Math 20-1

KEY

L17 – 4.3 Solving Quadratic Equations by Completing the Square

Key Ideas

Remember from 3.3 that "Completing the Square" is the process of converting the equation from Standard Form, $y = ax^2 + bx + c$, to Vertex Form, $y = (x - h)^2 + k$.

$y = 2x^2 - 5x - 3$

Step #1: Group the first two terms.

$y = (2x^2 - 5x) - 3$

Step #2: Factor out the "a" coefficient.

$y = 2\left(x^2 - \frac{5}{2}x\right) - 3$

Step #3: Split the second term into two equal parts.

$y = 2\left(x^2 - \frac{5}{4}x - \frac{5}{4}x\right) - 3$ where $\left(x - \frac{5}{4}\right)\left(x - \frac{5}{4}\right) = x^2 - \frac{5}{4}x - \frac{5}{4}x + \frac{25}{16}$

Step #4: It sure would be nice if the stuff in the brackets were factorable. So add a $\frac{25}{16}$ into the brackets. But remember... we actually added $2\left(\frac{25}{16}\right)$, so we need to subtract the same amount of $\frac{25}{8}$ from the end.

$y = 2\left(x^2 - \frac{5}{4}x - \frac{5}{4}x + \frac{25}{16}\right) - 3 - \frac{25}{8}$

Step #5: Now simplify to finish converting to Vertex Form.

$y = 2\left(x - \frac{5}{4}\right)^2 - \frac{49}{8}$

You can use *Completing the Square* to determine the roots of a Quadratic Equation.

KEY

Math 20-1

Part 1 – Why use this method?

Q1: Given the equation $x^2 + 8x + 15 = 0$, determine the roots.

Standard Form	*Vertex Form*

Standard Form

$x^2+8x+15=0$

$\underset{+3}{\square}+\underset{+5}{\square}=8$

$\square\times\square=15$

$(x+3)(x+5)=0$

$\boxed{x=-3}$ $\quad$ $\boxed{x=-5}$

Vertex Form

$(x^2+8x)+15=0$

$1(x^2+4x+4x)+15=0$

$1(x^2+4x+4x+16)+15-16=0$

$\boxed{1(x+4)^2-1=0}$ Vertex Form

$+1 \quad +1$

$(x+4)^2=1$

$x+4=\sqrt{1}$

$x+4=+1$ $\quad$ $x+4=-1$

$-4 \quad -4$ $\quad$ $-4 \quad -4$

$\boxed{x=-3}$ $\quad$ $\boxed{x=-5}$

Q2: Given the equation $x^2 + 6x + 7 = 0$, determine the roots.

Standard Form	*Vertex Form*

Standard Form

$x^2+6x+7=0$

$\square+\square=6$

$\square\times\square=7$

1,7

Not factorable! Can't solve until we learn the quadratic formula.

Vertex Form

$(x^2+6x)+7=0$

$1(x^2+3x+3x)+7=0$

$1(x^2+3x+3x+9)+7-9=0$

$\boxed{1(x+3)^2-2=0}$ Vertex Form

$+2 \quad +2$

$1(x+3)^2=2$

$x+3=\sqrt{2}$

$x+3=+\sqrt{2}$ $\quad$ $x+3=-\sqrt{2}$

-3 $\quad$ -3

$\boxed{x=+\sqrt{2}-3}$ $\quad$ $\boxed{x=-\sqrt{2}-3}$

KEY

Math 20-1

Part 2 – Solving Equations by Completing the Square (Decimal Approximation, Calculator Verification)

Q3: Solve the equation.

$x^2 - 6x + 7 = 0$

$1(x^2-6x)+7=0$

$1(x^2-3x-3x+9)+7-9=0$

$\boxed{1(x-3)^2-2=0}$ Vertex Form

$+2 \quad +2$

$(x-3)^2=2$

$(x-3)=\sqrt{2}$

$x-3 \doteq +1.41$ | $x-3 \doteq -1.41$

$+3 \quad +3$ | $+3 \quad +3$

$\boxed{x \doteq 4.41}$ | $\boxed{x \doteq 1.59}$

$x^2 - 10x + 22 = 0$

$1(x^2-10x)+22=0$

$1(x^2-5x-5x+25)+22-25=0$

$\boxed{1(x-5)^2-3=0}$ Vertex Form

$+3 \quad +3$

$(x-5)^2=3$

$(x-5)=\sqrt{3}$

$x-5 \doteq +1.73$ | $x-5 \doteq -1.73$

$+5 \quad +5$ | $+5 \quad +5$

$\boxed{x \doteq 6.73}$ | $\boxed{x \doteq 3.27}$

Q4: Solve the equation

$-\frac{1}{2}x^2 + 4x - 10 = 0$

$-\frac{1}{2}(x^2-8x)-10=0$

$-\frac{1}{2}(x^2-4x-4x+16)-10+8=0$

$\boxed{-\frac{1}{2}(x-4)^2-2=0}$ Vertex Form

$+2 \quad +2$

$-\frac{1}{2}(x-4)^2=2$

$\div(-\frac{1}{2}) \quad \div(-\frac{1}{2})$

$(x-4)^2=-4$

$x-4=\sqrt{-4}$ ← Doesn't work. No solution.

Graph to verify that it doesn't cross the x-axis.

$2x^2 + 16x + 35 = 0$

$2(x^2+8x)+35=0$

$2(x^2+4x+4x+16)+35-32=0$

$\boxed{2(x+4)^2+3=0}$ Vertex Form

$-3 \quad -3$

$2(x+4)^2=-3$

$\div 2 \quad \div 2$

$(x+4)^2=-3/2$

$x+4=\sqrt{-3/2}$ ← Doesn't work. No solution.

Graph to verify that it doesn't cross the x-axis.

Math 20-1

KEY

Part 3 – Solving Equations by Completing the Square (Exact Answers)

Q5: Solve the equation.

$\frac{1}{4}x^2 + 6 = 3x$

$-3x \quad -3x$

$\frac{1}{4}x^2 - 3x + 6 = 0$

$\frac{1}{4}(x^2 - 12x) + 6 = 0$

$\frac{1}{4}(x^2 - 6x - 6x) + 6 = 0$

$\frac{1}{4}(x^2 - 6x - 6x + 36) + 6 - 9 = 0$

$\boxed{\frac{1}{4}(x-6)^2 - 3 = 0}$ Vertex Form

$+3 \quad +3$

$\frac{1}{4}(x-6)^2 = 3$

$\div(1/4) \quad \div(1/4)$

$(x-6)^2 = 12$

$x - 6 = \sqrt{12}$

$+6 \quad +6$

$\boxed{x = \pm\sqrt{12} + 6}$ where $\sqrt{12} = \sqrt{2^2 \cdot 3} = 2\sqrt{3}$

or

$\boxed{x = \pm 2\sqrt{3} + 6}$

$\boxed{x_1 = +2\sqrt{3} + 6}$ $\boxed{x_2 = -2\sqrt{3} + 6}$

$2x + 5 = \frac{1}{2}x^2$

$-2x \quad -2x$

$5 = \frac{1}{2}x^2 - 2x$

$-5 \quad -5$

$0 = \frac{1}{2}x^2 - 2x - 5$

$0 = \frac{1}{2}(x^2 - 4x) - 5$

$0 = \frac{1}{2}(x^2 - 2x - 2x + 4) - 5 - 2$

$\boxed{0 = \frac{1}{2}(x-2)^2 - 7}$ Vertex Form

$+7 \quad +7$

$7 = \frac{1}{2}(x-2)^2$

$\div(1/2) \quad \div(1/2)$

$14 = (x-2)^2$

$\sqrt{14} = x - 2$

$+\sqrt{14} = x - 2$ $-\sqrt{14} = x - 2$

$+2 \quad +2$ $+2 \quad +2$

$\boxed{+\sqrt{14} + 2 = x_1}$ $\boxed{-\sqrt{14} + 2 = x_2}$