KEY

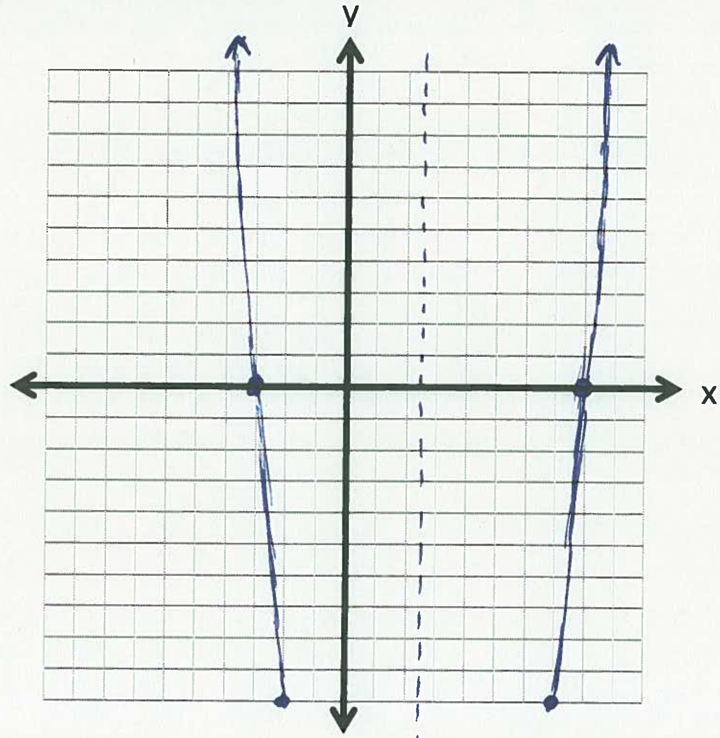
Lxx – Worksheet – 4.1 Graphical Solutions of Quadratic Equations

Pg 215 #3ac: Solve each equation by graphing the corresponding function.

a. $0 = x^2 - 5x - 24$

$y = x^2 - 5x - 24$

x	f(x)
-4	12
-3	0
-2	-10
-1	-18
0	-24
1	-28
2	-30
3	-30
4	-28
5	-
6	-18
7	-10
8	0



$x = 2.5$
(Halfway between zeroes)

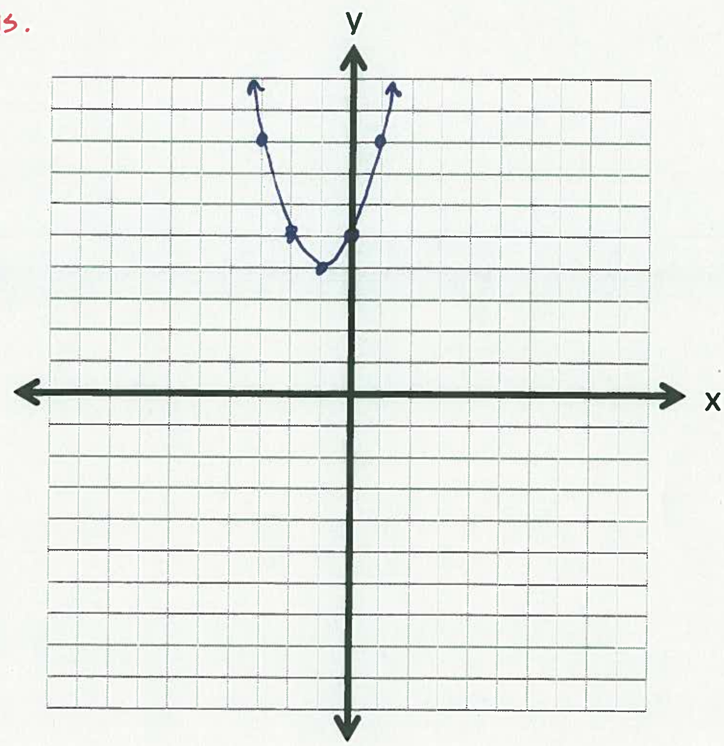
$\boxed{x = -3, 8}$

when $x = -3$ or 8, then $y = 0$. So these values are our answers.

c. $x^2 + 2x + 5 = 0$

$y = x^2 + 2x + 5$

x	f(x)
-3	8
-2	5
-1	4
0	5
1	8
2	13



$\boxed{\text{No soln}}$

At no point does $y = 0$.

KEY

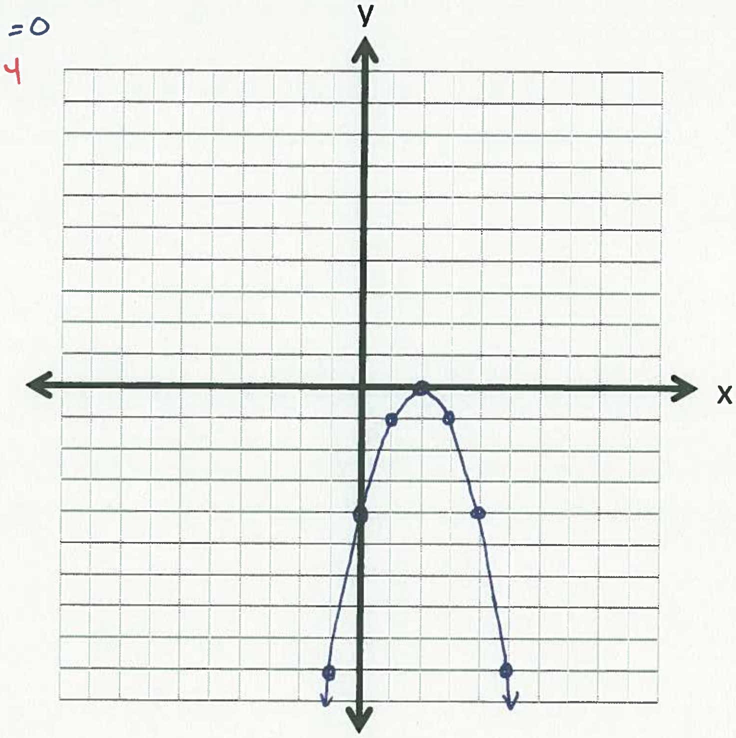
Math 20-1

Pg 215 #3e: Solve each equation by graphing the corresponding function.

e. $-x^2 + 4x = 4$ $\quad -x^2 + 4x - 4 = 0$

$y = -x^2 + 4x - 4$

x	f(x)
-1	-9
0	-4
1	-1
2	0
3	-1
4	-4
5	-9



$\boxed{x = 2}$

when $x = 2$, $y = 0$.

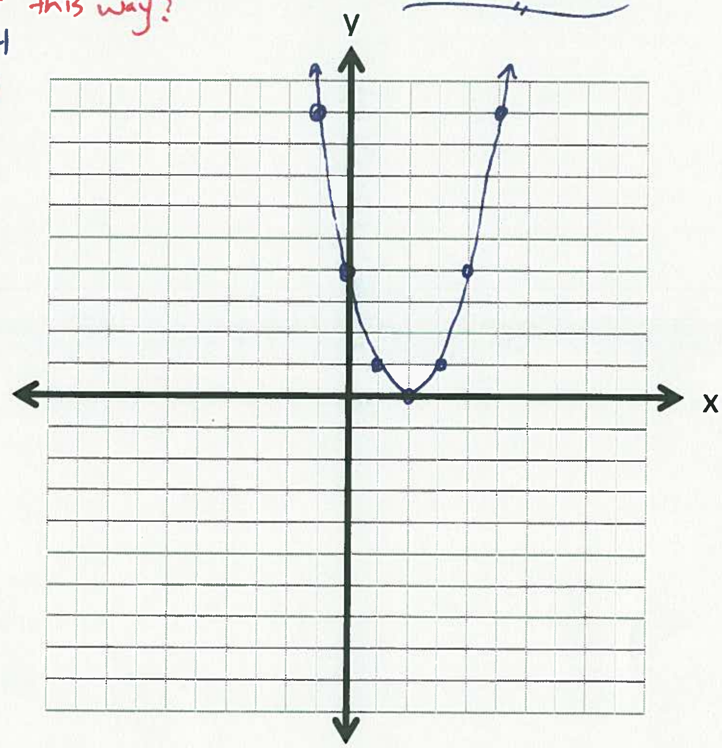
Pg 215 #3e: Solve each equation by graphing the corresponding function. ***(Alternate Method)***

what if we solved it this way?

e. $-x^2 + 4x = 4$ $\quad 0 = x^2 - 4x + 4$

$y = x^2 - 4x + 4$

x	f(x)
-1	9
0	4
1	1
2	0
3	1
4	4
5	9



$\boxed{x = 2}$

when $x = 2$, $y = 0$.

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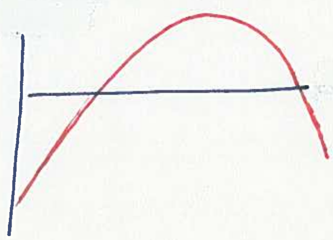
KEY

Pg 215 #5: In a Canadian Football League game, the path of the football at one particular kick-off can be modelled using the function $h(d) = -0.02d^2 + 2.6d - 66.5$, where h is the height of the ball and d is the horizontal distance from the kicking team's goal line, both in yards. A value of $h(d) = 0$ represents the height of the ball at ground level. What horizontal distance does the ball travel before it hits the ground?

Using a T.I. Calculator
w/ x : 0 → 100
y : -80 → 20



If $h(d) = -0.02d^2 + 2.6d - 66.5$
then $0 = -0.02d^2 + 2.6d - 66.5$ when $d = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$d = \frac{-2.6 \pm \sqrt{(2.6)^2 - 4(-0.02)(-66.5)}}{2(-0.02)}$$

$$d = \frac{-2.6 \pm \sqrt{6.76 - 5.32}}{-0.04} = \frac{-2.6 \pm 1.2}{-0.04}$$

$$d_1 = \frac{-2.6 + 1.2}{-0.04} = +35 \qquad d_2 = \frac{-2.6 - 1.2}{-0.04} = +95$$

Ball takes off from 35m and lands at 95m.
It travels 60m.

Pg 215 #6: Two numbers have a sum of 9 and a product of 20.

a. What single-variable quadratic equation in the form $ax^2 + bx + c = 0$ can be used to represent the product of the two numbers?

$x+y=9 \Rightarrow y = 9-x \Rightarrow (x)(9-x) = 20$
$(x)(y) = 20$

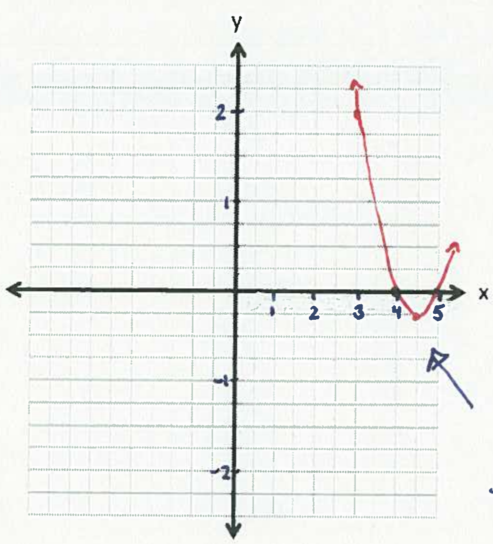
$-x^2+9x = 20$

$\boxed{0 = x^2 - 9x + 20}$ or $\boxed{-x^2+9x-20 = 0}$

Despite opening in different directions, both have the same x-intercepts (zeroes).
See Pg 215 # 3e with alternate method.

b. Determine the two numbers by graphing the corresponding quadratic function.

x	y
3	2
4	0
5	0
4.5	-0.25



... but I'd rather do it by putting it into factored form...

$0 = (x-4)(x-5)$

$x-4=0$ $\quad$ $x-5=0$
$x=4$ $\quad$ $x=5$

$y=0$ when $x=4$ and $x=5$.

So (A) If $x=4$, $y=5$.
(B) If $x=5$, $y=4$.

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Pg 215 #8: The path of a stream of water coming out of a fire hose can be approximated using the function $h(x) = -0.09x^2 + x + 1.2$, where *h* is the height of the water stream and *x* is the horizontal distance from the firefighter holding the nozzle, both in meters.

a. What does the equation $-0.09x^2 + x + 1.2 = 0$ represent in this situation?

The height at ground level... $h(x) = 0$.

b. At what maximum distance from the building could a firefighter stand and still reach the base of the fire with the water? Express your answer to the nearest tenth of a meter.

$0 = -0.09x^2 + x + 1.2$ when $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

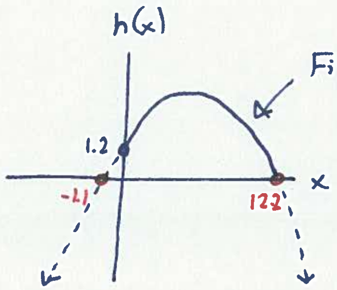
$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(-0.09)(1.2)}}{2(-0.09)}$$

$$x = \frac{-1 \pm \sqrt{1 + 0.432}}{-0.18} = \frac{-1 \pm 1.19666202413}{-0.18}$$

$$x_1 = \frac{-1 + 1.19666202413}{-0.18} = -1.1 \qquad x_2 = \frac{-1 - 1.19666202413}{-0.18} = 12.2$$

Doesn't make sense.

That's our answer.



Finite path modelled with a quadratic.

Be careful what information you extrapolate from the graph. The "dotted" portions of the graph do not represent the path of the water.

Also note, axis of symmetry occ exactly halfway between the zeroe Vertex lies on this line.

c. What assumptions did you make when solving this problem?

That water can't go backwards?

No wind or air resistance?

KEY

Pg 215 #11: Émilie Heymans is a three-time Canadian Olympic diving medalist. Suppose that for a dive off the 10-m tower, her height, *h*, in meters, above the surface of the water is given by the function $h(t) = -2d^2 + 3d + 10$, where *d* is the horizontal distance from the end of the tower platform, in meters.

a. Write a quadratic equation to represent the situation when Émilie enters the water.

when $h(t) = 0$

$$0 = -2d^2 + 3d + 10$$

b. What is Émilie's horizontal distance from the end of the tower platform when she enters the water? Express your answer to the nearest tenth of a meter.

$0 = -2d^2 + 3d + 10$ when $d = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$d = \frac{-3 \pm \sqrt{(-3)^2 - 4(-2)(10)}}{2(-2)} = \frac{-3 \pm \sqrt{9 + 80}}{-4}$$

$$d_1 = \frac{-3 + \sqrt{89}}{-4} = -1.6 \qquad d_2 = \frac{-3 - \sqrt{89}}{-4} = +3.1$$

Might be an x-intercept (or "zero"), but doesn't make sense for the word problem.

Yeppers, this one is our answer.

KEY

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Pg 215 #12: Matthew is investigating the old Borden Bridge, which spans the North Saskatchewan River about 50km west of Saskatoon. The three parabolic arches of the bridge can be modelled using quadratic functions, where h is the height of the arch above the bridge deck and x is the horizontal distance of the bridge deck from the beginning of the first arch, both in meters.

First arch: $h(x) = -0.01x^2 + 0.84x$

Second arch: $h(x) = -0.01x^2 + 2.52x - 141.12$

Third arch: $h(x) = -0.01x^2 + 4.2x - 423.36$

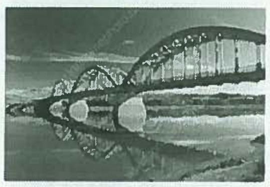
a. What are the zeroes of each quadratic function?
b. What is the significance of the zeroes in this situation?
c. What is the total span of the Borden Bridge?

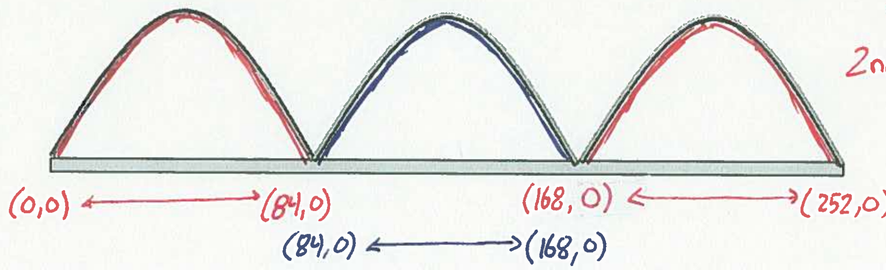
Option #1: Use quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for all three functions.

Option #2: Use a T.I. Graphing Calculator.

This one!

2nd CALC → Zero





Ⓐ The zeroes are

First arch: (0,0) and (84,0)

Second arch: $x = 84$, $x = 168$

Third arch: $x = 168$, $x = 252$

Ⓑ The zeroes represent where the arches attach to the bridge.

Ⓒ Total span is 252m

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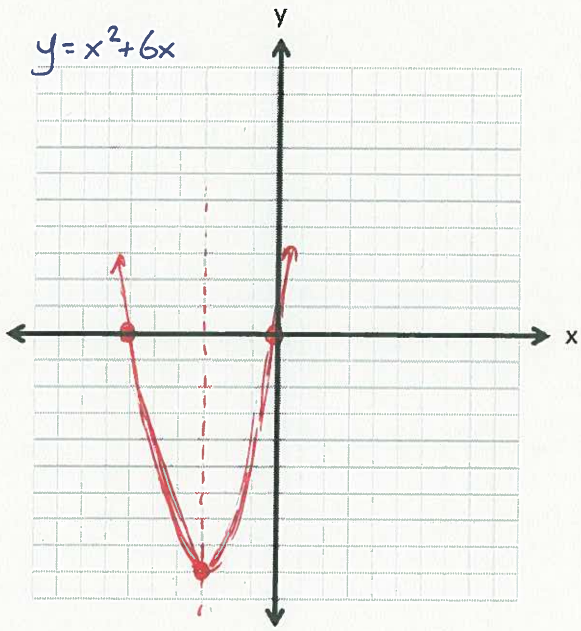
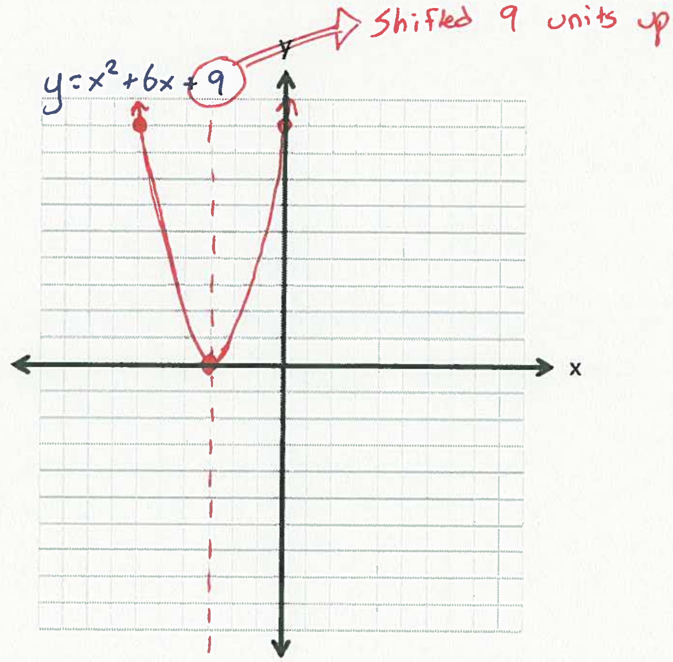
KEY

Pg 215 #13: For what values of *k* does the equation $x^2 + 6x + k = 0$ have

a. One real root?
b. Two distinct real roots?
c. No real roots?

Shifts up/down.
y-intercept.

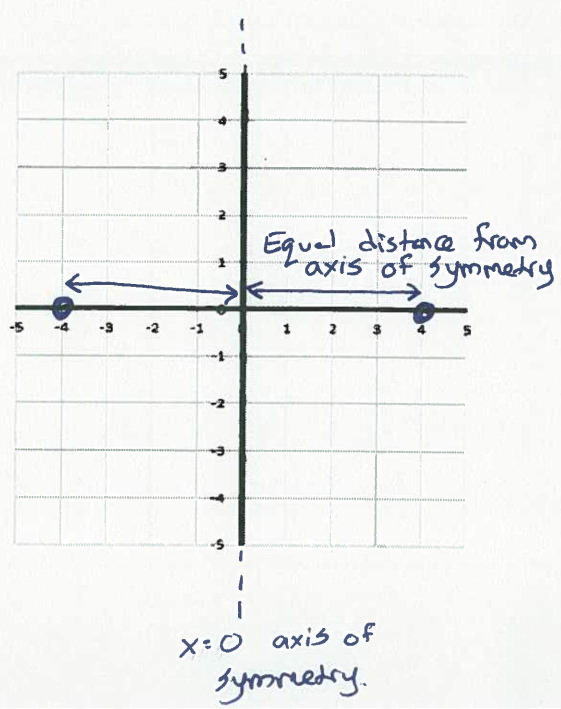




(A) If $k = 9$, only 1 real root.
(B) If $k < 9$, 2 real roots.
(C) If $k > 9$, no real roots.

Pg 215 #17: The equation of the axis of symmetry of a quadratic function is $x = 0$ and one of the x-intercepts is -4. What is the other x-intercept? Explain using a diagram.

$x = 4$ because it needs to be equal distance from the axis of symmetry.



For example, if axis of symmetry were at $x = -1$, and first intercept at $x = -4$, then