30&31 – 8.2 Solving Systems of Equations Algebraically

Key Ideas

Linear-Quadratic and Quadratic-Quadratic equations can be solved by elimination ***or*** substitution.

Substitution:

- Isolate one variable in one equation.
- Substitute it into the other equation for that variable and solve for the first variable.
- Plug solution back into any original equation and solve for variable 2.
- Verify answer by checking.

Elimination:

- Rearrange the equation so like terms align.
- Possibly multiply by a constant to make equivalent equations with variables with opposite coefficients.
- Add ***or*** subtract to eliminate a variable and solve for the other variable.
- Substitute the solution into any original equation to solve for the other variable.
- Verify by checking solutions.

Part 1 – Linear Systems of Equations (Substitution) – Math 10C Review

Q1: Solve the following system of equations.

$f(x) = -x + 3$

$g(x) = \frac{1}{2}x - 3$

$y = -x+3$ $\quad$ $y = \frac{1}{2}x - 3$

$y = y$

$-x+3 = \frac{1}{2}x - 3$

$+1x \quad +1x$

$3 = \frac{3}{2}x - 3$

$+3 \quad +3$

$6 = \frac{3}{2}x$

$\div 3/2 \quad \div 3/2$

$\boxed{4 = x}$ $\longrightarrow$ $y = -x+3$

$y = -(4)+3$

$\boxed{y = -1}$

$\boxed{\text{Soln is } (4, -1)}$

Part 2 – Linear-Quadratic Systems of Equations (Substitution) – Factoring

Q2: Solve the following system of equations.

$y = \frac{1}{2}x^2 + 2x - 3$

$y = \frac{1}{2}x - 3$

$y = y$

$\frac{1}{2}x^2 + 2x - 3 = \frac{1}{2}x - 3$

$-\frac{1}{2}x \quad -\frac{1}{2}x$

$\frac{1}{2}x^2 + \frac{3}{2}x - 3 = -3$

$+3 \quad +3$

$\frac{1}{2}x^2 + \frac{3}{2}x = 0$

$\frac{1}{2}(x^2 + 3x) = 0$

$\frac{1}{2}(x)(x+3) = 0$

$x = 0$ or $x + 3 = 0$, $x = -3$

$y = \frac{1}{2}(0) - 3$, $y = -3$

Soln is $(0, -3)$

$y = \frac{1}{2}(-3) - 3$, $y = -\frac{9}{2}$

Soln is $(-3, -9/2)$

Q3: Solve the following system of equations.

$y = -x^2 + 6x - 2$

$y = x - 8$

$y = y$

$-x^2 + 6x - 2 = x - 8$

$+x^2 \quad +x^2$

$6x - 2 = x^2 + 1x - 8$

$-6x \quad -6x$

$-2 = x^2 - 5x - 8$

$+2 \quad +2$

$0 = x^2 - 5x - 6$

$0 = (x-6)(x+1)$

$x - 6 = 0$, $x = 6$

$y = (6) - 8$, $y = -2$

Soln is $(6, -2)$

$x + 1 = 0$, $x = -1$

$y = (-1) - 8$, $y = -9$

Soln is $(-1, -9)$

Part 3 – Linear-Quadratic Systems of Equations (Substitution) – Quadratic Equation

20 → 2, 10 → 2, 5

Q4: Solve the following system of equations.

$y = -x^2 + 5x + 3$

$y = x + 2$

$y = y$

$x + 2 = -x^2 + 5x + 3$

$+x^2 \quad +x^2$

$x^2 + x + 2 = 5x + 3$

$-5x \quad -5x$

$x^2 - 4x + 2 = 3$

$-3 \quad -3$

$x^2 - 4x - 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{4 \pm \sqrt{4^2 - 4(1)(-1)}}{2(1)}$

$x = \frac{4 \pm \sqrt{16 + 4}}{2}$

$x = \frac{4 \pm \sqrt{20}}{2}$ or $\frac{4 \pm 2\sqrt{5}}{2}$

$x_1 = 2 + \sqrt{5}$ $\quad$ $x_2 = 2 - \sqrt{5}$

$y = x + 2$

$y = (2 + \sqrt{5}) + 2$

$y = 4 + \sqrt{5}$

Soln is $(2 + \sqrt{5}, 4 + \sqrt{5})$

$y = x + 2$

$y = (2 - \sqrt{5}) + 2$

$y = 4 - \sqrt{5}$

Soln is $(2 - \sqrt{5}, 4 - \sqrt{5})$

Q5: Solve the following system of equations.

ELIMINATE FRACTIONS

$y = x^2 + 2x - 4$

$y = \frac{1}{2}x + 4$

$y = y$

$x^2 + 2x - 4 = \frac{1}{2}x + 4$

$-\frac{1}{2}x \quad -\frac{1}{2}x$

$x^2 + \frac{3}{2}x - 4 = 4$

$-4 \quad -4$

$x^2 + \frac{3}{2}x - 8 = 0$

I hate fractions, so...

$2x^2 + 3x - 16 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-16)}}{2(2)}$

$x = \frac{-3 \pm \sqrt{9 + 128}}{4}$

$x = \frac{-3 \pm \sqrt{137}}{4}$

$x_1 = \frac{-3 + \sqrt{137}}{4}$ $\quad$ $x_2 = \frac{-3 - \sqrt{137}}{4}$

$y = \frac{1}{2}x + 4$

$y = \frac{-3 + \sqrt{137}}{8} + 4$

$y = \frac{29 + \sqrt{137}}{8}$

Soln is $\left(\frac{-3 + \sqrt{137}}{4}, \frac{29 + \sqrt{137}}{8}\right)$

$y = \frac{1}{2}x + 4$

$y = \frac{-3 - \sqrt{137}}{8} + 4$

$y = \frac{29 - \sqrt{137}}{8}$

Soln is $\left(\frac{-3 - \sqrt{137}}{4}, \frac{29 - \sqrt{137}}{8}\right)$

Key

Part 4 – Quadratic-Quadratic Systems of Equations (Substitution) - Easy

Q6: Solve the following system of equations.

$y = x^2 + 2x + 6$

$y = x^2 + x + 4$

$y = y$

$x^2+2x+6 = x^2+x+4$

$-x^2 \quad -x^2$

$2x+6 = x+4$

$-x \quad -x$

$x+6 = 4$

$-6 \quad -6$

$\boxed{x = -2}$

$y = (-2)^2 + (-2) + 4$

$= 4 - 2 + 4$

$\boxed{y = 6}$

$\boxed{\text{Soln is } (-2, 6)}$

Q7: Solve the following system of equations.

$y = 2x^2 + 3x - 5$

$y = 2x^2 - 8$

$y = y$

$2x^2+3x-5 = 2x^2-8$

$-2x^2 \quad -2x^2$

$3x-5 = -8$

$+5 \quad +5$

$3x = -3$

$\div 3 \quad \div 3$

$\boxed{x = -1}$

$y = 2(-1)^2 - 8$

$= 2(1) - 8$

$\boxed{y = -6}$

$\boxed{\text{Soln is } (-1, -6)}$

Part 5 – Quadratic-Quadratic Systems of Equations (Substitution) - Factoring

Q8: Solve the following system of equations.

$y = x^2 + 2x - 5$

$y = -2x^2 - 7x - 11$

$y = y$

$x^2+2x-5 = -2x^2-7x-11$

$+2x^2 \quad +2x^2$

$3x^2+2x-5 = -7x-11$

$+7x \quad +7x$

$3x^2+9x-5 = -11$

$+11 \quad +11$

$3x^2+9x+6 = 0$

$3(x^2+3x+2) = 0$

$3(x+2)(x+1) = 0$

$x+2 = 0$

$\boxed{x = -2}$

$y = (-2)^2 + 2(-2) - 5$

$= 4 - 4 - 5$

$\boxed{y = -5}$

$\boxed{\text{Soln is } (-2, -5)}$

$x+1 = 0$

$\boxed{x = -1}$

$y = (-1)^2 + 2(-1) - 5$

$= 1 - 2 - 5$

$\boxed{y = -6}$

$\boxed{\text{Soln is } (-1, -6)}$

Q9: Solve the following system of equations.

$y = 3x^2 + 2x + 5$

$y = 2x^2 + 4x + 13$

$y = y$

$3x^2+2x+5 = 2x^2+4x+13$

$-2x^2 \quad -2x^2$

$x^2+2x+5 = 4x+13$

$-4x \quad -4x$

$x^2-2x+5 = 13$

$-13 \quad -13$

$x^2-2x-8 = 0$

$(x-4)(x+2) = 0$

$x-4 = 0$

$\boxed{x = 4}$

$y = 2(4)^2 + 4(4) + 13$

$y = 32 + 16 + 13$

$\boxed{y = 61}$

$\boxed{\text{Soln is } (4, 61)}$

$x+2 = 0$

$\boxed{x = -2}$

$y = 2(-2)^2 + 4(-2) + 13$

$y = 8 - 8 + 13$

$\boxed{y = 13}$

$\boxed{\text{Soln is } (-2, 13)}$

Part 6 – Quadratic-Quadratic Systems of Equations (Substitution) – Quadratic Equation

Q10: Solve the following system of equations.

$y = x^2 - 2x + 3$

$y = -x^2 + 4x + 12$

$y = y$

$x^2 - 2x + 3 = -x^2 + 4x + 12$

$+x^2 \quad +x^2$

$2x^2 - 2x + 3 = 4x + 12$

$-4x \quad -4x$

$2x^2 - 6x + 3 = 12$

$-12 \quad -12$

$2x^2 - 6x - 9 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{6 \pm \sqrt{6^2-4(2)(-9)}}{2(2)}$

$x = \frac{6 \pm \sqrt{36+72}}{4} = \frac{6 \pm \sqrt{108}}{4}$

$108 \to 2, 54 \to 2, 27 \to 3, 9 \to 3, 3$

$x = \frac{6 \pm 6\sqrt{3}}{4}$

$x_1 = \frac{6+6\sqrt{3}}{4}$ $\quad$ $x_2 = \frac{6-6\sqrt{3}}{4}$

$y_1 = \left(\frac{6+6\sqrt{3}}{4}\right)\left(\frac{6+6\sqrt{3}}{4}\right) - 2\left(\frac{6+6\sqrt{3}}{4}\right) + 3$

$y_1 = \frac{36+72\sqrt{3}+108}{16} + \frac{-12-12\sqrt{3}}{4} + \frac{12}{4}$

$y_1 = \frac{144+24\sqrt{3}}{16} = 9 + \frac{3}{2}\sqrt{3}$

Soln is $\left(\frac{6+6\sqrt{3}}{4}, 9+\frac{3}{2}\sqrt{3}\right)$

$y_2 = \left(\frac{6-6\sqrt{3}}{4}\right)\left(\frac{6-6\sqrt{3}}{4}\right) - 2\left(\frac{6-6\sqrt{3}}{4}\right) + 3$

$y_2 = \frac{36-72\sqrt{3}+108}{16} - \frac{12+12\sqrt{3}}{4} + \frac{12}{4}$

$y_2 = \frac{144-24\sqrt{3}}{16} = 9 - \frac{3}{2}\sqrt{3}$

Soln is $\left(\frac{6-6\sqrt{3}}{4}, 9-\frac{3}{2}\sqrt{3}\right)$

Q11: Solve the following system of equations.

$y = x^2 + 5x + 8$

$y = -x^2 + 6x + 10$

$y = y$

$x^2 + 5x + 8 = -x^2 + 6x + 10$

$+x^2 \quad +x^2$

$2x^2 + 5x + 8 = 6x + 10$

$-6x \quad -6x$

$2x^2 - 1x + 8 = 10$

$-10 \quad -10$

$2x^2 - 1x - 2 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{1 \pm \sqrt{1^2-4(2)(-2)}}{2(2)}$

$x = \frac{1 \pm \sqrt{17}}{4}$

$x_1 = \frac{1+\sqrt{17}}{4}$ $\quad$ $x_2 = \frac{1-\sqrt{17}}{4}$

$y_1 = \left(\frac{1+\sqrt{17}}{4}\right)\left(\frac{1+\sqrt{17}}{4}\right) + 5\left(\frac{1+\sqrt{17}}{4}\right) + 8$

$y_1 = \frac{1+2\sqrt{17}+17}{16} + \frac{5+5\sqrt{17}}{4} + 8$

$y_1 = \frac{1+2\sqrt{17}+17}{16} + \frac{20+20\sqrt{17}}{16} + \frac{128}{16}$

$y_1 = \frac{166+22\sqrt{17}}{16} = \frac{83}{8} + \frac{11\sqrt{17}}{8}$

Soln is $\left(\frac{1+\sqrt{17}}{4}, \frac{83+11\sqrt{17}}{8}\right)$

$y_2 = \left(\frac{1-\sqrt{17}}{4}\right)\left(\frac{1-\sqrt{17}}{4}\right) + 5\left(\frac{1-\sqrt{17}}{4}\right) + 8$

$y_2 = \frac{1-2\sqrt{17}+17}{16} + \frac{5-5\sqrt{17}}{4} + 8$

$y_2 = \frac{1-2\sqrt{17}+17}{16} + \frac{20-20\sqrt{17}}{16} + \frac{128}{16}$

$y_2 = \frac{166-22\sqrt{17}}{16} = \frac{83}{8} - \frac{11\sqrt{17}}{8}$

Soln is $\left(\frac{1-\sqrt{17}}{4}, \frac{83-11\sqrt{17}}{8}\right)$

Today's Assignment: *Q1-Q3, Pg451 #3ac,8,9*

Tomorrow's Assignment: *Pg451#4ac,6,10,17*

Part 8 – Linear Systems of Equations (Elimination) – Math 10C Review

Q12: Solve the following system of equations.

$f(x) = -x + 3$

$g(x) = \frac{1}{2}x - 3$

$y = -1x + 3$
$-(y = \frac{1}{2}x - 3)$
$0 = -\frac{3}{2}x + 6$
$-6 \quad -6$
$-6 = -\frac{3}{2}x$
$\div(-\frac{3}{2}) \quad \div(-\frac{3}{2})$
$\boxed{4 = x}$ ⟶

$y = \frac{1}{2}(4) - 3$
$y = 2 - 3$
$\boxed{y = -1}$
$\boxed{\text{Soln is } (4, -1)}$

Part 9 – Linear-Quadratic Systems of Equations (Elimination)

Q13: Solve the following system of equations using Substitution.

$y = x^2 + 2x - 4$

$y = -x$

$y = x^2 + 2x - 4$
$-(y = -1x)$
$0 = x^2 + 3x - 4$
$0 = (x+4)(x-1)$

$\boxed{x = -4}$
$y = -(-4)$
$\boxed{y = 4}$
$\boxed{\text{Soln is } (-4, 4)}$

$\boxed{x = 1}$
$y = -(1)$
$\boxed{y = -1}$
$\boxed{\text{Soln is } (1, -1)}$

Q14: Solve the following system of equations using Elimination

$y = x^2 - 2x - 8$

$y = -x + 4$

$y = x^2 - 2x - 8$
$-(y = -x + 4)$
$0 = x^2 - x - 12$
$0 = (x-4)(x+3)$

$\boxed{x = 4}$
$y = -(4) + 4$
$\boxed{y = 0}$
$\boxed{\text{Soln is } (4, 0)}$

$\boxed{x = -3}$
$y = -(-3) + 4$
$\boxed{y = 7}$
$\boxed{\text{Soln is } (-3, 7)}$

KEY

Part 10 – Quadratic-Quadratic Systems of Equations (Elimination)

Q15: Solve the following system of equations using Substitution.

$y = x^2 - 4x + 5$

$-(y = -x^2 + 3x + 9)$

$0 = 2x^2 - 7x - 4$

$-8 + 1$
$\square + \square = -7$
$\square \times \square = -8$

$0 = 2x^2 - 8x + 1x - 4$
$0 = (2x^2 - 8x) + (1x - 4)$
$0 = 2x(x-4) + 1(x-4)$
$0 = (x-4)(2x+1)$

$x - 4 = 0$
$\boxed{x = 4}$

$y = (4)^2 - 4(4) + 5$
$y = 16 - 16 + 5$
$\boxed{y = 5}$

$\boxed{\text{Soln is } (4, 5)}$

$2x + 1 = 0$
$\boxed{x = -\frac{1}{2}}$

$y = (-\frac{1}{2})^2 - 4(-\frac{1}{2}) + 5$
$y = \frac{1}{4} + 2 + 5$
$\boxed{y = \frac{29}{4}}$

$\boxed{\text{Soln is } (-\frac{1}{2}, \frac{29}{4})}$

Q16: Solve the following system of equations using Elimination.

$y = 2x^2 + x + 6$

$-(y = x^2 + 3x + 9)$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x - 3 = 0$
$\boxed{x = 3}$

$y = (3)^2 + 3(3) + 9$
$y = 9 + 9 + 9$
$\boxed{y = 27}$

$\boxed{\text{Soln is } (3, 27)}$

$x + 1 = 0$
$\boxed{x = -1}$

$y = (-1)^2 + 3(-1) + 9$
$y = 1 - 3 + 9$
$\boxed{y = 7}$

$\boxed{\text{Soln is } (-1, 7)}$

Today's Assignment: *Pg451#4ac,6,10,17*