KEY

L21 – Worksheet – Solving Quadratic Equations   /24 marks

Part 1a – Finding Zeroes in Standard Form using Factoring

Q1: With the equation $f(x) = 2x^2 + 4x - 6$, the zeroes are located at $x_1 = -a$ and $x_2 = b$, where a and b are ___ and ___.

*(Record your **two-digit** answer in the Numerical Response boxes below)*

| 3 | 1 | | |

$0 = 2(x^2 + 2x - 3)$
$0 = 2(x+3)(x-1)$

$x+3 = 0 \qquad x-1 = 0$
$x = -3 \qquad\quad x = +1$

Q2: With the equation $y = 2x^2 - 3x - 9$, the zeroes are located at $x_1 = -\dfrac{a}{b}$ and $x_2 = c$, where a, b, and c are ___, ___, and ___.

*(Record your **three-digit** answer in the Numerical Response boxes below)*

| 3 | 2 | 3 | |

$0 = 2x^2 - 3x - 9$
$0 = 2x^2 + 3x - 6x - 9$
$0 = (2x^2 + 3x) + (-6x - 9)$
$0 = x(2x+3) - 3(2x+3)$
$0 = (2x+3)(x-3)$

$2x+3 = 0 \qquad\qquad x-3 = 0$
$x = -\frac{3}{2} \qquad\qquad\quad x = 3$

$+3 \quad -6$
$\square + \square = -3$
$\square \times \square = -18$

$1, 18$
$2, 9$
$\underline{3, 6}$

Part 1b – Solving Equations in Standard Form using Factoring

Q3: Solve $x^2 = 2x + 15$ using factoring. *(1 mark)*

$-2x \qquad -2x$
$x^2 - 2x = 15$
$-15 \qquad -15$
$x^2 - 2x - 15 = 0$
$(x-5)(x+3) = 0$

$x-5 = 0 \qquad\qquad x+3 = 0$
$\boxed{x = 5} \qquad\qquad \boxed{x = -3}$

Q4: Solve $x^2 + x = 6$ using factoring. *(1 mark)*

$-6 \quad -6$
$x^2 + x - 6 = 0$
$(x+3)(x-2) = 0$

$x+3 = 0 \qquad\qquad x-2 = 0$
$\boxed{x = -3} \qquad\qquad \boxed{x = 2}$

KEY

Part 2a – Finding Zeroes in Vertex Form

Q5: Determine the zeroes of the function $f(x) = (x-2)^2 - 6$ algebraically. *(1 mark)*

$$0 = (x-2)^2 - 6$$
$$+6 \qquad\qquad +6$$
$$6 = (x-2)^2$$
$$\sqrt{6} = x-2$$

$+\sqrt{6} = x-2$
$+2 \qquad +2$

$$\boxed{\sqrt{6} + 2 = x_1}$$

$-\sqrt{6} = x-2$
$+2 \qquad +2$

$$\boxed{-\sqrt{6} + 2 = x_2}$$

Q6: Convert the equation $y = -\frac{1}{2}x^2 + 2x + 1$ to vertex form, and determine the zeroes algebraically. *(2 marks)*

$$y = \left(-\frac{1}{2}x^2 + 2x\right) + 1$$
$$= -\frac{1}{2}(x^2 - 4x) + 1$$
$$= -\frac{1}{2}(x^2 - 2x - 2x) + 1$$
$$= -\frac{1}{2}(x^2 - 2x - 2x + 4) + 1 + 2$$
$$\boxed{y = -\frac{1}{2}(x-2)^2 + 3} \text{ Vertex Form}$$

$$0 = -\frac{1}{2}(x-2)^2 + 3$$
$$-3 \qquad\qquad\qquad -3$$
$$-3 = -\frac{1}{2}(x-2)^2$$
$$\div\left(-\frac{1}{2}\right) \quad \div\left(-\frac{1}{2}\right)$$
$$6 = (x-2)^2$$
$$\sqrt{6} = x-2$$

$+\sqrt{6} = x-2$
$+2 \qquad +2$

$$\boxed{\sqrt{6} + 2 = x_1}$$

$-\sqrt{6} = x-2$
$+2 \qquad +2$

$$\boxed{-\sqrt{6} + 2 = x_2}$$

KEY

Part 2b – Solving Equations by ~~Converting to Vertex Form~~ Completing the Square

Q7: Solve $2x + 5 = x^2$ by completing the square. *(2 marks)*

$$-2x \qquad\qquad -2x$$
$$5 = x^2 - 2x$$
$$-5 \qquad\qquad -5$$
$$0 = x^2 - 2x - 5$$
$$0 = (x^2 - 2x) - 5$$
$$0 = (x^2 - 1x - 1x) - 5$$
$$0 = (x^2 - 1x - 1x + 1) - 5 - 1$$
$$\boxed{0 = (x-1)^2 - 6}$$
$$+6 \qquad\qquad +6$$
$$6 = (x-1)^2$$
$$\sqrt{6} = x - 1$$

$$+\sqrt{6} = x - 1 \qquad\qquad -\sqrt{6} = x - 1$$
$$+1 \qquad +1 \qquad\qquad\qquad +1 \qquad +1$$
$$\boxed{\sqrt{6} + 1 = x_1} \qquad\qquad \boxed{-\sqrt{6} + 1 = x_2}$$

Q8: Solve $\frac{1}{2}x^2 + 3x = 2$ by completing the square. *(2 marks)*

$$-2 \qquad -2$$
$$\tfrac{1}{2}x^2 + 3x - 2 = 0$$
$$\left(\tfrac{1}{2}x^2 + 3x\right) - 2 = 0$$
$$\tfrac{1}{2}\left(x^2 + 6x\right) - 2 = 0$$
$$\tfrac{1}{2}\left(x^2 + 3x + 3x\right) - 2 = 0$$
$$\tfrac{1}{2}\left(x^2 + 3x + 3x + 9\right) - 2 - \tfrac{9}{2} = 0$$
$$\boxed{\tfrac{1}{2}(x+3)^2 - \tfrac{13}{2} = 0}$$
$$+ \tfrac{13}{2} \qquad + \tfrac{13}{2}$$
$$\tfrac{1}{2}(x+3)^2 = \tfrac{13}{2}$$
$$\div(\tfrac{1}{2}) \qquad\qquad \div(\tfrac{1}{2})$$
$$(x+3)^2 = 13$$
$$x + 3 = \sqrt{13}$$

$$x + 3 = +\sqrt{13} \qquad\qquad x + 3 = -\sqrt{13}$$
$$-3 \qquad -3 \qquad\qquad\qquad -3 \qquad -3$$
$$\boxed{x_1 = \sqrt{13} - 3} \qquad\qquad \boxed{x_2 = -\sqrt{13} - 3}$$

KEY

Part 3a – Finding Zeroes in Standard Form using Quadratic Equation

Q9: Determine the zeroes of the function $f(x) = 2x^2 + 4x - 1$ using the Quadratic Equation. Give your answer as an exact value. *(2 marks)*

$y = ax^2 + bx + c$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4(2)(-1)}}{2(2)}$

$x = \dfrac{-4 \pm \sqrt{24}}{4}$

$x = \dfrac{-4 \pm 2\sqrt{6}}{4}$

$\boxed{X_1 = -1 + \tfrac{1}{2}\sqrt{6}}$ $\boxed{X_2 = -1 - \tfrac{1}{2}\sqrt{6}}$

24
② 12
② 6
② ③

$\sqrt{24} = \sqrt{2^2 \cdot 2 \cdot 3} = 2\sqrt{6}$

Q10: Determine the zeroes of the function $f(x) = -3x^2 - 5x + 6$ using the Quadratic Equation. Give your answer as an exact value. *(2 marks)*

$y = ax^2 + bx + c$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{5 \pm \sqrt{(-5)^2 - 4(-3)(6)}}{2(-3)}$

$x = \dfrac{5 \pm \sqrt{25 + 72}}{-6}$

$x = \dfrac{5 \pm \sqrt{97}}{-6}$

$\boxed{X_1 = \dfrac{5 + \sqrt{97}}{-6}}$ $\boxed{X_2 = \dfrac{5 - \sqrt{97}}{-6}}$

Part 3b – Solving Equations in Standard Form using Quadratic Equation

Q11: Solve the equation $2x^2 + 4x = 6$ using the Quadratic Equation. *(2 marks)*

$$-6 \quad -6$$

$$2x^2 + 4x - 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-6)}}{2(2)}$$

$$x = \frac{-4 \pm \sqrt{16 + 48}}{4}$$

$$x = \frac{-4 \pm \sqrt{64}}{4}$$

$$x = \frac{-4 \pm 8}{4}$$

$$x_1 = \frac{-4 + 8}{4} \qquad x_2 = \frac{-4 - 8}{4}$$

$$\boxed{x_1 = 1} \qquad \boxed{x_2 = -3}$$

Q12: Solve the equation $5x^2 = x + 2$ using the Quadratic Equation. *(2 marks)*

$$-x \quad -x$$

$$5x^2 - 1x = 2$$

$$-2 \quad -2$$

$$5x^2 - 1x - 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(-2)}}{2(5)}$$

$$x = \frac{1 \pm \sqrt{1 + 40}}{10}$$

$$x = \frac{1 \pm \sqrt{41}}{10}$$
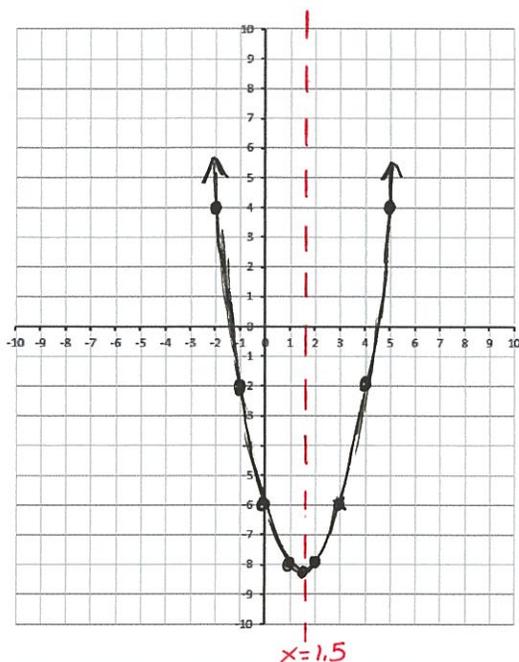
$$\boxed{x_1 = \frac{1 + \sqrt{41}}{10}} \qquad \boxed{x_2 = \frac{1 - \sqrt{41}}{10}}$$

$$\boxed{x_1 \approx 0.74} \qquad \boxed{x_2 \approx -0.54}$$

$\blacksquare$ *Key* $\blacksquare$

Part 4a – Finding Zeroes using Graphing

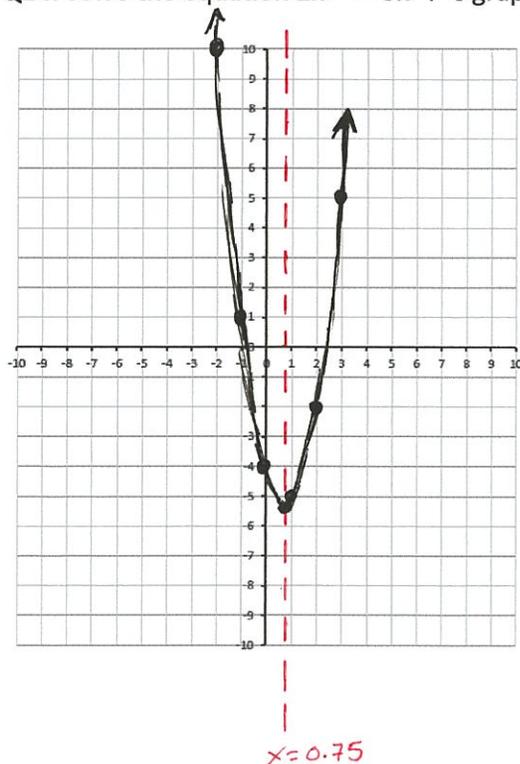**Q13:** Determine the zeroes (approximate value) of the function $y = x^2 - 3x - 6$ by graphing the function. *(2 marks)*

$x = 1.5$

y-intercept $= -6$
Vertex $= (1.50, -8.25)$
Axis of symmetry $x = 1.50$

X	y
-2	4
-1	-2
0	-6
1	-8
2	-8
3	-6
4	-2
5	4

$\boxed{x \doteq -1.4, \; 4.4}$

Part 4b – Solving Equations using Graphing

Q14: Solve the equation $2x^2 = 3x + 4$ graphically. *(3 marks)*

$x = 0.75$

$2x^2 - 3x - 4 = 0$
Graph $y = 2x^2 - 3x - 4$ and look for zeroes.

Vertex at $(0.75, -5.13)$
y-intercept $y = -4$
Axis of symmetry $x = 0.75$

X	y
-2	10
-1	1
0	-4
1	-5
2	-2
3	5
4	16

$\boxed{x \doteq -0.9, \; 2.4}$