KEY

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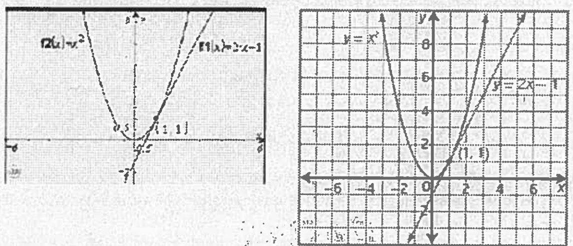
Lxx – Worksheet – Review of Systems of Equations

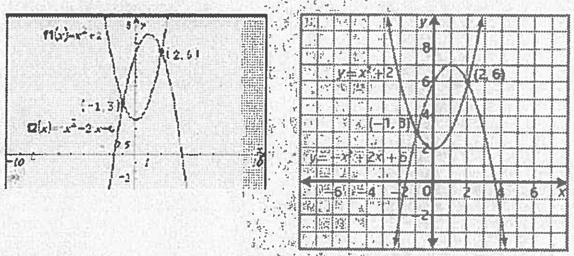
Part 1 – Concept Review

Key Ideas

- Any ordered pair (x, y) that satisfies both equations in a linear-quadratic system or in a quadratic-quadratic system is a solution to the system.
- The solution to a system can be found graphically by graphing both equations on the same coordinate grid and finding the point(s) of intersection.



Since there is only one point of intersection, the linear-quadratic system shown has one solution, (1, 1).



Since there are two points of intersection, the quadratic-quadratic system shown has two solutions, approximately $(-1, 3)$ and $(2, 6)$.

- Systems of linear-quadratic equations may have no real solution, one real solution, or two real solutions.
- Systems of quadratic-quadratic equations may have no real solution, one real solution, two real solutions, or an infinite number of real solutions.

Key Ideas

- Solve systems of linear-quadratic or quadratic-quadratic equations algebraically by using either a substitution method or an elimination method.
- To solve a system of equations in two variables using substitution,
  - isolate one variable in one equation
  - substitute the expression into the other equation and solve for the remaining variable
  - substitute the value(s) into one of the original equations to determine the corresponding value(s) of the other variable
  - verify your answer by substituting into both original equations
- To solve a system of equations in two variables using elimination,
  - if necessary, rearrange the equations so that the like terms align
  - if necessary, multiply one or both equations by a constant to create equivalent equations with a pair of variable terms with opposite coefficients
  - add or subtract to eliminate one variable and solve for the remaining variable
  - substitute the value(s) into one of the original equations to determine the corresponding value(s) of the other variable
  - verify your answer(s) by substituting into both original equations

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Part 2 – Easy Questions

Use the following equations for Q1-Q5:

$$f(x) = 2x^2 + 3x - 10$$

$$g(x) = x^2 + 3x - 6$$

Q1: Convert each equation into Vertex form.

$f(x) = (2x^2+3x) - 10$
$= 2(x^2+1.5x) - 10$
$= 2(x^2+0.75x+0.75x+0.5625) - 10 \quad -1.125$
$= 2(x+0.75)^2 - 11.125$

$g(x) = (x^2+3x) - 6$
$= (x^2+1.5x+1.5x) - 6$
$= (x^2+1.5x+1.5x+2.25) - 6 \quad -2.25$
$= (x+1.5)^2 - 8.25$

Q2: Determine the coordinates of the vertex and the zeroes for each function.

$f(x) = 2(x+0.75)^2 - 11.125$

Vertex at $(-0.75, -11.125)$

$0 = 2(x+0.75)^2 - 11.125$
$+11.125 \qquad +11.125$
$11.125 = 2(x+0.75)^2$
$\div 2 \qquad \div 2$
$5.5625 = (x+0.75)^2$
$\pm 2.358 = x + 0.75$
$x_1 = +1.61 \qquad x_2 = -3.11$

$g(x) = (x+1.5)^2 - 8.25$

Vertex at $(-1.5, -8.25)$

$0 = (x+1.5)^2 - 8.25$
$+8.25 \qquad +8.25$
$8.25 = (x+1.5)^2$
$\pm 2.872 = x + 1.5$
$x_1 = +1.37 \qquad x_2 = -4.37$

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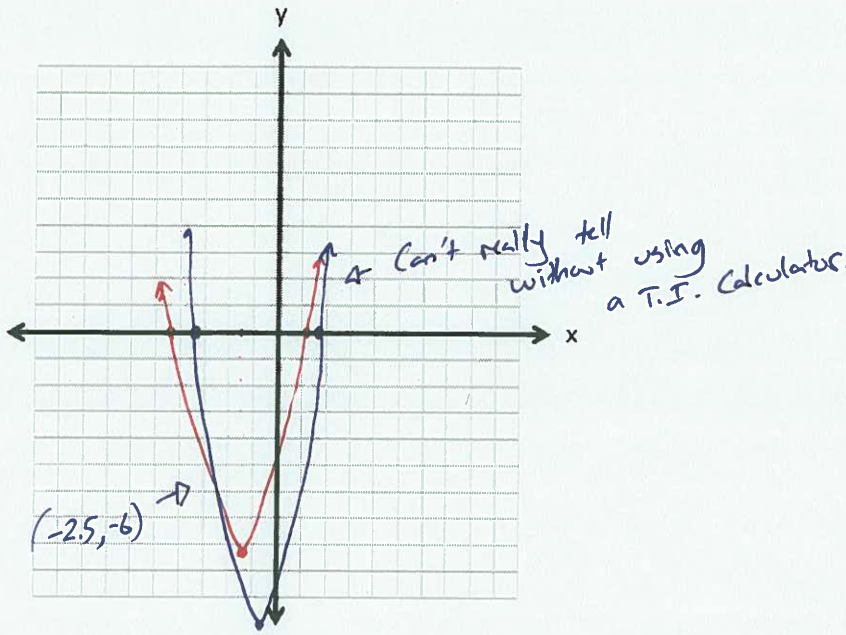
Q3: Graph each function to determine the solution to the system of equations.



Q4: Use *Elimination* to determine the solution to the system of equations.

$y = 2x^2 + 3x - 10$

$-(y = x^2 + 3x - 6)$

$0 = x^2 - 4$

$4 = x^2$

$x = \pm 2$

so $x_1 = +2$

$x_2 = -2$

$g(+2) = (+2)^2 + 3(+2) - 6$

$= 4 + 6 - 6$

$= 4$

$g(-2) = (-2)^2 + 3(-2) - 6$

$= 4 - 6 - 6$

$= -8$

so solutions are $(-2, -8)$ and $(2, 4)$

Q5: Use *Substitution* to determine the solution to the system of equations.

$y = 2x^2 + 3x - 10$

$y = x^2 + 3x - 6$

$x^2 + 3x - 6 = 2x^2 + 3x - 10$

$-x^2 \qquad -x^2$

$3x - 6 = x^2 + 3x - 10$

$-3x \qquad -3x$

$-6 = x^2 - 10$

$+10 \qquad +10$

$4 = x^2$

$x = \pm 2$

so $x_1 = +2$

$x_2 = -2$

$f(+2) = 2(+2)^2 + 3(+2) - 10$

$= 8 + 6 - 10$

$= 4$

$f(-2) = 2(-2)^2 + 3(-2) - 10$

$= 8 - 6 - 10$

$= -8$

so solutions are $(-2, -8)$ and $(2, 4)$.

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Part 3 – Medium Questions

Use the following equations for Q6-Q10:

$$f(x) = -2x^2 + 6x + 5$$

$$g(x) = x + 2$$

Q6: Convert $f(x) = -2x^2 + 6x + 5$ into Vertex form.

$f(x) = (-2x^2 + 6x) + 5$

$= -2(x^2 - 3x) + 5$

$= -2(x^2 - 1.5x - 1.5x) + 5$

$= -2(x^2 - 1.5x - 1.5x + 2.25) + 5 + 4.5$

$= -2(x - 1.5)^2 + 9.5$

Q7: Determine the coordinates of the vertex and the zeroes of *f(x)*.

Vertex at $(1.5, 9.5)$

Zeroes...

$0 = -2(x-1.5)^2 + 9.5$

$-9.5 = -2(x-1.5)^2$

$4.75 = (x-1.5)^2$

$\pm 2.179 = x - 1.5$

$x_1 = +3.68$

$x_2 = -0.68$

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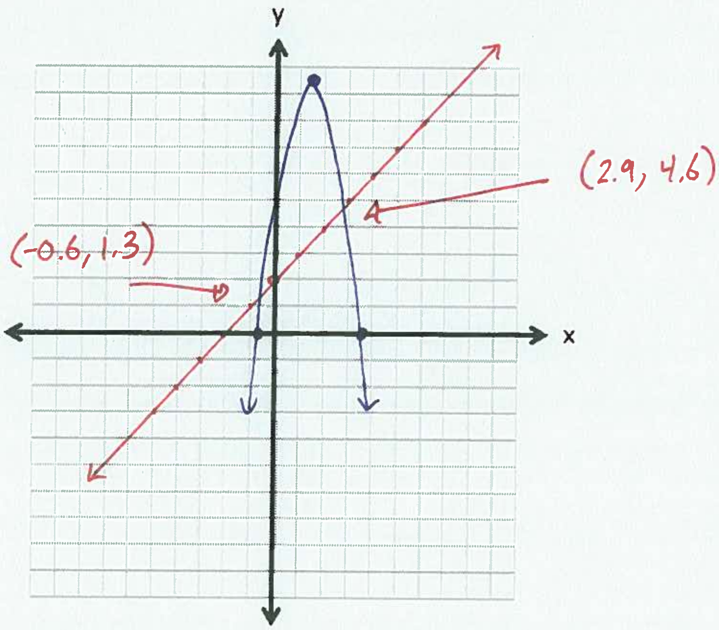
Q8: Graph each function to determine the solution to the system of equations.



Q9: Use *Elimination* to determine the solution to the system of equations.

$$\begin{aligned} & y = -2x^2+6x+5 \\ -(& y = \quad\quad\quad x+2) \\ \hline & 0 = -2x^2+5x+3 \end{aligned}$$

or $2x^2-5x-3=0$

$(x-3)(2x+1)=0$

$x-3=0$ $2x+1=0$

$x=3$ $x=-\frac{1}{2}$

$g(3)=(3)+2$ $=5$

$g(-\frac{1}{2})=(-\frac{1}{2})+2$ $=3/2$

Solns are $(-\frac{1}{2}, \frac{3}{2})$ and $(3,$

Q10: Use *Substitution* to determine the solution to the system of equations.

$y=-2x^2+6x+5$ $y=x+2$

$x+2=-2x^2+6x+5$

$2x^2-5x-3=0$

Okay... now same as last one...

Solns are $(-\frac{1}{2}, \frac{3}{2})$ and $(3,5)$.

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Part 4 – Harder Questions (Exact Values)

Use the following equations for Q11-Q12:

$$f(x) = 2x^2 + 5x - 10$$

$$g(x) = -x^2 + 3x + 7$$

Q11: Use *Elimination* to determine the ***exact*** solution to the system of equations.

$y = 2x^2 + 5x - 10$

$-(y = -x^2 + 3x + 7)$

$0 = 3x^2 + 2x - 17$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 4(3)(-17)}}{2(3)}$

$x = \frac{-2 \pm \sqrt{4 + 204}}{6} = \frac{-2 \pm \sqrt{208}}{6}$

$= \frac{-2 \pm 4\sqrt{13}}{6} = \frac{-1 \pm 2\sqrt{13}}{3}$

$x_1 = \frac{-1 + 2\sqrt{13}}{3}$ $\quad x_2 = \frac{-1 - 2\sqrt{13}}{3}$

$g(x_1) = -(x_1)^2 + 3(x_1) + 7$

$= -\left(\frac{-1 + 2\sqrt{13}}{3}\right)\left(\frac{-1 + 2\sqrt{13}}{3}\right) + 3\left(\frac{-1 + 2\sqrt{13}}{3}\right) + 7$

$= -1\left(\frac{1 - 4\sqrt{13} + 4(13)}{9}\right) + (-1 + 2\sqrt{13}) + 7$

$= -\frac{1}{9}(53 - 4\sqrt{13}) + 2\sqrt{13} + 6$

$= -\frac{53}{9} + \frac{4}{9}\sqrt{13} + 2\sqrt{13} + 6$

$= \frac{1}{9} + \frac{22}{9}\sqrt{13}$

Solution is $\left(\frac{-1}{3} + \frac{2}{3}\sqrt{13}, \ \frac{1}{9} + \frac{22}{9}\sqrt{13}\right)$

$g(x_2) = -(x_2)^2 + 3(x_2) + 7$

$= -\left(\frac{-1 - 2\sqrt{13}}{3}\right)\left(\frac{-1 - 2\sqrt{13}}{3}\right) + 3\left(\frac{-1 - 2\sqrt{13}}{3}\right) + 7$

$= -1\left(\frac{1 + 4\sqrt{13} + 4(13)}{9}\right) + (-1 - 2\sqrt{13}) + 7$

$= -\frac{1}{9}(53 + 4\sqrt{13}) - 2\sqrt{13} + 6$

$= -\frac{53}{9} - \frac{4}{9}\sqrt{13} - 2\sqrt{13} + 6$

$= \frac{1}{9} - \frac{22}{9}\sqrt{13}$

Solution is

$\left(-\frac{1}{3} - \frac{2}{3}\sqrt{13}, \ \frac{1}{9} - \frac{22}{9}\sqrt{13}\right)$

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Part 5 – Word Problems

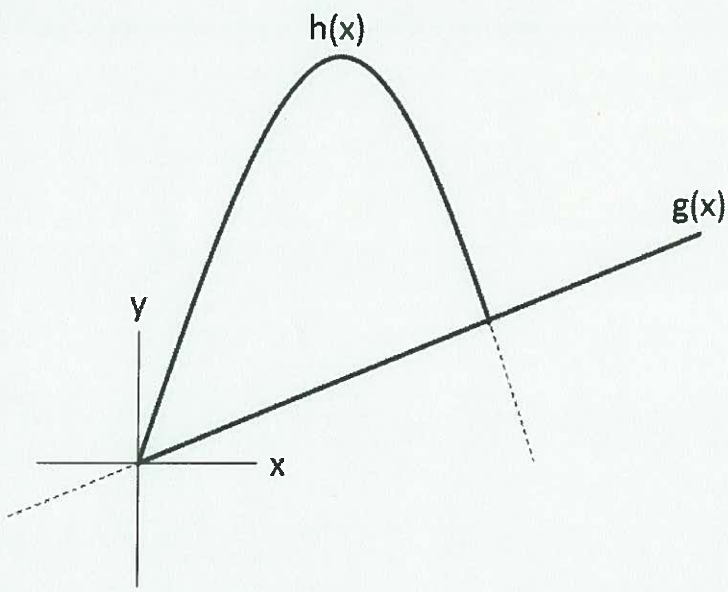
A kangaroo is jumping horizontally. The height as a function of horizontal distance, in meters, is:

$$h(x) = -x^2 + 5x$$

The kangaroo is jumping uphill. The height of the hill as a function of horizontal distance, in meters, is:

$$g(x) = \frac{1}{2}x$$

The quick sketch of the jump is as follows:



Q12: Algebraically determine the horizontal distance travelled by the kangaroo.

$$\begin{array}{rl} & y = -x^2 + 5x + 0 \\ - & \left(y = \quad \tfrac{1}{2}x + 0\right) \\ \hline & 0 = -x^2 + 4.5x + 0 \end{array}$$

$$x^2 \quad 4.5x = 0$$

$$(x)(x - 4.5) = 0$$

$x = 0$ $\qquad$ $x = 4.5$

Kangaroo travels a horizontal distance of 4.5 m

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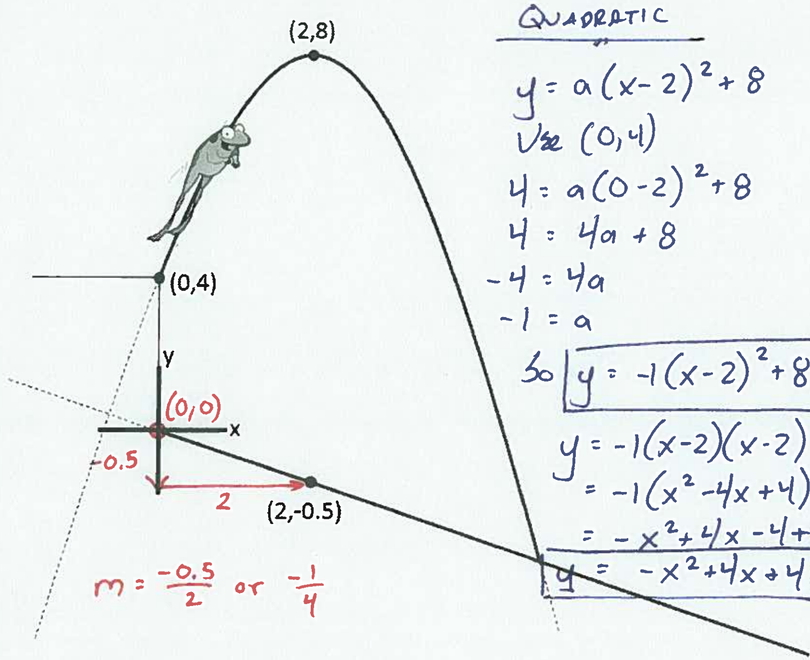
A frog jumps off of a 4cm tall ledge onto a downward sloped hill, per the diagram below.



QUADRATIC

$y = a(x-2)^2 + 8$

Use $(0,4)$

$4 = a(0-2)^2 + 8$

$4 = 4a + 8$

$-4 = 4a$

$-1 = a$

So $\boxed{y = -1(x-2)^2 + 8}$

$y = -1(x-2)(x-2) + 8$

$= -1(x^2 - 4x + 4) + 8$

$= -x^2 + 4x - 4 + 8$

$\boxed{y = -x^2 + 4x + 4}$

LINEAR

$y = mx + b$

$\boxed{y = -\frac{1}{4}x + 0}$

All coordinates are in centimeters.

Q13: The frog lands on the hill after travelling a horizontal distance of ***a.bc*** centimeters, where ***a, b,*** and ***c*** are ___, ___, and ___.

(Record your 3-digit answer in the Numerical Response boxes below)

5	0	4	

$y = -x^2 + 4x + 4$

$-(y = -\frac{1}{4}x)$

$0 = -x^2 + 4.25x + 4$

$x^2 - 4.25x - 4 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{4.25 \pm \sqrt{18.0625 - 4(1)(-4)}}{2(1)}$

$= \frac{4.25 \pm \sqrt{34.0625}}{2} = \frac{4.25 \pm 5.83631}{2}$

$x_1 = 5.04$ — This one works.

$x_2 = -0.79$ — Doesn't make sense.

Frog jumps horizontal distance of 5.04 cm