Math 20-1

KEY

Lxx – Worksheet – 4.3 Solving Quadratic Equations by Completing the Square

Pg 240 #1: What value of c makes each expression a perfect square?

$x^2 + x + c$

$x^2 + \frac{1}{2}x + \frac{1}{2}x + c$

where $(x+\frac{1}{2})(x+\frac{1}{2})$
$= x^2 + \frac{1}{2}x + \frac{1}{2}x + \frac{1}{4}$

so $c = \frac{1}{4}$

$x^2 - 5x + c$

$x^2 - \frac{5}{2}x - \frac{5}{2}x + c$

where $(x-\frac{5}{2})(x-\frac{5}{2})$
$= x^2 - \frac{5}{2}x - \frac{5}{2}x + \frac{25}{4}$

so $c = \frac{25}{4}$

$x^2 - 0.5x + c$

$x^2 - 0.25x - 0.25x + c$

where $(x-0.25)(x-0.25)$
$= x^2 - 0.25x - 0.25x + 0.0625$

so $c = 0.0625$

$x^2 + 0.2x + c$

$x^2 + 0.1x + 0.1x + c$

where $(x+0.1)(x+0.1)$
$= x^2 + 0.1x + 0.1x + 0.01$

so $c = 0.01$

$x^2 + 15x + c$

$x^2 + \frac{15}{2}x + \frac{15}{2}x + c$

where $(x+\frac{15}{2})(x+\frac{15}{2})$
$= x^2 + \frac{15}{2}x + \frac{15}{2}x + \frac{225}{4}$

so $c = \frac{225}{4}$

$x^2 - 9x + c$

$x^2 - \frac{9}{2}x - \frac{9}{2}x + c$

where $(x-\frac{9}{2})(x-\frac{9}{2})$
$= x^2 - \frac{9}{2}x - \frac{9}{2}x + \frac{81}{4}$

so $c = \frac{81}{4}$

Pg 240 #2: Complete the square to write each quadratic equation in the form $(x+p)^2 = q$

$2x^2 + 8x + 4 = 0$

$(2x^2+8x) + 4 = 0$
$2(x^2+4x) + 4 = 0$
$2(x^2+2x+2x) + 4 = 0$
$2(x^2+2x+2x+4) + 4 - 8 = 0$
$2(x+2)^2 - 4 = 0$
$2(x+2)^2 = 4$
$\div 2 \quad \div 2$
$(x+2)^2 = 2$

$-3x^2 + 12x + 5 = 0$

$(-3x^2+12x) + 5 = 0$
$-3(x^2-4x) + 5 = 0$
$-3(x^2-2x-2x) + 5 = 0$
$-3(x^2-2x-2x+4) + 5 + 12 = 0$
$-3(x-2)^2 + 17 = 0$
$-3(x-2)^2 = -17$
$(x-2)^2 = \frac{17}{3}$

$\frac{1}{2}x^2 - 3x + 5 = 0$

$(\frac{1}{2}x^2 - 3x) + 5 = 0$
$\frac{1}{2}(x^2-6x) + 5 = 0$
$\frac{1}{2}(x^2-3x-3x) + 5 = 0$
$\frac{1}{2}(x^2-3x-3x+9) + 5 - \frac{9}{2} = 0$
$\frac{1}{2}(x-3)^2 + \frac{1}{2} = 0$
$\frac{1}{2}(x-3)^2 = -\frac{1}{2}$
$\cdot 2 \quad \cdot 2$
$(x-3)^2 = -1$

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Pg 240 #3: Write each equation in the form $a(x-p)^2+q=0$

$x^2-12x+9=0$

$(x^2-12x)+9=0$

$(x^2-6x-6x)+9=0$

$(x^2-6x-6x+36)+9-36=0$

$(x+6)^2-27=0$

$5x^2-20x-1=0$

$(5x^2-20x)-1=0$

$5(x^2-4x)-1=0$

$5(x^2-2x-2x)-1=0$

$5(x^2-2x-2x+4)-1-20=0$

$5(x-2)^2-21=0$

$-2x^2+x-1=0$

$(-2x^2+1x)-1=0$

$-2(x^2-\frac{1}{2}x)-1=0$

$-2(x^2-\frac{1}{4}x-\frac{1}{4}x)-1=0$

$-2(x^2-\frac{1}{4}x-\frac{1}{4}x+\frac{1}{16})-1+\frac{1}{8}=0$

$-2(x-\frac{1}{4})^2-\frac{7}{8}=0$

$0.5x^2+2.1x+3.6=0$

$(0.5x^2+2.1x)+3.6=0$

$0.5(x^2+4.2x)+3.6=0$

$0.5(x^2+2.1x+2.1x)+3.6=0$

$0.5(x^2+2.1x+2.1x+4.41)+3.6-2.205=0$

$0.5(x+2.1)^2+1.395=0$

$-1.2x^2-5.1x-7.4=0$

$(-1.2x^2-5.1x)-7.4=0$

$-1.2(x^2+4.25x)-7.4=0$

$-1.2(x^2+2.125x+2.125x)-7.4=0$

$-1.2(x^2+2.125x+2.125x+4.515625)-7.4+5.415=0$

$-1.2(x+2.125)^2-1.98125=0$

$\frac{1}{2}x^2+3x-6=0$

$(\frac{1}{2}x^2+3x)-6=0$

$\frac{1}{2}(x^2+6x)-6=0$

$\frac{1}{2}(x^2+3x+3x)-6=0$

$\frac{1}{2}(x^2+3x+3x+9)-6-\frac{9}{2}=0$

$\frac{1}{2}(x+3)^2-\frac{21}{2}=0$

Pg 240 #4ab: Solve each quadratic equation. Express your answers as exact roots.

$x^2=64$

$\sqrt{x^2}=\sqrt{64}$

$x=\pm 8$

$2x^2-8=0$

$+8 \quad +8$

$2x^2=8$

$\div 2 \quad \div 2$

$x^2=4$

$\sqrt{x^2}=\sqrt{4}$

$x=\pm 2$

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Pg 240 #4cd: Solve each quadratic equation. Express your answers as exact roots.

$\frac{1}{3}t^2 - 1 = 11$

$+1 \quad +1$

$\frac{1}{3}t^2 = 12$

$\cdot 3 \quad \cdot 3$

$t^2 = 36$

$\sqrt{t^2} = \sqrt{36}$

$t = \pm 6$

BEDMAS

$-y^2 + 5 = -6$

$-5 \quad -5$

$-1y^2 = -11$

$\div(-1) \quad \div(-1)$

$y^2 = 11$

$y = \pm\sqrt{11}$

BEDMAS

Pg 240 #6: Solve each quadratic equation by completing the square. Express your answers as exact roots.

$x^2 + 10x + 4 = 0$

$(x^2+10x) + 4 = 0$

$(x^2+5x+5x) + 4 = 0$

$(x^2+5x+5x+25) + 4 - 25 = 0$

$(x+5)^2 - 21 = 0$

$+21 \quad +21$

$(x+5)^2 = 21$

$x+5 = \pm\sqrt{21}$

$x_1 = -5+\sqrt{21} \qquad x_2 = -5-\sqrt{21}$

$x^2 - 8x + 13 = 0$

$(x^2-8x) + 13 = 0$

$(x^2-4x-4x) + 13 = 0$

$(x^2-4x-4x+16) + 13 - 16 = 0$

$(x-4)^2 - 3 = 0$

$(x-4)^2 = 3$

$x-4 = \pm\sqrt{3}$

$x_1 = 4+\sqrt{3} \qquad x_2 = 4-\sqrt{3}$

$3x^2 + 6x + 1 = 0$

$(3x^2+6x) + 1 = 0$

$3(x^2+2x) + 1 = 0$

$3(x^2+1x+1x+1) + 1 - 3 = 0$

$3(x+1)^2 - 2 = 0$

$3(x+1)^2 = 2$

$(x+1)^2 = \frac{2}{3}$

$x+1 = \sqrt{2/3}$

$x_1 = -1+\sqrt{2/3} \qquad x_2 = -1-\sqrt{2/3}$

$-2x^2 + 4x + 3 = 0$

$(-2x^2+4x) + 3 = 0$

$-2(x^2-2x) + 3 = 0$

$-2(x^2-1x-1x) + 3 = 0$

$-2(x^2-1x-1x+1) + 3 + 2 = 0$

$-2(x-1)^2 + 5 = 0$

$-2(x-1)^2 = -5$

$(x-1)^2 = 5/2$

$x-1 = \sqrt{5/2}$

$x = +1 \pm \sqrt{5/2}$

$-0.1x^2 - 0.6x + 0.4 = 0$

$(-0.1x^2-0.6x) + 0.4 = 0$

$-0.1(x^2+6x) + 0.4 = 0$

$-0.1(x^2+3x+3x) + 0.4 = 0$

$-0.1(x^2+3x+3x+9) + 0.4 + 0.9 = 0$

$-0.1(x+3)^2 + 1.3 = 0$

$-0.1(x+3)^2 = -1.3$

$(x+3)^2 = 13$

$x+3 = \sqrt{13}$

$x = -3 \pm \sqrt{13}$

$0.5x^2 - 4x - 6 = 0$

$(0.5x^2-4x) - 6 = 0$

$0.5(x^2-8x) - 6 = 0$

$0.5(x^2-4x-4x) - 6 = 0$

$0.5(x^2-4x-4x+16) - 6 - 8 = 0$

$0.5(x-4)^2 - 14 = 0$

$0.5(x-4)^2 = 14$

$(x-4)^2 = 28$

$x-4 = \sqrt{28}$

$x = 4 \pm \sqrt{28}$

$x = 4 \pm 2\sqrt{7}$

28 → 2, 14 → 2, 7

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KEY

Pg 240 #7: Solve each quadratic equation by completing the square. Express your to the nearest tenth.

$x^2 - 8x - 4 = 0$

$(x^2-8x) - 4 = 0$

$(x^2-4x-4x) - 4 = 0$

$(x^2-4x-4x+16) - 4 - 16 = 0$

$(x-4)^2 - 20 = 0$

$(x-4)^2 = 20$

$x-4 = \sqrt{20}$

$x = 4 \pm \sqrt{20}$

$x_1 = 4+\sqrt{20} = 8.5$ $\quad x_2 = 4-\sqrt{20} = -0.5$

$-3x^2 + 4x + 5 = 0$

$(-3x^2+4x) + 5 = 0$

$-3(x^2 - \frac{4}{3}x) + 5 = 0$

$-3(x^2 - \frac{2}{3}x - \frac{2}{3}x) + 5 = 0$

$-3(x^2 - \frac{2}{3}x - \frac{2}{3}x + \frac{4}{9}) + 5 + \frac{4}{3} = 0$

$-3(x-\frac{2}{3})^2 + \frac{19}{3} = 0$

$-3(x-\frac{2}{3})^2 = -\frac{19}{3}$

$(x-\frac{2}{3})^2 = +19$

$x - \frac{2}{3} = \sqrt{19}$

$x = \frac{2}{3} \pm \sqrt{19}$

$x_1 = \frac{2}{3} + \sqrt{19} = 2.1$ $\quad x_2 = \frac{2}{3} - \sqrt{19} = -0.8$

$\frac{1}{2}x^2 - 6x - 5 = 0$

$(\frac{1}{2}x^2 - 6x) - 5 = 0$

$\frac{1}{2}(x^2 - 12x) - 5 = 0$

$\frac{1}{2}(x^2 - 6x - 6x) - 5 = 0$

$\frac{1}{2}(x^2 - 6x - 6x + 36) - 5 - 18 = 0$

$\frac{1}{2}(x-6)^2 - 23 = 0$

$\frac{1}{2}(x-6)^2 = 23$

$(x-6)^2 = 46$

$x - 6 = \pm\sqrt{46}$

$x = 6 \pm \sqrt{46}$

$x_1 = 6+\sqrt{46} = 12.8$ $\quad x_2 = 6-\sqrt{46} = -0.8$

$0.2x^2 + 0.12x - 11 = 0$

$(0.2x^2 + 0.12x) - 11 = 0$

$0.2(x^2 + 0.6x) - 11 = 0$

$0.2(x^2 + 0.3x + 0.3x) - 11 = 0$

$0.2(x^2 + 0.3x + 0.3x + 0.09) - 11 - 0.018 = 0$

$0.2(x+0.3)^2 - 11.018 = 0$

$0.2(x+0.3)^2 = 11.018$

$(x+0.3)^2 = 55.09$

$x + 0.3 = \pm\sqrt{55.09}$

$x_1 = -0.3 + \sqrt{55.09} = 7.1$ $\quad x_2 = -0.3 - \sqrt{55.09} = -7.7$

$-\frac{2}{3}x^2 - x + 2 = 0$

$(-\frac{2}{3}x^2 - x) + 2 = 0$

$-\frac{2}{3}(x^2 + \frac{3}{2}x) + 2 = 0$

$-\frac{2}{3}(x^2 + \frac{3}{4}x + \frac{3}{4}x) + 2 = 0$

$-\frac{2}{3}(x^2 + \frac{3}{4}x + \frac{3}{4}x + \frac{9}{16}) + 2 + \frac{3}{8} = 0$

$-\frac{2}{3}(x+\frac{3}{4})^2 + \frac{19}{8} = 0$

$-\frac{2}{3}(x+\frac{3}{4})^2 = -\frac{19}{8}$

$(x+\frac{3}{4})^2 = \frac{57}{16}$

$x + \frac{3}{4} = \pm\sqrt{\frac{57}{16}}$

$x_1 = -\frac{3}{4} + \sqrt{\frac{57}{16}}$ $\quad x_2 = -\frac{3}{4} - \sqrt{\frac{57}{16}}$

$x_1 = -2.6$ $\quad x_2 = +1.1$

$\frac{3}{4}x^2 + 6x + 1 = 0$

$(\frac{3}{4}x^2 + 6x) + 1 = 0$

$\frac{3}{4}(x^2 + 8x) + 1 = 0$

$\frac{3}{4}(x^2 + 4x + 4x) + 1 = 0$

$\frac{3}{4}(x^2 + 4x + 4x + 16) + 1 - 12 = 0$

$\frac{3}{4}(x+4)^2 - 11 = 0$

$\frac{3}{4}(x+4)^2 = 11$

$(x+4)^2 = \frac{44}{3}$

$x + 4 = \pm\sqrt{\frac{44}{3}}$

$x_1 = -4 + \sqrt{\frac{44}{3}}$ $\quad x_2 = -4 - \sqrt{\frac{44}{3}}$

$x_1 = -0.2$ $\quad x_2 = -7.8$

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Pg 240 #9: Evan passes a flying disc to a teammate during a competition at the Flatland Ultimate and Cups Tournament in Winnipeg. The flying disc follows the path $h(t) = -0.02d^2 + 0.4d + 1$, where h is the height, in meters, and d is the horizontal distance, in meters, that the flying disc has travelled from the thrower. If no one catches the flying disc, the height of the disc above the ground when it lands can be modelled by $h(d) = 0$.

a. What quadratic equation can you use to determine how far the disc will travel if no one catches it?
b. How far will the disc travel if no one catches it? Express your answer to the nearest tenth of a meter.

(A) $h(t) = -0.02d^2 + 0.4d + 1$

$0 = -0.02d^2 + 0.4d + 1$

(B) Option #1: Complete the square

$(-0.02d^2 + 0.4d) + 1 = 0$

$-0.02(d^2 - 20d) + 1 = 0$

$-0.02(d^2 - 10d - 10d) + 1 = 0$

$-0.02(d^2 - 10d - 10d + 100) + 1 + 2 = 0$

$-0.02(d-10)^2 + 3 = 0$

$-0.02(d-10)^2 = -3$

$(d-10)^2 = 150$

$d - 10 = \pm\sqrt{150}$

$d_1 = 10 + \sqrt{150}$ $\quad d_2 = 10 - \sqrt{150}$

$d_1 = 22.2$ $\quad d_2 = -2.2$

Doesn't make sense in context of problem.

So $d = 22.2$m

Option #2: Quadratic Formula

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-0.4 \pm \sqrt{0.16 + 0.08}}{2(-0.02)}$

$= \dfrac{-0.4 \pm \sqrt{0.24}}{-0.04}$

$x_1 = \dfrac{-0.4 + \sqrt{0.24}}{-0.04}$ $\quad x_2 = \dfrac{-0.4 - \sqrt{0.24}}{-0.04}$

$x_1 = -2.2$ $\quad x_2 = 22.2$

So $d = 22.2$m

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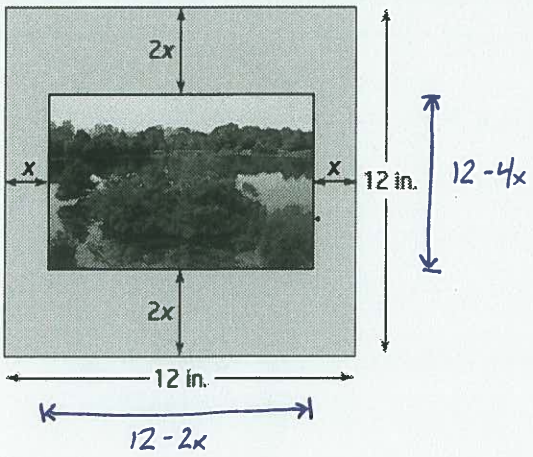
Pg 240 #11: Brian is placing a photograph behind a 12-in. by 12-in. piece of matting. He positions the photograph so that the matting is twice as wide at the top and bottom as it is at the sides.

The visible area of the photograph is 54 sq. in. What are the dimensions of the photograph?



$A = (L)(w)$

$54 = (12-4x)(12-2x)$

$54 = 144 - 72x + 8x^2$

$0 = 8x^2 - 72x + 90$

$8x^2 - 72x + 90 = 0$

$(8x^2 - 72x) + 90 = 0$

$8(x^2 - 9x) + 90 = 0$

$8\left(x^2 - \frac{9}{2}x - \frac{9}{2}x\right) + 90 = 0$

$8\left(x^2 - \frac{9}{2}x - \frac{9}{2}x + \frac{81}{4}\right) + 90 - 162 = 0$

$8\left(x - \frac{9}{2}\right)^2 - 72 = 0$

$8\left(x - \frac{9}{2}\right)^2 = 72$

$\left(x - \frac{9}{2}\right)^2 = 9$

$x - \frac{9}{2} = \pm\sqrt{9}$

$x_1 = \frac{9}{2} + 3$

$x_1 = \frac{15}{2}$

$x_1 = 7.5\text{in}$

Results in negative dimensions.

$x_2 = \frac{9}{2} - 3$

$x_2 = \frac{3}{2}$

$x_2 = 1.5\text{cm}$

Pg 240 #15: Determine the roots of $ax^2 + bx + c = 0$ by completing the square. Can you use this result to solve any quadratic equation? Explain.

$ax^2 + bx + c = 0$

$(ax^2 + bx) + c = 0$

$a\left(x^2 + \frac{b}{a}x\right) + c = 0$

$a\left(x^2 + \frac{b}{2a}x + \frac{b}{2a}x\right) + c = 0$

$a\left(x^2 + \frac{b}{2a}x + \frac{b}{2a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a} = 0$

$a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} = 0$

$a\left(x + \frac{b}{2a}\right)^2 + \frac{c}{1}\left(\frac{4a}{4a}\right) - \frac{b^2}{4a} = 0$

$a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac}{4a} - \frac{b^2}{4a} = 0$

$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

$\left(x + \frac{b}{2a}\right) = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

$\left(x + \frac{b}{2a}\right) = \frac{\pm\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$

$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$

$x = -\frac{b}{2a} + \frac{\pm\sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Oooo... it's the quadratic formula! This only gives real roots (i.e. the quadratic crosses the x-axis) if the stuff under the root is positive so $b^2 - 4ac \geq 0$.