KEY

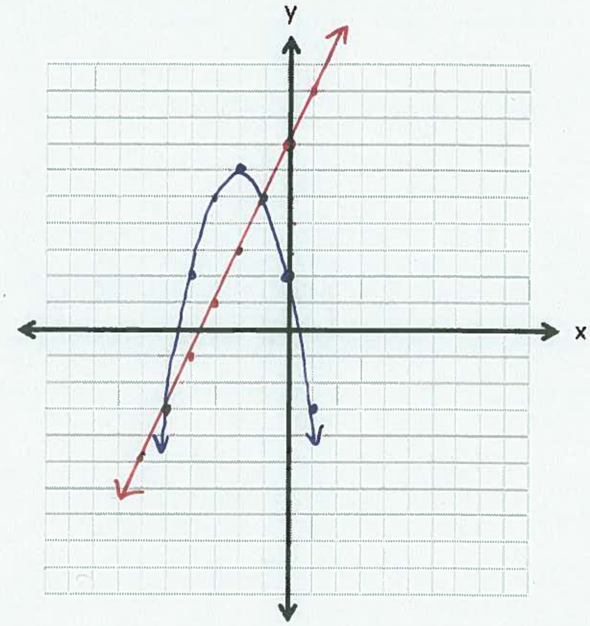
Math 20-1

Lxx – Worksheet – 8.2 Solving Systems of Equations Algebraically

Part 1 – Easy Questions (Factorable)

Q1: Graph the following equations:

$$f(x) = -1(x+2)^2 + 6$$
$$g(x) = 2x + 7$$



Intersects at $(-5,-3)$ and $(-1,5)$
These are my "solutions" to the system of equations.

Q2: Using *Substitution*, algebraically determine the solution(s) to this system of equations (*i.e. the coordinates of the points of intersection*).

$y = -1(x+2)^2+6$ and $y = 2x+7$

$2x+7 = -1(x+2)^2+6$

$2x+7 = -x^2-4x+2$

$x^2+6x+5 = 0$

$(x+5)(x+1) = 0$

$x+5=0$ $\quad$ $x+1=0$

$x=-5$ $\quad$ $x=-1$

$g(-5) = 2(-5)+7 = -10+7 = -3$

$g(-1) = 2(-1)+7 = -2+7 = 5$

Soln is $(-5,-3)$ and $(-1,5)$

Math 20-1

KEY

Part 2 – Hard Questions (Radicals)

Q3: Using *Substitution,* algebraically determine the ***exact*** solution(s) to this system of equations:

$$f(x) = \frac{1}{2}x^2 + 2x + 4$$
$$g(x) = -x + 2$$

$y = \frac{1}{2}x^2 + 2x + 4 \qquad y = -x + 2$

$\frac{1}{2}x^2 + 2x + 4 = -x + 2$

$\frac{1}{2}x^2 + 3x + 2 = 0$ (Multiply by 2 to remove fractions)

$x^2 + 6x + 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 - 4(1)(4)}}{2(1)} = \frac{-6 \pm \sqrt{36-16}}{2}$$

$$= \frac{-6 \pm \sqrt{20}}{2} = \frac{-6 \pm 2\sqrt{5}}{2} = -3 \pm \sqrt{5}$$

$x_1 = -3 + \sqrt{5}$

$g(-3+\sqrt{5}) = -(-3+\sqrt{5}) + 2$
$= 3 - \sqrt{5} + 2$
$= 5 - \sqrt{5}$

$x_2 = -3 - \sqrt{5}$

$g(-3-\sqrt{5}) = -(-3-\sqrt{5}) + 2$
$= 3 + \sqrt{5} + 2$
$= 5 + \sqrt{5}$

Solutions are $(-3+\sqrt{5},\ 5-\sqrt{5})$ and $(-3-\sqrt{5},\ 5+\sqrt{5})$

Math 20-1 KEY

Part 3 – Textbook Questions

Pg 451 #3ac: Solve each system of equations by substitution, and verify your solution(s).

$x^2 - y + 2 = 0$ ⇨ $x^2 + 2 = y$
$4x = 14 - y$

$4x = 14 - (x^2 + 2)$
$4x = 14 - x^2 - 2$
$x^2 + 4x - 12 = 0$
$(x+6)(x-2) = 0$

$x + 6 = 0$
$x = -6$

$x - 2 = 0$
$x = 2$

$y = (-6)^2 + 2$
$= 36 + 2$
$= 38$

$y = (2)^2 + 2$
$= 4 + 2$
$= 6$

Soln is $(-6, 38)$

Soln is $(2, 6)$

Verify by graphing $y = x^2 + 2$
$y = -4x + 14$

$7d^2 + 5d - t - 8 = 0$ ⇨ $7d^2 + 5d - 8 = t$
$10d - 2t = -40$

$10d - 2(7d^2 + 5d - 8) = -40$
$10d - 14d^2 - 10d + 16 = -40$
$-14d^2 + 0d + 56 = 0$ Divide every term by -2
$7d^2 + 0d - 28 = 0$
$7d^2 = 28$
$d^2 = 4$
$d = \pm 2$

$10d - 2t = -40$ ⇨ $10d + 40 = 2t$ or $t = 5d + 20$

$t(d) = 5d + 20$

$t(+2) = 5(+2) + 20$
$= +30$

$t(-2) = 5(-2) + 20$
$= 10$

Soln is $(2, 30)$

Soln is $(-2, 10)$

Verify by graphing $t = 7d^2 + 5d - 8$
$t = 5d + 20$

KEY

Pg 451 #4ac: Solve each system of equations by elimination, and verify your solution(s).

$$6x^2 - 3x = 2y - 5$$
$$2x^2 + x = y - 4$$

$\Rightarrow$ $6x^2 - 3x + 5 = 2y$, $2x^2 + x + 4 = y$

$6x^2 - 3x + 5 = 2y$
$2(2x^2 + x + 4 = y)$

$\Rightarrow$

$6x^2 - 3x + 5 = 2y$
$-(4x^2 + 2x + 8 = 2y)$

$2x^2 - 5x - 3 = 0$

$(x-3)(2x+1) = 0$

$x - 3 = 0$
$x = 3$

$2x + 1 = 0$
$x = -1/2$

$y(x) = 2x^2 + x + 4$
$y(3) = 2(3)^2 + (3) + 4$
$= 18 + 3 + 4$
$= 25$

Soln is $(3, 25)$

$y(x) = 2x^2 + x + 4$
$y(-1/2) = 2(-1/2)^2 + (\frac{1}{2}) + 4$
$= \frac{1}{2} + \frac{1}{2} + 4$
$= 5$

Soln is $(-1/2, 5)$

$$2p^2 = 4p - 2m + 6$$
$$5m + 8 = 10p + 5p^2$$

$\Rightarrow$ $2p^2 - 4p - 6 = -2m$, $5m = 5p^2 + 10p - 8$

$\frac{5}{2}(-2m = 2p^2 - 4p - 6)$
$5m = 5p^2 + 10p - 8$

$\Rightarrow$

$-5m = 5p^2 - 10p - 15$
$+(5m = 5p^2 + 10p - 8)$

$0 = 10p^2 + 0p - 23$
$23 = 10p^2$
$2.3 = p^2$
$p = \pm 1.52$

$-2m = 2p^2 - 4p - 6$
$m = -p^2 + 2p + 3$
$m(p) = -p^2 + 2p + 3$
$m(+1.52) = -(1.52)^2 + 2(1.52) + 3$
$= 3.73$

Soln is $(1.52, 3.73)$

$m(p) = -p^2 + 2p + 3$
$m(-1.52) = -(-1.52)^2 + 2(-1.5$
$= -2.35$

Soln is $(-1.52, -2.35)$

KEY

Math 20-1

Pg 451 #6: Alex and Kaela are considering the two equations $n - m^2 = 7$ and $2m^2 - 2n = -1$. Without making any calculations, they both claim that the system involving these two equations has no solution.

Alex's reasoning:

If I double every term in the first equation and then use the elimination method, both of the variables will disappear, so the system does not have a solution.

Kaela's reasoning:

If I solve the first equation for *n* and substitute into the second equation, I will end up with an equation without any variables, so the system does not have a solution.

a. Is each person's reasoning correct?

b. Verify the conclusion graphically.

A

Alex

$-2m^2 + 2n = 14$

$+(2m^2 - 2n = -1)$

$0 = 13$

Yep, both are correct.

Kayla

$n = 7 + m^2$

$2m^2 - 2(7 + m^2) = -1$

$2m^2 - 14 - 2m^2 = -1$

$-14 = -1$

B

$n = m^2 + 0m + 7$ or $y = x^2 + 0x + 7$

$-2n = -2m^2 - 1$

$n = m^2 + 1/2$ or $y = x^2 + 0x + 1/2$

Same equation, just vertically shifted. Won't intersect.

Pg 451 #8: Determine the values of *m* and *n* if (2,8) is a solution to the following system of equations.

$mx^2 - y = 16$

$mx^2 + 2y = n$

$m(2)^2 - (8) = 16$

$4m - 8 = 16$

$4m = 24$

$\boxed{m = 6}$

$6x^2 + 2y = n$

$6(2)^2 + 2(8) = n$

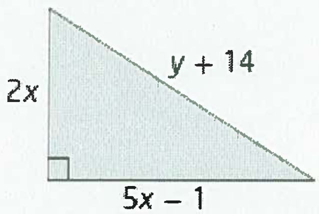
$24 + 16 = n$

$\boxed{n = 40}$

Math 20-1

KEY

Pg 451 #9: The perimeter of the right triangle is 60 m. The area of the triangle is 10y square meters.



a. Write a simplified expression for the triangle's perimeter in terms of *x* and *y*.

$P = (2x) + (5x-1) + (y+14)$

$\boxed{P = 7x + y + 13}$

b. Write a simplified expression for the triangle's area in terms of *x* and *y*.

$A = \frac{1}{2}(b)(h)$

$= \frac{1}{2}(5x-1)(2x)$

$= \frac{1}{2}(10x^2 - 2x)$

$\boxed{A = 5x^2 - x}$

c. Write a system of equations and explain how it relates to this problem.

$60 = 7x + y + 13$

$0 = 7x + y - 47$

$\boxed{y = -7x + 47}$

$\boxed{10y = 5x^2 - x}$

d. Solve the system for *x* and *y*. What are the dimensions of the triangle?

$10(-7x+47) = 5x^2 - x$

$-70x + 470 = 5x^2 - x$

$0 = 5x^2 + 69x - 470$

$0 = (x-5)(5x+94)$

$x - 5 = 0$

$x = 5$

This one works.

$5x + 94 = 0$

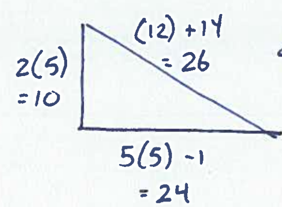
$x = \frac{-94}{5}$

Can't have a negative height.

$y = -7(5) + 47$

$= -35 + 47$

$= 12$



e. Verify your solution.

$a^2 + b^2 = c^2$

$10^2 + 24^2 = c^2$

$c = 26$ Yep!

KEY

Pg 451 #10: Two integers have a difference of -30. When the larger integer is increased by 3 and added to the square of the smaller integer, the results is 189.

a. Model the given information with a system of equations.
b. Determine the value of the integers by solving the system.
c. Verify your solution.

HINT: Let *y* be the larger integer and *x* be the smaller integer. This way you are assigning the same variables as the answer key.

Let y = larger number
x = smaller number

(A) $x - y = -30$
$y + 3 + x^2 = 189$

(B) $x - y = -30 \Rightarrow x + 30 = y$
$y + 3 + x^2 = 189 \Rightarrow y = -x^2 + 186$

$(x+30) = -x^2 + 186$

$x^2 + x - 156 = 0$

$(x+13)(x-12) = 0$

$x + 13 = 0$
$x = -13$

$x - 12 = 0$
$x = 12$

$y = (-13) + 30$
$= 17$

$y = (12) + 30$
$= 42$

Soln is $(-13, 17)$

Soln is $(12, 42)$

So integers are -13 and 17, or 12 and 42.

(C) Using $(-13, 17)$

$y = (-13) + 30$
$= 17$

$y = -(-13)^2 + 186$
$= 17$

Yep, this works.

Using $(12, 42)$

$y = (12) + 30$
$= 42$

$y = -(12)^2 + 186$
$= 42$

Yep, this works.

Math 20-1

KEY

Pg 451 #17: The monthly economic situation of a manufacturing firm is given by the following equations.

$$R = 5000x - 10x^2$$
$$R_M = 5000 - 20x$$
$$C = 300x + \frac{1}{12}x^2$$
$$C_M = 300 + \frac{1}{4}x^2$$

Where x represents the quantity sold, R represents the firm's total revenue, R_M represents marginal revenue, C represents total cost, and C_M represents the marginal cost. All costs are in dollars.

a. Maximum profit occurs when marginal revenue is equal to marginal cost. How many items should be sold to maximize profit?

b. Profit is total revenue minus total cost. What is the firm's maximum monthly profit?

A

$R_M = C_M$

$5000 - 20x = 300 + \frac{1}{4}x^2$

$0 = \frac{1}{4}x^2 + 20x - 4700$

Times by 4 to remove fractions

$0 = x^2 + 80x - 18{,}800$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-80 \pm \sqrt{6400 - 4(1)(-18{,}800)}}{2(1)}$

$= \frac{-80 \pm \sqrt{81600}}{2}$

$= \frac{-80 \pm 285.6571}{2}$

$x_1 = 102.83$ $\quad$ $x_2 = -182.83$

So 103 items sold.

B

$P = R - C$

$P = (5000x - 10x^2) - (300x + \frac{1}{12}x^2)$

$P = 5000x - 10x^2 - 300x - \frac{1}{12}x^2$

$P = \frac{-121}{12}x^2 + 4700x$

$P(103) = \frac{-121}{12}(103)^2 + 4700(103)$

$= -106{,}974.08\bar{3} + 484100$

$= 377{,}125.92$

Max monthly profit is \$377,125.92.

Careful! It is tempting to convert this to vertex form $P = -10.08(x - 233.06)^2 + 547{,}685.95$ but the other eqn already told us that max profit occurs at 103 items sold.