Math 20-1 KEY

L17 – 4.2 Factoring Quadratic Equations

Key Ideas

1. Quadratic equations can be solved by factoring.
2. The solutions to a quadratic equation are called the roots of the equation.
3. You can factor polynomials in quadratic form.

$$4x^2 + 12x + 9$$

$$(2x+3)(2x+3)$$

$$(2x+3)^2$$

4. You can also factor more complicated expressions using substitution:

$$2(x+3)^2 - 11(x+3) + 15 \qquad \text{Let } r = (x+3)$$

$$2r^2 - 11r + 15$$

$$[2r-5][r-3]$$

$$[2(x+3)-5][(x+3)-3]$$

$$[2x+6\quad -5][x+3\quad -3]$$

$$[2x+1][x]$$

5. You can factor a difference of squares $P^2 - Q^2$ as $(P+Q)(P-Q)$.

$$\frac{4}{9}x^2 - 16y^2$$

$$\left(\frac{2}{3}x + 4y\right)\left(\frac{2}{3}x - 4y\right)$$

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Part 1 – Factoring Practice

Q1: Factor the following:

$x^2 + 12x + 35$

$+5 \quad +7$

$\square + \square = 12$

$\square \times \square = 35$

1, 35

$\boxed{5, 7}$

$(x+5)(x+7)$

$x^2 - 8x + 12$

$-2 \quad -6$

$\square + \square =$

$\square \times \square =$

1, 12

$\boxed{2, 6}$

3, 4

$(x-2)(x-6)$

Q2: Factor the following:

$\frac{1}{2}x^2 + 7x + \frac{45}{2}$

$\frac{1}{2}(x^2+14x+45)$

$\frac{1}{2}(x+5)(x+9)$

$+5 \quad +9$

$\square + \square = 14$

$\square \times \square = 45$

1, 45

3, 15

$\boxed{5, 9}$

$-\frac{1}{4}x^2 + \frac{5}{4}x + 6$

$-\frac{1}{4}(x^2+5x-24)$

$-\frac{1}{4}(x-3)(x+8)$

$-3 \quad +8$

$\square + \square = 5$

$\square \times \square = -24$

1, 24

2, 12

$\boxed{3, 8}$

Q3: Factor the following:

$(x-2)^2 + 5(x-2) - 6$

Let $(x-2) = y$

$y^2 + 5y - 6$

$-1 \quad +6$

$\square + \square = 5$

$\square \times \square = -6$

1, 6

2, 3

$(y-1)(y+6)$

$(x-2-1)(x-2+6)$

$(x-3)(x+4)$

$\frac{1}{2}(x+3)^2 + \frac{1}{2}(x+3) - 10$

$\frac{1}{2}\left[(x+3)^2 + (x+3) - 20\right]$

Let $y = x+3$

$\frac{1}{2}\left[y^2 + 1y - 20\right]$

$-4 \quad +5$

$\square + \square = 1$

$\square \times \square = -20$

1, 20

2, 10

4, 5

$\frac{1}{2}\left[(y-4)(y+5)\right]$

$\frac{1}{2}\left[(x+3-4)(x+3+5)\right]$

$\frac{1}{2}\left[(x-1)(x+8)\right]$

$\frac{1}{2}(x-1)(x+8)$

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Part 2 – Solving Quadratic Equations by Factoring

Q4: Solve for x.

$2x^2 + 5x + 2 = 0$

$\square + \square = 5$ (+1, +4)

$\square \times \square = 4$

$\boxed{1,4}$

2,2

$2x^2 + 1x + 4x + 2 = 0$

$(2x^2 + 1x) + (4x + 2) = 0$

$x(2x+1) + 2(2x+1) = 0$

$(2x+1)(x+2) = 0$

$2x+1 = 0$; -1 -1 ; $2x = -1$; $\div 2$ $\div 2$; $\boxed{x = -\frac{1}{2}}$

$x+2 = 0$; -2 -2 ; $\boxed{x = -2}$

$2x^2 + 9x + 9 = 0$

$\square + \square = 9$ (+3, +6)

$\square \times \square = 18$

1,18

2,9

$\boxed{3,6}$

$2x^2 + 3x + 6x + 9 = 0$

$(2x^2 + 3x) + (6x + 9) = 0$

$x(2x+3) + 3(2x+3) = 0$

$(2x+3)(x+3) = 0$

$2x+3 = 0$; -3 -3 ; $2x = -3$; $\div 2$ $\div 2$; $\boxed{x = -3/2}$

$x+3 = 0$; -3 -3 ; $\boxed{x = -3}$

$2x^2 + 12x = -10$

$+10$ $+10$

$2x^2 + 12x + 10 = 0$

$2(x^2 + 6x + 5) = 0$

$\square + \square = 6$ (+1, +5)

$\square \times \square = 5$

$\boxed{1,5}$

$2(x+1)(x+5) = 0$

$x+1 = 0$; -1 -1 ; $\boxed{x = -1}$

$x+5 = 0$; -5 -5 ; $\boxed{x = -5}$

$x^2 = -8x - 15$

$+8x$ $+8x$

$x^2 + 8x = -15$

$+15$ $+15$

$x^2 + 8x + 15 = 0$

$\square + \square = 8$ (+3, +5)

$\square \times \square = 15$

1,15

3,5

$(x+3)(x+5) = 0$

$x+3 = 0$; -3 -3 ; $\boxed{x = -3}$

$x+5 = 0$; -5 -5 ; $\boxed{x = -5}$

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Part 3 – Quadratic Equations Word Problems

Use the following information to answer Q5:

In Physics 20, we learn that the height of an object is dependent on the time it is in the air. The equation that governs this relationship is given by

$$\Delta d = v_i t + \frac{1}{2}at^2$$

Where $\Delta d = d_f - d_i$

$$d_f - d_i = v_i t + \frac{1}{2}at^2$$

And we can add d_i to both sides

$$d_f = \frac{1}{2}at^2 + v_i t + d_i$$

Where the acceleration due to gravity is approximately -10 m/s^2. In Math 20-1, we can write this function as follows:

$$h(t) = -5t^2 + v_i t + h_i$$

Where h_i is our initial height, or y-intercept.

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Q5: For a given grasshopper, it is sitting on a 50cm tall ledge when it jumps off with an initial velocity of 15m/s. The equation governing its height, as a function of time, is

$$h(t) = -5t^2 + 15t + 50$$

a. Algebraically determine the zeroes of the function by factoring. What do they represent?

$0 = -5t^2 + 15t + 50$

$\div(-5) \quad \div(-5) \quad \div(-5) \quad \div(-5)$

$0 = t^2 - 3t - 10$

$+2 \quad -5$

$\square + \square = -3$

$\square \times \square = -10$

1,10

2,5

$0 = (t+2)(t-5)$

$t + 2 = 0$ → $\boxed{t = -2}$ ← Garbage

$t - 5 = 0$ → $\boxed{t = 5}$ ← Landing time.

b. Algebraically determine the coordinates of the vertex, and explain what it represents.

$\frac{(-2)+(5)}{2} = 1.5$

Axis of symmetry at $t = 1.5$ sec

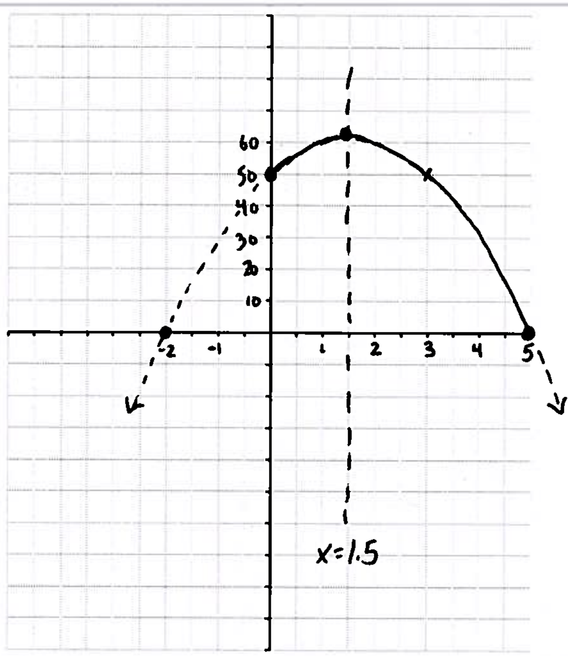
$h(1.5) = -5(1.5)^2 + 15(1.5) + 50$

$h(1.5) = 61.25$

Vertex at $(1.5, 61.25)$

At 1.5 sec, grasshopper is 61.25 cm high.

c. Sketch the function below.



d. State the Domain and Range of the function.

Domain: $\{t \mid 0 \le t \le 5, t \in \mathbb{R}\}$

Range: $\{h \mid 0 \le h \le 61.25, h \in \mathbb{R}\}$