximum area of the pen?



Area $= (L)(w)$

$A(x) = (x)(50-2x)$

$= 50x - 2x^2$

$\boxed{A(x) = -2x^2 + 50x + 0}$

Standard Form

$A(x) = -2x^2 + 50x + 0$

$= (-2x^2 + 50x) + 0$

$= -2(x^2 - 25x) + 0$

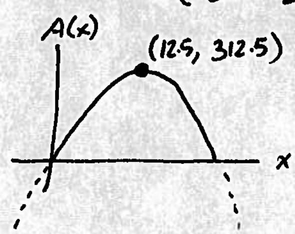
$= -2(x^2 - \frac{25}{2}x - \frac{25}{2}x) + 0$

$= -2(x^2 - \frac{25}{2}x - \frac{25}{2}x + \frac{625}{4}) + 0 + \frac{625}{2}$

$\boxed{A(x) = -2(x - \frac{25}{2})^2 + \frac{625}{2}}$

Vertex Form

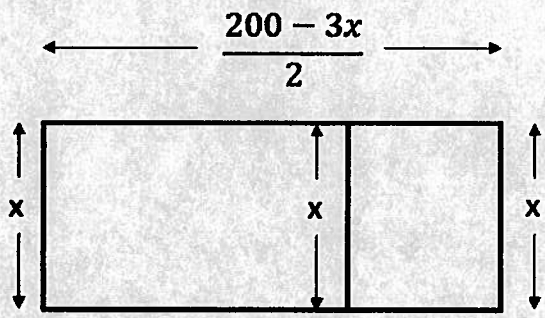
Vertex at $(\frac{25}{2}, \frac{625}{2})$ or $(12.5, 312.5)$



$\boxed{\text{Maximum area is } 312.5 \text{ m}^2}$

[redacted] Key [redacted]

Q3: **The same farmer, bored out of his mind, decides to create a playpen for his two infant twins, Timothy and Tomothy. Because the farmer loves Timothy more, Timothy will have 2/3 of the pen. Tomothy only gets 1/3, because he threw up on the farmer last week. What is the maximum area that Timothy's playpen will have?**



Area $= (L)(W)$

$A(x) = (x)\left(\frac{200 \cdot 3x}{2}\right)$

$= \frac{200x \cdot 3x^2}{2}$

$\boxed{A(x) = -\frac{3}{2}x^2 + 100x + 0}$

Standard Form

$A(x) = -\frac{3}{2}x^2 + 100x + 0$

$= \left(-\frac{3}{2}x^2 + 100x\right) + 0$

$= -\frac{3}{2}\left(x^2 - \frac{200}{3}x\right) + 0$

$= -\frac{3}{2}\left(x^2 - \frac{200}{6}x - \frac{200}{6}x + \frac{40{,}000}{36}\right) + 0 + \frac{20{,}000}{12}$

$= -\frac{3}{2}\left(x - \frac{200}{6}\right)^2 + \frac{20{,}000}{12}$

$\boxed{A(x) = -\frac{3}{2}\left(x - \frac{100}{3}\right)^2 + \frac{5000}{3}}$

Vertex Form

Vertex is $\left(\frac{100}{3}, \frac{5000}{3}\right)$

Maximum area is $\frac{5000}{3}$ m^2 or $1666.\bar{6}$ m^2.

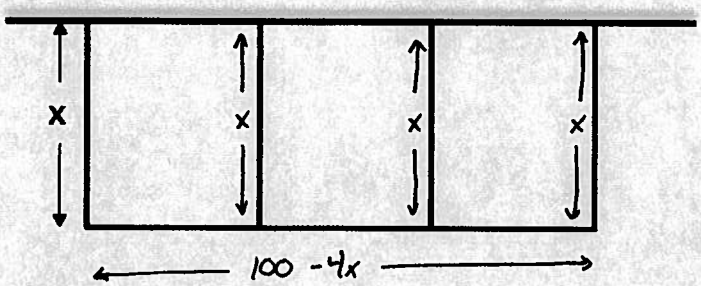
Timothy gets $\frac{2}{3}$rd of this area.

Timothy's area is $\frac{2}{3}\left(\frac{5000}{3}\right) = \frac{10{,}000}{9}$ m^2 or $1111.\bar{1}$ m^2

Note: Did you remember to look at Timothy's area, not the total area?

KEY

Q4: A large retaining wall against the back of the property can be used to house three identical areas for tasty, tasty geese. The farmer has 100m of fence. What is the maximum area of each section?



Area = $(L)(w)$
$= (x)(100 - 4x)$
$= 100x - 4x^2$
$\boxed{A(x) = -4x^2 + 100x + 0}$
Standard Form

$A(x) = -4x^2 + 100x + 0$
$= (-4x^2 + 100x) + 0$
$= -4(x^2 - 25x) + 0$
$= -4\left(x^2 - \frac{25}{2}x - \frac{25}{2}x + \frac{625}{4}\right) + 0 + 625$
$\boxed{A(x) = -4\left(x - \frac{25}{2}\right)^2 + 625}$
Vertex Form

Vertex at $\left(\frac{25}{2}, 625\right)$

Maximum area is $625 m^2$

$\boxed{\text{Each section is } 208.\overline{3}\ m^2}$

Note: Did you remember to calculate the area of each section?

KEY

Part 2 – Maximizing Profit

Q5: **Bob sells tickets to see "Paw Patrol on Ice". Bob can sell 500 tickets at a cost of $25, but Bob discovered that every time he increases the ticket price by $1, the ticket sales only reduce by 10 people. How much should Bob sell tickets for to maximize profits?**

Normal Profit $= (500)(25)$ Let n = number of $1 price increases.

New Profit $= (500 - 10n)(25 + 1n)$

$= 12500 + 500n - 250n - 10n^2)$

$P(n) = -10n^2 + 250n + 12500$

$= (-10n^2 + 250n) + 12500$

$= -10(n^2 - 25n) + 12500$

$= -10(n^2 - 12.5n - 12.5n) + 12500$

$= -10(n^2 - 12.5n - 12.5n + 156.25) + 12500 + 1562.5$

$\boxed{P(n) = -10(n - 12.5)^2 + 14062.5}$

Vertex/Max at $(12.5, 14062.5)$

So a $12.50 increase results in $14,062.50

$\boxed{\text{Tickets should be sold for \$37.50}}$

Q6: **The next season Bob sells tickets for "Extreme SeaWorld: Orcas Attack!" Bob can sell 1200 tickets at a cost of $50, but Bob discovered that every time he decreases ticket cost by $1, ticket sales increase by 25. How much should Bob sell tickets for to maximize profits?**

Normal Profit $= (1200)(50)$ Let n = number of $1 price increases.

New Profit $= (1200 + 25n)(50 - 1n)$

$= 60{,}000 - 1200n + 1250n - 20n^2$

$P(n) = -20n^2 + 50n + 60{,}000$

$= (-20n^2 + 50n) + 60{,}000$

$= -20(n^2 - 2.5n) + 60{,}000$

$= -20(n^2 - 1.25n - 1.25n) + 60{,}000$

$= -20(n^2 - 1.25n - 1.25n + 1.5625) + 60{,}000 + 31.25$

$= -20(n - 1.25)^2 + 60{,}031.25$

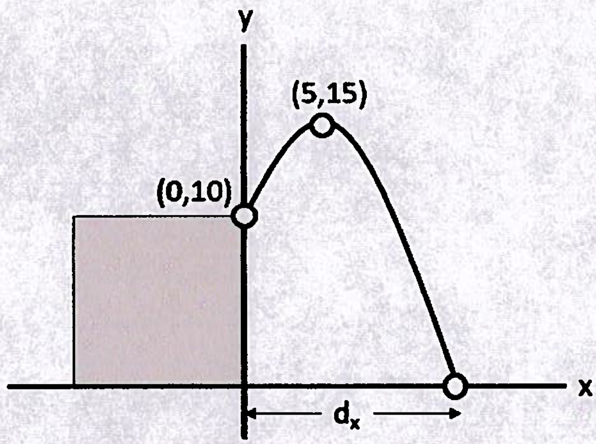
Vertex at $(1.25, 60{,}031.25)$

$\boxed{\text{Best profits when tickets sell at \$48.75}}$

KEY

Part 3 – Projectile Motion

Q7: A cat jumps off a 10ft tall ledge, reaching a maximum height of 15m after travelling a horizontal distance of 5m. What horizontal distance, d_x, does the cat cover before landing?



$h(x) = a(x-h)^2 + K$ or $h(x) = a(x-p)^2 + q$

$h(x) = a(x-5)^2 + 15$

Use point $(0,10)$

$10 = a(0-5)^2 + 15$

$10 = a(-5)^2 + 15$

$-15 \qquad -15$

$-5 = a(25)$

$\div 25 \qquad \div 25$

$-\frac{1}{5} = a$

$h(x) = -\frac{1}{5}(x-5)^2 + 15$

Now find the zeroes.

$0 = -\frac{1}{5}(x-5)^2 + 15$

$-15 \qquad -15$

$-15 = -\frac{1}{5}(x-5)^2$

$\cdot(-5) \qquad \cdot(-5)$

$75 = (x-5)^2$

$\sqrt{75} = (x-5)$

$\sqrt{75} = (x-5)$

$+8.66 = x-5$

$+5 \qquad +5$

$\boxed{13.66 = x}$

$-8.66 = x-5$

$+5 \qquad +5$

$\boxed{-3.66 = x}$

$\boxed{\text{Cat covers } 13.66\text{m}}$

Q8: The equation of a height of a football, as a function of time, is given by $h(t) = -4.905t^2 + 15t + 3$, where *t* is measured in seconds and *h(t)* is measured in meters. What is the maximum height that this football attains?

$h(t) = -4.905t^2 + 15t + 3$

$= (-4.905t^2 + 15t) + 3$

$= -4.905(t^2 - 3.058t) + 3$

$= -4.905(t^2 - 1.529t - 1.529t) + 3$

$= -4.905(t^2 - 1.529t - 1.529t + 2.338) + 3 + 11.468$

$= -4.905(t - 1.529)^2 + 14.468$

Vertex at $(1.529, 14.468)$

Football attains a maximum height of 14.468m