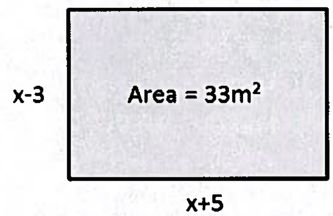
## L22 – Word Problems

### *Area Problems*

*Use the following information to answer Q1:*

A patio has a total area of 33m², and is proportioned per the diagram below.




**Q1:** Set up a quadratic equation and solve to determine the value(s) of ***x***.

$A(x) = (x-3)(x+5)$
$33 = (x-3)(x+5)$
$33 = x^2 + 2x - 15$
$-33 \qquad -33$
$0 = x^2 + 2x - 48$
$0 = (x+8)(x-6)$
$x = -8 \qquad x = +6$

Can't have negative side length, so $\boxed{x = 6}$ is the only value that works.

**Q2:** A slightly smaller patio has the same proportions, but has a total area of 30m². Determine the value of ***x***, to the nearest tenth.

*(Record your answer in the Numerical Response boxes below)*

| 5 | . | 8 | |
|---|---|---|---|

$A(x) = x^2 + 2x - 15$
$30 = x^2 + 2x - 15$
$-30 \qquad -30$
$\boxed{0 = x^2 + 2x - 45}$

Method #1: Complete the Square

$0 = (x^2 + 2x) - 45$
$0 = (x^2 + 1x + 1x) - 45$
$0 = (x^2 + 1x + 1x + 1) - 45 - 1$
$\boxed{0 = (x+1)^2 - 46}$
$+46 \qquad +46$
$46 = (x+1)^2$
$\sqrt{46} = x+1$

$+\sqrt{46} = x+1 \qquad -\sqrt{46} = x+1$
$-1 \quad -1 \qquad -1 \quad -1$
$\sqrt{46} - 1 = x_1 \qquad -\sqrt{46} - 1 = x_2$
$x_1 \approx 5.78 \qquad x_2 \approx -7.78$
$\boxed{\text{So } x = 5.78}$

Method #2: Quadratic Formula

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(-45)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{184}}{2}$

$x_1 \approx \dfrac{-2 + \sqrt{184}}{2} \qquad x_2 \approx \dfrac{-2 - \sqrt{184}}{2}$

$x_1 \approx 5.78 \qquad x_2 \approx -7.78$

$\boxed{\text{So } x = 5.78}$

KEY

Math 20-1

## Projectile Motion

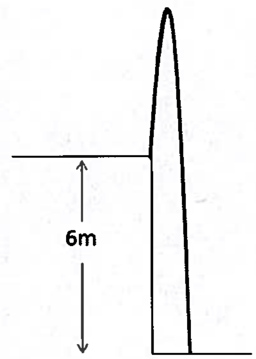
*Use the following information to answer Q3:*

An object is launched vertically at 13m/s off a 6m tall ledge. The *Physics 20* equation that can be used to determine time is given by:

$$\Delta d = v_i t + \frac{1}{2}at^2$$

Where the acceleration due to gravity is approximately -10 m/s². In Math 20-1, we can write this function as follows:

$$h(t) = -5t^2 + 13t + 6$$




**Q3:** How long does it take the object to land? Support your work algebraically.

### *Using Factoring*

$0 = -5t^2 + 13t + 6$

$0 = -1(5t^2 \quad 13t - 6)$

$+2 \quad -15$

$\square + \square = -13$

$\square \times \square = -30$

1,30
2,15
3,10

$0 = -1[5t^2 + 2t - 15t - 6]$

$0 = -1[(5t^2 + 2t) + (-15t - 6)]$

$0 = -1[t(5t+2) - 3(5t+2)]$

$0 = -1(5t+2)(t-3)$

$5t + 2 = 0 \qquad t - 3 = 0$

$t = -2/5 \qquad \boxed{t = 3}$

It takes 3 sec to hit the ground.

### *Using Vertex Form*

$h(t) = (-5t^2 + 13t) + 6$

$= -5(t^2 - \frac{13}{5}t) + 6$

$= -5(t^2 - \frac{13}{10}t - \frac{13}{10}t) + 6$

$= -5(t^2 - \frac{13}{10}t - \frac{13}{10}t + \frac{169}{100}) + 6 + \frac{169}{20}$

$\boxed{h(t) = -5(t - \frac{13}{10})^2 + \frac{289}{20}}$

$0 = -5(t - \frac{13}{10})^2 + \frac{289}{20}$

$-\frac{289}{20} \qquad -\frac{289}{20}$

$-\frac{289}{20} = -5(t - \frac{13}{10})^2$

$\div(-5) \qquad \div(-5)$

$\frac{289}{100} = (t - \frac{13}{10})^2$

$\sqrt{\frac{289}{100}} = t - \frac{13}{10}$

$+\frac{17}{10} = t - \frac{13}{10} \qquad -\frac{17}{10} = t - \frac{13}{10}$

$+\frac{13}{10} \quad +\frac{13}{10} \qquad +\frac{13}{10} \quad +\frac{13}{10}$

$\boxed{3 = t} \qquad -\frac{2}{5} = t$

It takes 3 sec to hit the ground.

### *Using Quadratic Formula*

$h(t) = -5t^2 + 13t + 6$

$0 = -5t^2 + 13t + 6$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-13 \pm \sqrt{13^2 - 4(-5)(6)}}{2(-5)}$

$t = \frac{-13 \pm \sqrt{289}}{-10}$

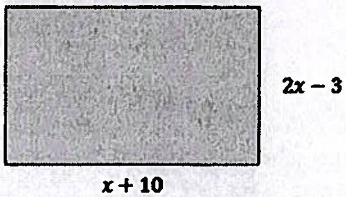
$t_1 = \frac{-13 + 17}{-10} \qquad t_2 = \frac{-13 - 17}{-10}$

$t_1 = -2/5 \qquad \boxed{t_2 = 3}$

It takes 3 sec to hit the ground.

Math 20-1 Key

**Pg 230 #11:** A rectangle has dimension $x + 10$ and $2x - 3$, where $x$ is in centimeters. The area of the rectangle is 54 cm$^2$.




a. What equation could you use to determine the value of $x$?
b. What is the value of $x$?

(A) Area = $LW$

$54 = (x+10)(2x-3)$

$54 = 2x^2 - 3x + 20x - 30$

$54 = 2x^2 + 17x - 30$

$0 = 2x^2 + 17x - 84$

(B) $\square + \square = +17$

$\square \times \square = 168$

1, 168
2, 84
4, 42
6, 28
3, 56
$\boxed{7, 24}$
8, 21

$2x^2 + 17x - 84 = 0$

$2x^2 - 7x + 24x - 84 = 0$

$(2x^2 - 7x) + (+24x - 84) = 0$

$x(2x-7) + 12(2x-7) = 0$

$(x+12)(2x-7) = 0$

$x + 12 = 0$, $x = -12$

$2x - 7 = 0$, $x = +7/2$

$x = -12$ doesn't give positive length or width.

So $x = +7/2$ cm

or $x = +3.5$ cm.

**Pg 230 #13:** A flare is launched from a boat. The height, $h$, in meters, of the flare above the water is approximately modelled by the function $h(t) = 150t - 5t^2$, where $t$ is the number of seconds after the flare is launched.

a. What equation could you use to determine the time it takes for the flare to return to the water?
b. How many seconds will it take for the flare to return to the water?

(A) $0 = 150t - 5t^2$

$5t^2 - 150t + 0 = 0$

(B) $5t(t - 30) = 0$

$5t = 0$, $t = 0$

$t - 30 = 0$, $t = 30$

So at water level when $t = 0$ (launched) and $t = 30$ (landed).

So lands after 30 sec.

As a side note, the vertex is halfway between these "zeroes" (x-intercepts), so it reaches max height at 15 seconds.

KEY

**Pg 240 #9:** Evan passes a flying disc to a teammate during a competition at the Flatland Ultimate and Cups Tournament in Winnipeg. The flying disc follows the path $h(t) = -0.02d^2 + 0.4d + 1$, where $h$ is the height, in meters, and $d$ is the horizontal distance, in meters, that the flying disc has travelled from the thrower. If no one catches the flying disc, the height of the disc above the ground when it lands can be modelled by $h(d) = 0$.

a. What quadratic equation can you use to determine how far the disc will travel if no one catches it?

b. How far will the disc travel if no one catches it? Express your answer to the nearest tenth of a meter.

A) $h(t) = -0.02d^2 + 0.4d + 1$

$0 = -0.02d^2 + 0.4d + 1$

B) Option #1: Complete the square

$(-0.02d^2 + 0.4d) + 1 = 0$

$-0.02(d^2 - 20d) + 1 = 0$

$-0.02(d^2 - 10d - 10d) + 1 = 0$

$-0.02(d^2 - 10d - 10d + 100) + 1 + 2 = 0$

$-0.02(d-10)^2 + 3 = 0$

$-0.02(d-10)^2 = -3$

$(d-10)^2 = 150$

$d - 10 = \pm\sqrt{150}$

$d_1 = 10 + \sqrt{150}$ $\quad d_2 = 10 - \sqrt{150}$

$d_1 = 22.2$ $\quad d_2 = -2.2$

Doesn't make sense in context of problem.

$\boxed{\text{So } d = 22.2\text{m}}$

Option #2: Quadratic Formula

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-0.4 \pm \sqrt{0.16 + 0.08}}{2(-0.02)}$

$= \dfrac{-0.4 \pm \sqrt{0.24}}{-0.04}$

$x_1 = \dfrac{-0.4 + \sqrt{0.24}}{-0.04}$ $\quad x_2 = \dfrac{-0.4 - \sqrt{0.24}}{-0.04}$

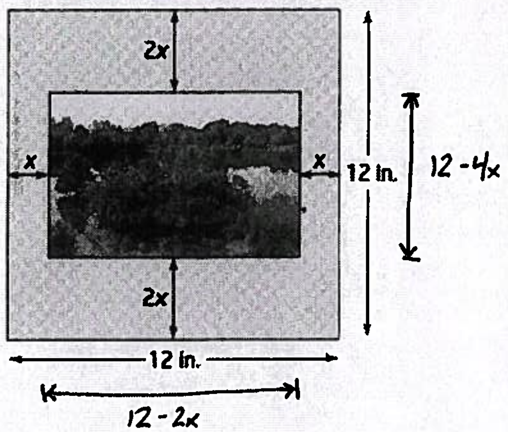
$x_1 = -2.2$ $\quad x_2 = 22.2$

$\boxed{\text{So } d = 22.2\text{m}}$

Math 20-1 ~~Key~~

**Pg 240 #11:** Brian is placing a photograph behind a 12-in. by 12-in. piece of matting. He positions the photograph so that the matting is twice as wide at the top and bottom as it is at the sides.

The visible area of the photograph is 54 sq. in. What are the dimensions of the photograph?

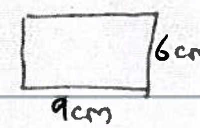



$$A = (L)(w)$$
$$54 = (12-4x)(12-2x)$$
$$54 = 144 - 72x + 8x^2$$
$$0 = 8x^2 - 72x + 90$$

$$8x^2 - 72x + 90 = 0$$
$$(8x^2 - 72x) + 90 = 0$$
$$8(x^2 - 9x) + 90 = 0$$
$$8\left(x^2 - \tfrac{9}{2}x - \tfrac{9}{2}x\right) + 90 = 0$$
$$8\left(x^2 - \tfrac{9}{2}x - \tfrac{9}{2}x + \tfrac{81}{4}\right) + 90 - 162 = 0$$
$$8\left(x - \tfrac{9}{2}\right)^2 - 72 = 0$$
$$8\left(x - \tfrac{9}{2}\right)^2 = 72$$
$$\left(x - \tfrac{9}{2}\right)^2 = 9$$
$$x - \tfrac{9}{2} = \pm\sqrt{9}$$

| | |
|---|---|
| $x_1 = \frac{9}{2} + 3$ | $x_2 = \frac{9}{2} - 3$ |
| $x_1 = \frac{15}{2}$ | $x_2 = \frac{3}{2}$ |
| $x_1 = 7.5\text{in}$ | $x_2 = 1.5\text{cm}$ |

Results in negative dimensions. ⇐ $x_1 = 7.5\text{in}$  $x_2 = 1.5\text{cm}$ ⇒




**Pg 240 #15:** Determine the roots of $ax^2 + bx + c = 0$ by completing the square. Can you use this result to solve any quadratic equation? Explain.

$$ax^2 + bx + c = 0$$
$$(ax^2 + bx) + c = 0$$
$$a\left(x^2 + \tfrac{b}{a}x\right) + c = 0$$
$$a\left(x^2 + \tfrac{b}{2a}x + \tfrac{b}{2a}x\right) + c = 0$$
$$a\left(x^2 + \tfrac{b}{2a}x + \tfrac{b}{2a}x + \tfrac{b^2}{4a^2}\right) + c - \tfrac{b^2}{4a} = 0$$
$$a\left(x + \tfrac{b}{2a}\right)^2 + c - \tfrac{b^2}{4a} = 0$$
$$a\left(x + \tfrac{b}{2a}\right)^2 + \tfrac{c}{1}\left(\tfrac{4a}{4a}\right) - \tfrac{b^2}{4a} = 0$$
$$a\left(x + \tfrac{b}{2a}\right)^2 + \tfrac{4ac}{4a} - \tfrac{b^2}{4a} = 0$$
$$a\left(x + \tfrac{b}{2a}\right)^2 = \tfrac{b^2 - 4ac}{4a}$$
$$\left(x + \tfrac{b}{2a}\right)^2 = \tfrac{b^2 - 4ac}{4a^2}$$

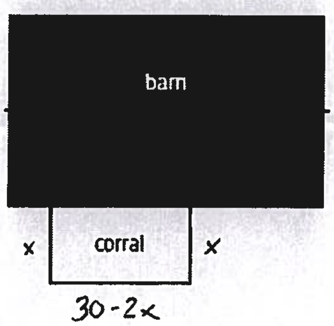
$$\left(x + \tfrac{b}{2a}\right) = \pm\sqrt{\tfrac{b^2 - 4ac}{4a^2}}$$
$$\left(x + \tfrac{b}{2a}\right) = \frac{\pm\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$
$$x + \tfrac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-b}{2a} + \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Oooo... it's the quadratic formula! This only gives real roots (i.e. the quadratic crosses the x-axis) if the stuff under the root is positive so $b^2 - 4ac \geq 0$.

Math 20-1

KEY

**Pg 254 #8:** To save materials, Choma decides to build a horse corral using the barn for one side. He has 30 m of fencing materials and wants the corral to have an area of 100 m$^2$. What are the dimensions of the corral?

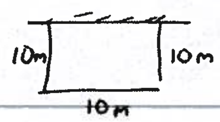


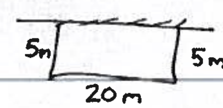

$$A = (L)(w)$$
$$100 = (x)(30-2x)$$
$$100 = 30x - 2x^2$$
$$2x^2 - 30x + 100 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{30 \pm \sqrt{900-4(2)(100)}}{2(2)}$$

$$x = \frac{30 \pm \sqrt{100}}{4}$$

$$x_1 = \frac{30+10}{4} = 10 \qquad x_2 = \frac{30-10}{4} = 5$$




**Pg 254 #10:** Subtracting a number from half its square gives a result of 11. What is the number? Express your answers as exact values and to the nearest hundredth.

$$\tfrac{1}{2}x^2 - x = 11$$
$$\tfrac{1}{2}x^2 - 1x - 11 = 0$$
$$\cdot 2 \quad \cdot 2 \quad \cdot 2 \quad \cdot 2$$
$$x^2 - 2x - 22 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{2 \pm \sqrt{4-4(1)(-22)}}{2(1)}$$

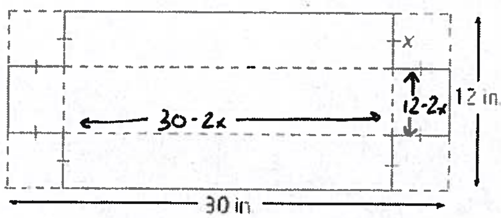
$$= \frac{2 \pm \sqrt{92}}{2}$$

$$x_1 = \frac{2+\sqrt{92}}{2} = 1+\sqrt{23} \approx 5.80 \qquad x_2 = \frac{2-\sqrt{92}}{2} = 1-\sqrt{23} \approx -3.80$$

KEY

Math 20-1

**Pg 254 #12:** An open-topped box is being made for a piece of cardboard measuring 12 in. by 30 in. The sides of the box are formed when four congruent squares are cut from the corners, as shown in the diagram. The base of the box has an area of 208 sq. in.




a. What equation represents the surface area of the base of the box? A $30-2x$ by $12-2x$
b. What is the side length, $x$, of the square cut from each corner?
c. What are the dimensions of the box?

B
$$A = (l)(w)$$
$$208 = (30-2x)(12-2x)$$
$$208 = 360 - 84x + 4x^2$$
$$0 = 4x^2 - 84x + 152$$
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$= \frac{84 \pm \sqrt{7056 - 4(4)(152)}}{2(4)}$$
$$= \frac{84 \pm \sqrt{4624}}{8}$$
$$= \frac{84 \pm 68}{8}$$

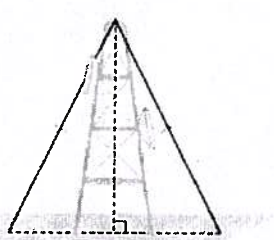
Longer than actual box.

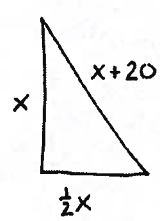
$$x_1 = \frac{84+68}{8} = 19 \text{ in}$$
$$x_2 = \frac{84-68}{8} = 2 \text{ in}$$

So $x = 2$ in.

C Box is 26" by 8", and 2" high.

**Pg 254 #16:** Two guy wires are attached to the top of a telecommunications tower and anchored to the ground on opposite sides of the tower, as shown. The length of the guy wire is 20 m more than the height of the tower. The horizontal distance from the base of the tower to where the guy wire is anchored to the ground is one-half the height of the tower. How tall is the tower, to the nearest tenth of a meter?






$$a^2 + b^2 = c^2$$
$$(x)^2 + (\tfrac{1}{2}x)^2 = (x+20)^2$$
$$x^2 + \tfrac{1}{4}x^2 = x^2 + 40x + 400$$
$$\tfrac{1}{4}x^2 - 40x - 400 = 0$$
$$\cdot 4 \quad \cdot 4 \quad \cdot 4 \quad \cdot 4$$
$$x^2 - 160x - 1600 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{160 \pm \sqrt{25{,}600 - 4(1)(-1600)}}{2(1)} = \frac{160 \pm \sqrt{32{,}000}}{2}$$

$$x_1 = \frac{160 + \sqrt{32{,}000}}{2} = 169.4 \text{ m}$$

$$x_2 = \frac{160 - \sqrt{32{,}000}}{2} = -9.4 \text{ m}$$

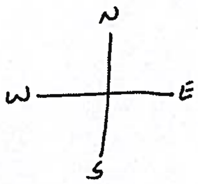
Doesn't make sense.

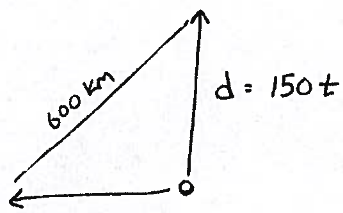
Tower is 169.4 m tall.

Math 20-1

KEY

**Pg 254 #20:** Two small private planes take off from the same airport. One plane flies north at 150 km/h. Two hours later, the second plane flies west at 200 km/h. How long after the first plane takes off will the two planes be 600 km apart? Express your answer to the nearest tenth of an hour.




$v = \frac{d}{t}$ (from Sc10)

so $d = vt$

$d = 200(t-2)$
$= 200t - 400$

$$a^2 + b^2 = c^2$$

$$(150t)^2 + (200t - 400)^2 = 600^2$$

$$(22{,}500t^2) + (40{,}000t^2 - 160{,}000t + 160{,}000) = 360{,}000$$

$$\underset{\div 100}{62{,}500t^2} \; \underset{\div 100}{- 160{,}000t} \; \underset{\div 100}{- 200{,}000} = \underset{\div 100}{0}$$

I don't really want to be dealing with these giant numbers...

$$\underset{\div 25}{625t^2} - \underset{\div 25}{1600t} - \underset{\div 25}{2000} = \underset{\div 25}{0}$$

$$25t^2 - 64t - 80 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{64 \pm \sqrt{4096 - 4(25)(-80)}}{2(25)} = \frac{64 \pm 109.981}{50}$$

$t_1 = \frac{64 + 109.981}{50}$ $\qquad$ $t_2 = \frac{64 - 109.981}{50}$

$t_1 = 3.5$ hrs $\qquad$ $t_2 = -0.9$ hrs

Doesn't make sense.

So it takes 3.5 hrs.