Math 20-1

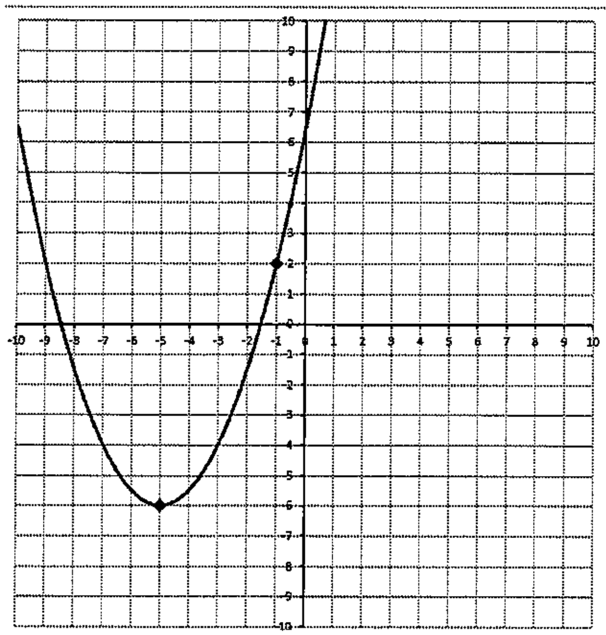
KEY

**L19 – Worksheet – 3.1-3.3 and 4.1-4.3** **/40 marks**

## *3.1 – Vertex Form (L11)*

*Use the following information to answer Q1-Q2:*

The function, f(x), is graphed below:



**Q1:** The equation of the function can be written in Vertex Form as $f(x) = \frac{a}{b}(x + c)^2 - d$, where ***a, b, c,*** and ***d*** are ___, ___, ___, and ___.

*(Record your answer in the Numerical Response boxes below)*

| 1 | 2 | 5 | 6 |
|---|---|---|---|

$f(x) = a(x-h)^2 + k$

$f(x) = a(x+5)^2 - 6$ Use $(-1, 2)$

$2 = a(-1+5)^2 - 6$

$2 = a(4)^2 - 6$

$+6 \qquad +6$

$8 = a(16)$

$\div 16 \qquad \div 16$

$\frac{1}{2} = a$

$f(x) = \frac{1}{2}(x+5)^2 - 6$

$a = 1$ $c = 5$ $d = 6$

$b = 2$

**Q2:** Determine the ***exact*** values of the x-intercepts. *(2 marks)*

$0 = \frac{1}{2}(x+5)^2 - 6$

$+6 \qquad +6$

$6 = \frac{1}{2}(x+5)^2$

$\div(\frac{1}{2}) \quad \div(\frac{1}{2})$

$12 = (x+5)^2$

$\sqrt{12} = x + 5$

$+\sqrt{12} = x + 5$

$-5 \qquad -5$

$\boxed{\sqrt{12} - 5 = x_1}$

or

$\boxed{x_1 = 2\sqrt{3} - 5}$

$-\sqrt{12} = x + 5$

$-5 \qquad -5$

$\boxed{-\sqrt{12} - 5 = x_2}$

or

$\boxed{x_2 = -2\sqrt{3} - 5}$

KEY

*Use the following information to answer Q3-Q5:*

$$g(x) = -2(x-3)^2 + 8$$

**Q3:** The vertex of the function is located at

a. (3,8)
b. (-3,8)
c. (3,-8)
d. (-3,-8)

$g(x) = a(x-h)^2 + k$

Vertex at $(3,8)$

**Q4:** Algebraically determine the location of the zeroes. *(2 marks)*

$0 = -2(x-3)^2 + 8$

$-8 \qquad -8$

$-8 = -2(x-3)^2$

$\div(-2) \qquad \div(-2)$

$4 = (x-3)^2$

$\sqrt{4} = x-3$

$+2 = x-3$

$+3 \qquad +3$

$\boxed{5 = x_1}$

$-2 = x-3$

$+3 \qquad +3$

$\boxed{1 = x_2}$

**Q5:** The y-intercept has a value of

a. -10
b. -8
c. 8
d. 10

$g(x) = -2(x-3)(x-3) + 8$

$= -2(x^2 - 6x + 9) + 8$

$= -2x^2 + 12x - 18 + 8$

$= -2x^2 + 12x - 10$

y-int.

KEY

## 3.2 – Standard Form (L10)

*Use the following information to answer Q6-Q9:*

$$g(x) = 3x^2 - 2x - 8$$

y-int

**Q6:** Determine the value of the y-intercept. *(1 mark)*

$\boxed{y = -8}$

**Q7:** Algebraically determine the values of the zeroes by factoring. *(2 marks)*

$0 = 3x^2 - 2x - 8$

$+4 \quad -6$

$\square + \square = -2$

$\square \times \square = -24$

1, 24
2, 12
3, 8
4, 6

$0 = 3x^2 + 4x - 6x - 8$

$0 = (3x^2 + 4x) + (-6x - 8)$

$0 = x(3x+4) - 2(3x+4)$

$0 = (3x+4)(x-2)$

$3x + 4 = 0$ $\quad$ $x - 2 = 0$

$\boxed{x = -4/3}$ $\quad$ $\boxed{x = 2}$

**Q8:** Determine the coordinates of the Vertex by using the Zeroes. *(2 marks)*

$\dfrac{(-4/3) + (2)}{2} = \dfrac{1}{3} \Rightarrow$ Vertex occurs at $x = 1/3$

$g(1/3) = 3(1/3)^2 - 2(1/3) - 8$

$= \frac{1}{3} - \frac{2}{3} - 8$

$= -\frac{25}{3}$

Vertex occurs at $(1/3, -25/3)$

**Q9:** Determine the coordinates of the Vertex by converting to Vertex Form. *(2 marks)*

$g(x) = (3x^2 - 2x) - 8$

$= 3(x^2 - 2/3x) - 8$

$= 3(x^2 - 1/3x - 1/3x) - 8$

$= 3(x^2 - 1/3x - 1/3x + 1/9) - 8 - 1/3$

$= 3(x - \frac{1}{3})^2 - \frac{25}{3}$

Vertex occurs at $(\frac{1}{3}, -\frac{25}{3})$

KEY

## 3.3 – Completing the Square (L12)

*Use the following to answer Q10-Q12:*

$$f(x) = -x^2 - 2x + 6$$

y-int = 6

**Q10:** Convert the function to Vertex Form. *(2 marks)*

$f(x) = (-x^2 - 2x) + 6$

$= -1(x^2 + 2x) + 6$

$= -1(x^2 + 1x + 1x) + 6$

$= -1(x^2 + 1x + 1x + 1) + 6 + 1$

$= -1(x+1)^2 + 7$

Vertex is $(-1, 7)$

Axis of symmetry at $x = -1$

**Q11:** Complete the table below, using the method of your choice. *(1/2 mark each; 2 marks total)*

| Vertex Coordinates | Y-Intercept | Zeroes (Decimal Approximation) | Equation for Axis of Symmetry |
|---|---|---|---|
| $(-1, 7)$ | $+6$ | Can use calculator. I used algebra (below). $x_1 = 1.65$ $x_2 = -3.65$ | $x = -1$ |

$0 = -1(x+1)^2 + 7$

$-7 \qquad -7$

$-7 = -1(x+1)^2$

$\div(-1) \quad \div(-1)$

$7 = (x+1)^2$

$\sqrt{7} = x + 1$

$+2.65 = x + 1$

$-1 \qquad -1$

$\boxed{1.65 = x_1}$

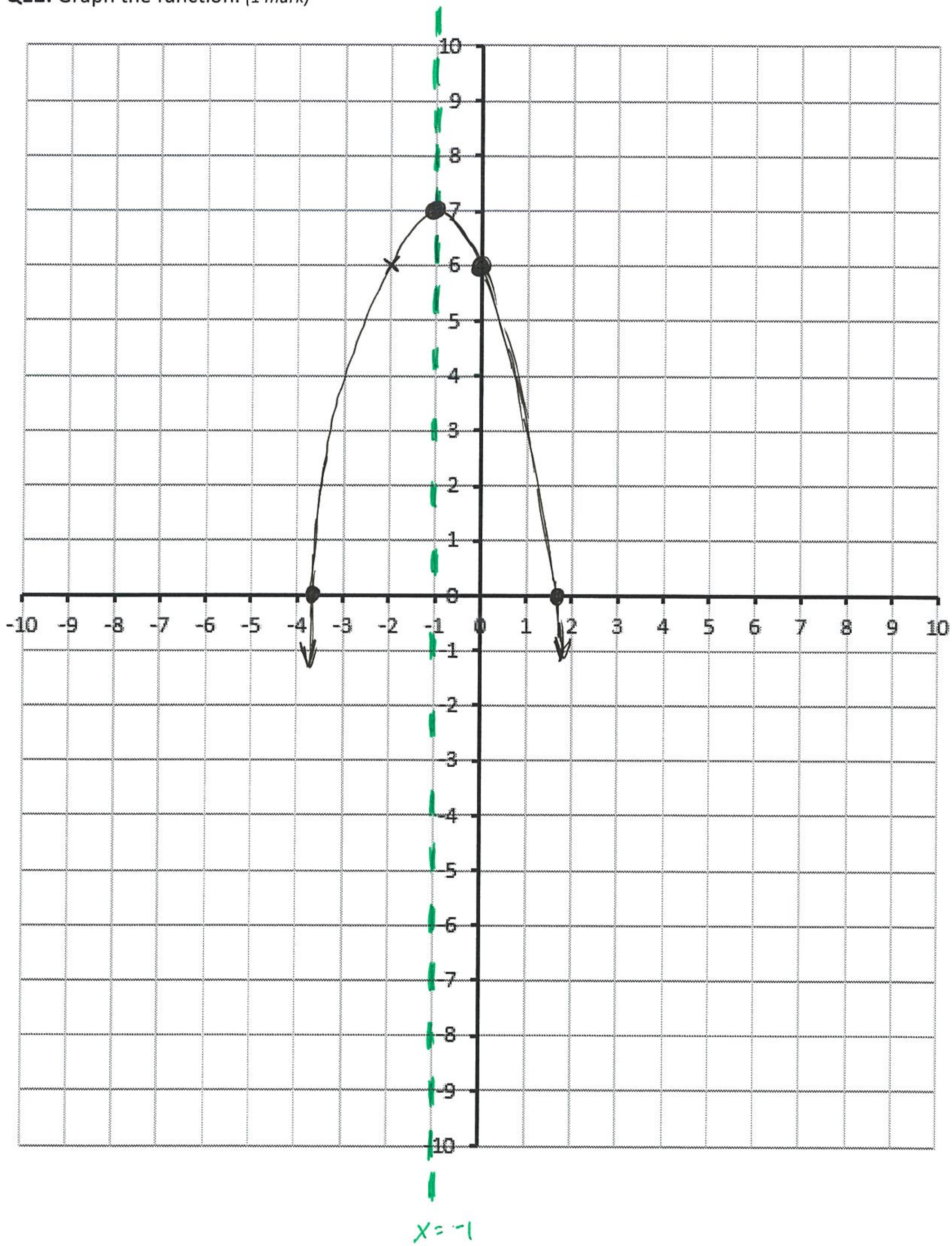
$-2.65 = x + 1$

$-1 \qquad -1$

$\boxed{-3.65 = x_2}$

KEY

**Q12:** Graph the function. *(1 mark)*



KEY

**Q13:** Convert $y = 2x^2 + 6x - 5$ to Vertex Form. *(2 marks)*

$$y = (2x^2 + 6x) - 5$$
$$y = 2(x^2 + 3x) - 5$$
$$y = 2(x^2 + 3/2x + 3/2x) - 5$$
$$y = 2(x^2 + \tfrac{3}{2}x + \tfrac{3}{2}x + \tfrac{9}{4}) - 5 - \tfrac{9}{2}$$
$$y = 2(x + \tfrac{3}{2})^2 - \tfrac{19}{2}$$

**Q14:** Convert $f(x) = -\frac{1}{4}x^2 + 4x - 10$ to Vertex Form. *(2 marks)*

$$y = (-\tfrac{1}{4}x^2 + 4x) - 10$$
$$y = -\tfrac{1}{4}(x^2 - 16x) - 10$$
$$y = -\tfrac{1}{4}(x^2 - 8x - 8x) - 10$$
$$y = -\tfrac{1}{4}(x^2 - 8x - 8x + 64) - 10 + 16$$
$$y = -\tfrac{1}{4}(x - 8)^2 + 6$$

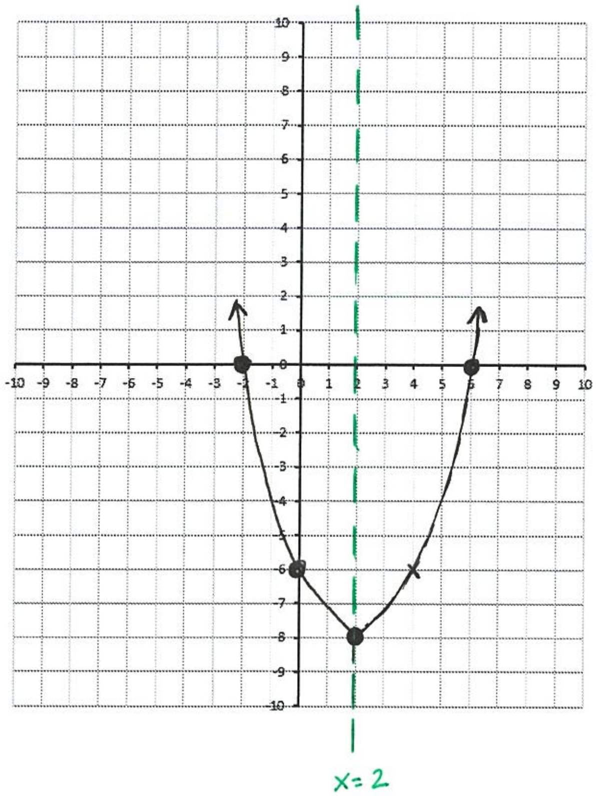
KEY

## 4.1 – Graphical Solutions of Quadratic Equations (L16)

**Q15:** Solve the equation $\frac{1}{2}x^2 = 2x + 6$ by graphing. Label all relevant points. State the solution clearly.

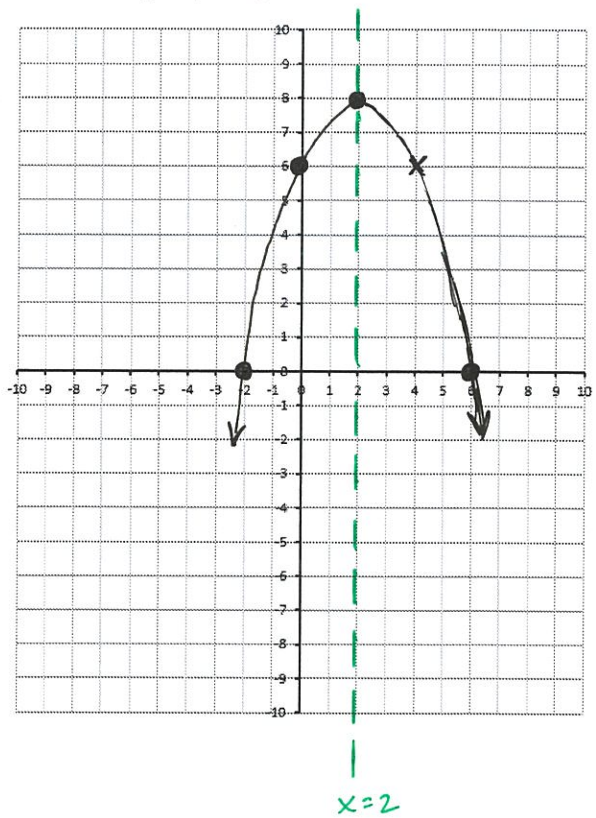
*(2 marks)*

If solved as $\frac{1}{2}x^2 - 2x - 6 = 0$
then graph $y = \frac{1}{2}x^2 - 2x - 6$




Soln is $x = -2, +6$

If solved as $0 = -\frac{1}{2}x^2 + 2x + 6$
then graph $y = -\frac{1}{2}x^2 + 2x + 6$




Soln is $x = -2, +6$

KEY

## 4.2 – Factoring Quadratic Equations (L17)

**Q16:** Solve $\frac{1}{2}x^2 - 2x = 6$ using factoring. *(2 marks)*

$-6 \quad -6$

$\frac{1}{2}x^2 - 2x - 6 = 0$

$\frac{1}{2}(x^2 - 4x - 12) = 0$

$\frac{1}{2}(x+2)(x-6) = 0$

$+2 \quad -6$

$\square + \square = -4$

$\square \times \square = -12$

1, 12

$\boxed{2, 6}$

3, 4

$x + 2 = 0$

$-2 \quad -2$

$\boxed{x = -2}$

$x - 6 = 0$

$+6 \quad +6$

$\boxed{x = 6}$

**Q17:** Solve $3x^2 + 30x + 72 = 0$ using factoring. *(2 marks)*

$3(x^2 + 10x + 24) = 0$

$3(x+4)(x+6) = 0$

$+4 \quad +6$

$\square + \square = 10$

$\square \times \square = 24$

1, 24

2, 12

3, 8

$\boxed{4, 6}$

$x + 4 = 0$

$-4 \quad -4$

$\boxed{x = -4}$

$x + 6 = 0$

$-6 \quad -6$

$\boxed{x = -6}$

**Q18:** Solve $\frac{1}{4}x^2 = \frac{1}{2}x + \frac{35}{4}$ using factoring. *(2 marks)*

$\frac{1}{4}x^2 - \frac{1}{2}x - \frac{35}{4} = 0$

$\frac{1}{4}(x^2 - 2x - 35) = 0$

$\frac{1}{4}(x+5)(x-7) = 0$

$+5 \quad -7$

$\square + \square = -2$

$\square \times \square = -35$

1, 35

$\boxed{5, 7}$

$x + 5 = 0$

$-5 \quad -5$

$\boxed{x = -5}$

$x - 7 = 0$

$+7 \quad +7$

$\boxed{x = 7}$

## 4.3 – Solving Quadratic Equations by Completing the Square (L18)

**Q19:** Solve $\frac{1}{2}x^2 + 2x - 5 = 0$ by completing the square. Give your answer as an exact value. *(3 marks)*

$$\left(\tfrac{1}{2}x^2+2x\right)-5=0$$
$$\tfrac{1}{2}(x^2+4x)-5=0$$
$$\tfrac{1}{2}(x^2+2x+2x)-5=0$$
$$\tfrac{1}{2}(x^2+2x+2x+4)-5-2=0$$
$$\boxed{\tfrac{1}{2}(x+2)^2-7=0}$$ Vertex Form

$+7 \quad +7$

$$\tfrac{1}{2}(x+2)^2=7$$

$\div(\tfrac{1}{2}) \quad \div(\tfrac{1}{2})$

$$(x+2)^2=14$$
$$(x+2)=\sqrt{14}$$

$x+2=+\sqrt{14}$ or $x+2=-\sqrt{14}$

$-2 \quad -2$

$\boxed{x_1=+\sqrt{14}-2}$ $\quad$ $\boxed{x_2=-\sqrt{14}-2}$

**Q20:** Solve $6 = 2x^2 + 5x$ by completing the square. Give your answer as an exact value. *(3 marks)*

$$0=2x^2+5x-6$$
$$0=(2x^2+5x)-6$$
$$0=2(x^2+5/2x)-6$$
$$0=2(x^2+\tfrac{5}{4}x+\tfrac{5}{4}x)-6$$
$$0=2\left(x^2+\tfrac{5}{4}x+\tfrac{5}{4}x+\tfrac{25}{16}\right)-6-\tfrac{25}{8}$$
$$\boxed{0=2\left(x+\tfrac{5}{4}\right)^2-\tfrac{73}{8}}$$ Vertex Form

$+\frac{73}{8} \quad +\frac{73}{8}$

$$\tfrac{73}{8}=2(x+5/4)^2$$

$\div 2 \quad \div 2$

$$\tfrac{73}{16}=\left(x+\tfrac{5}{4}\right)^2$$

$$\sqrt{\tfrac{73}{16}}=x+\tfrac{5}{4}$$

$+\frac{\sqrt{73}}{4}=x+\frac{5}{4}$ $\quad$ $-\frac{\sqrt{73}}{4}=x+\frac{5}{4}$

$-\frac{5}{4} \quad -\frac{5}{4}$

$\boxed{\frac{\sqrt{73}-5}{4}=x_1}$ $\quad$ $\boxed{\frac{-\sqrt{73}-5}{4}=x_2}$

**Q21:** Solve $\frac{1}{3}x^2 + 2x = 4$ by completing the square. Give your answer as an exact value. *(3 marks)*

$-4 \quad -4$

$$\tfrac{1}{3}x^2+2x-4=0$$
$$\left(\tfrac{1}{3}x^2+2x\right)-4=0$$
$$\tfrac{1}{3}(x^2+6x)-4=0$$
$$\tfrac{1}{3}(x^2+3x+3x)-4=0$$
$$\tfrac{1}{3}(x^2+3x+3x+9)-4-3=0$$
$$\boxed{\tfrac{1}{3}(x+3)^2-7=0}$$ Vertex Form

$+7 \quad +7$

$$\tfrac{1}{3}(x+3)^2=7$$

$\div(1/3) \quad \div(1/3)$

$$(x+3)^2=21$$

$$x+3=\sqrt{21}$$

$x+3=+\sqrt{21}$ $\quad$ $x+3=-\sqrt{21}$

$-3 \quad -3$

$\boxed{x_1=+\sqrt{21}-3}$ $\quad$ $\boxed{x_2=-\sqrt{21}-3}$