L35 – Quadratic Inequalities in One Variable

Key Ideas

The solution to a quadratic inequality in one variable is:

a. No values
b. One value
c. Infinite values

If the roots are easy to identify, use those values to identify the domain.

If roots are not easy to identify, then use the Quadratic Formula to solve for the roots, and use those roots to identify the domain.

KEY

Part 1 – Solving a Quadratic Inequality in One Variable using Graphing

Q1: The equation $y = 0.5x^2 + 2x - 6$ is graphed below:
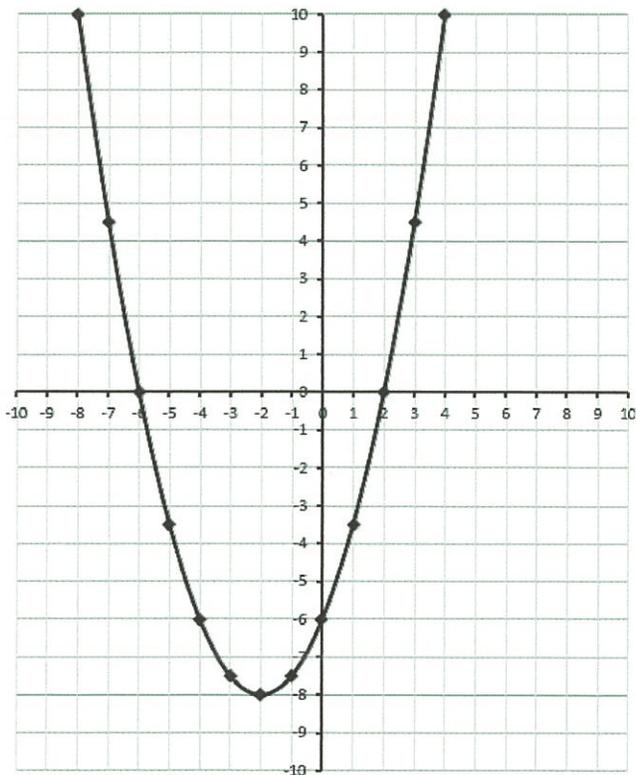
Use this graph to solve the following linear inequalities:

$0.5x^2 + 2x - 6 \geq 0$

$\{x \mid x \leq -6 \text{ or } x \geq 2, x \in \mathbb{R}\}$

$0.5x^2 + 2x - 6 \leq 0$

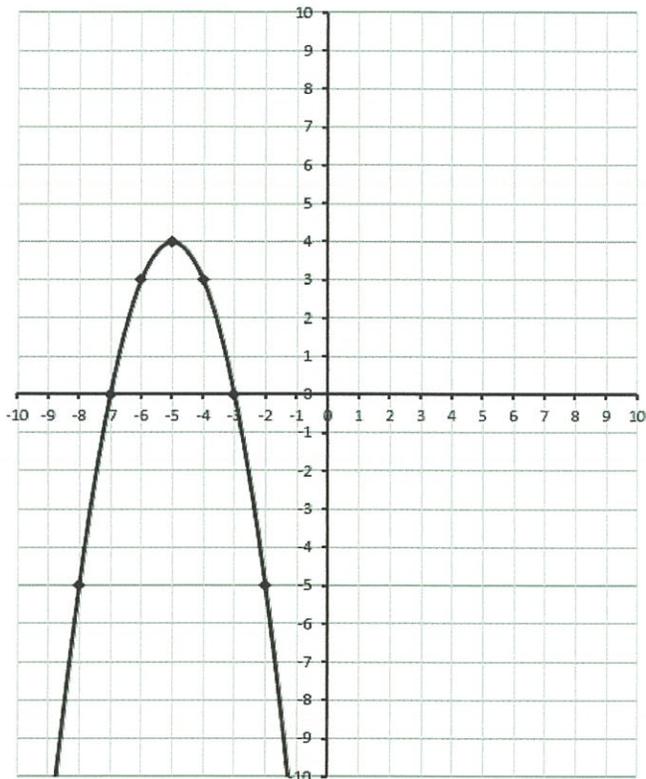$\{x \mid -6 \leq x \leq 2, x \in \mathbb{R}\}$

$0.5x^2 + 2x - 6 > 0$

$\{x \mid x < -6 \text{ or } x > 2, x \in \mathbb{R}\}$

$0.5x^2 + 2x - 6 < 0$

$\{x \mid -6 < x < 2, x \in \mathbb{R}\}$

Q2: The equation $y = -x^2 - 10x - 21$ is graphed below:

Use this graph to solve the following linear inequalities:

$-x^2 - 10x - 21 \geq 0$

$\{x \mid -7 \leq x \leq -3, x \in \mathbb{R}\}$

$-x^2 - 10x - 21 \leq 0$

$\{x \mid x \leq -7 \text{ or } x \geq -3, x \in \mathbb{R}\}$

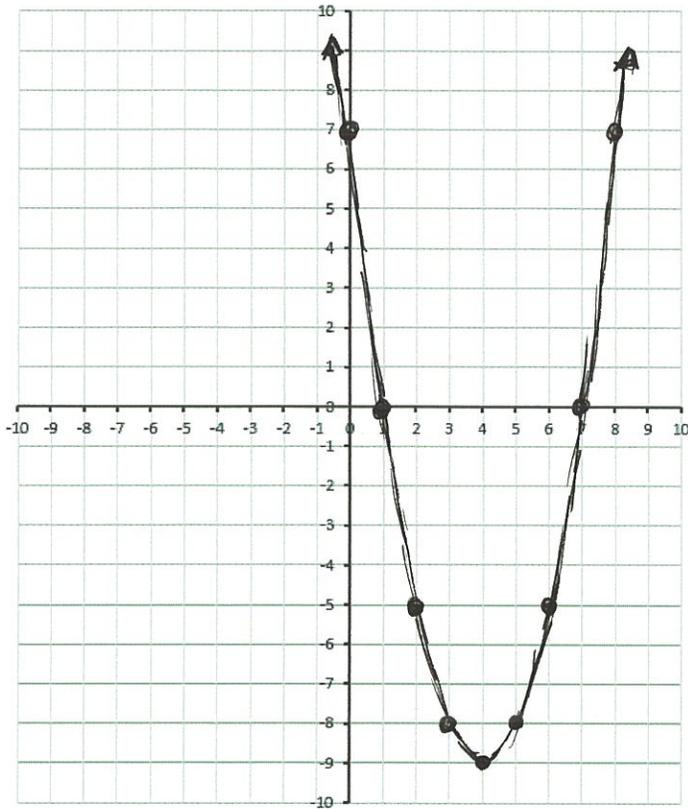$-x^2 - 10x - 21 > 0$

$\{x \mid -7 < x < -3, x \in \mathbb{R}\}$

$-x^2 - 10x - 21 < 0$

$\{x \mid x < -7 \text{ or } x > -3, x \in \mathbb{R}\}$

Q3: Graph the equation $y = x^2 - 8x + 7$ to solve several linear inequalities.

Use this graph to solve the following linear inequalities:

$x^2 - 8x + 7 \geq 0$

$$\{x \mid x \leq 1 \text{ or } x \geq 7, \ x \in \mathbb{R}\}$$

$x^2 - 8x + 7 \leq 0$

$$\{x \mid 1 \leq x \leq 7, \ x \in \mathbb{R}\}$$
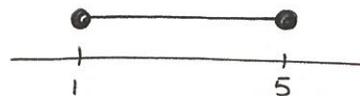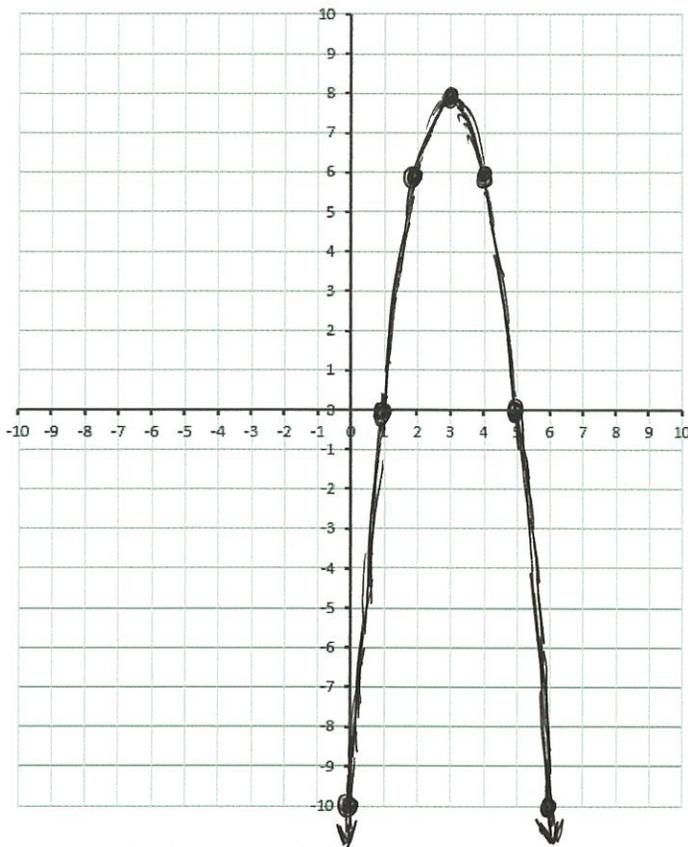
$x^2 - 8x + 7 > 0$

$$\{x \mid x < 1 \text{ or } x > 7, \ x \in \mathbb{R}\}$$

$x^2 - 8x + 7 < 0$

$$\{x \mid 1 < x < 7, \ x \in \mathbb{R}\}$$

🔳 KEY 🔳

Q4: Use graphing to solve the inequality $-2x^2 + 12x - 10 \geq 0$.

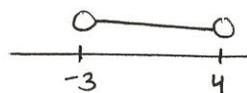$\{x \mid 1 \leq x \leq 5, \, x \in \mathbb{R}\}$
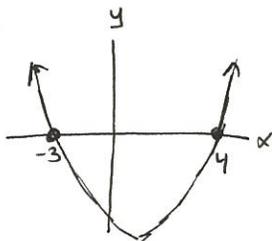
Part 2 – Solving a Quadratic Inequality in One Variable using Factoring and Sketching

Q5: Use sketching to solve the inequality $x^2 - x - 12 < 0$.

$y = x^2 - x - 12$

$0 = (x-4)(x+3)$

$x - 4 = 0 \qquad x + 3 = 0$

$x = 4 \qquad\qquad x = -3$

$\{x \mid -3 < x < 4, \, x \in \mathbb{R}\}$

Q6: Use sketching to solve the inequality $2x^2 - x - 15 \geq 0$.

$y = 2x^2 - x - 15$

$\begin{array}{l} +5 \quad -6 \\ \square + \square = -1 \\ \square \times \square = -30 \end{array}$
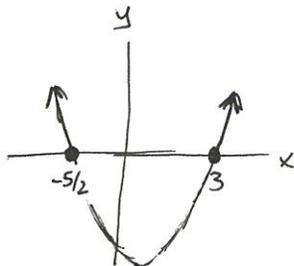
$y = 2x^2 + 5x - 6x - 15$

$y = (2x^2 + 5x) + (-6x - 15)$

$y = x(2x+5) - 3(2x+5)$

$y = (2x+5)(x-3)$

$0 = (2x+5)(x-3)$

$2x+5 = 0 \qquad x - 3 = 0$

$x = -\frac{5}{2} \qquad\quad x = 3$

$\{x \mid x \leq -\frac{5}{2} \text{ or } x \geq 3, \, x \in \mathbb{R}\}$

Q7: Use sketching to solve the inequality $2x^2 - 2x - 24 \leq 0$.
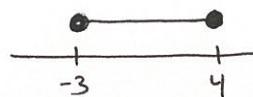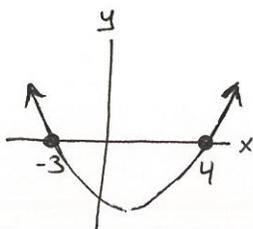
$y = 2(x^2 - x - 12)$
$y = 2(x-4)(x+3)$
$0 = 2(x-4)(x+3)$

$x - 4 = 0 \qquad x + 3 = 0$
$x = 4 \qquad\quad x = -3$

$\{x \mid -3 \leq x \leq 4, x \in \mathbb{R}\}$

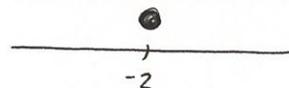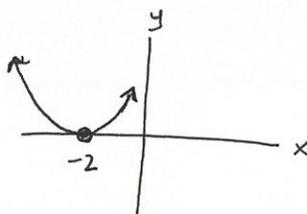**Q8:** Use sketching to solve the inequality $x^2 + 4x + 4 \leq 0$.

$y = x^2 + 4x + 4$
$y = (x+2)(x+2)$
$0 = (x+2)(x+2)$

$x + 2 = 0 \qquad x + 2 = 0$
$x = -2 \qquad\quad x = -2$

$\{x = -2\}$

Part 3 – Solving a Quadratic Inequality in One Variable using Factoring and Test Points

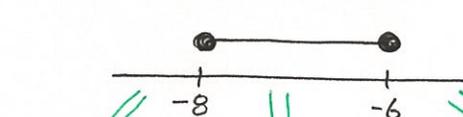**Q9:** Use test points to solve the inequality $x^2 + 12x + 32 \leq 0$.

$y = x^2 + 12x + 32$
$y = (x+4)(x+8)$
$0 = (x+4)(x+8)$

$x + 4 = 0 \qquad x + 8 = 0$
$x = -4 \qquad\quad x = -8$

$\{x \mid -8 \leq x \leq -6, x \in \mathbb{R}\}$

Try $x = -10$
$(-10)^2 + 12(-10) + 32 \leq 0$
$100 - 120 + 32 \leq 0$
$12 \leq 0$
FALSE

Try $x = -8$
$(-8)^2 + 12(-8) + 32 \leq 0$
$64 - 96 + 32 \leq 0$
$0 \leq 0$
TRUE

Try $x = 0$
$(0)^2 + 12(0) + 32 \leq 0$
$0 + 0 + 32 \leq 0$
$32 \leq 0$
FALSE

Q10: Use test points to solve the inequality $-x^2 + 13x - 40 < 0$.

$y = -x^2 + 13x - 40$
$y = -1(x^2 - 13x + 40)$
$y = -1(x-5)(x-8)$
$0 = -1(x-5)(x-8)$

$x - 5 = 0 \qquad x - 8 = 0$
$x = 5 \qquad\quad x = 8$

$\{x \mid x < 5 \text{ or } x > 8, x \in \mathbb{R}\}$

Try $x = 0$
$-(0)^2 + 13(0) - 40 < 0$
$0 + 0 - 40 < 0$
$-40 < 0$
TRUE

Try $x = 6$
$-(6)^2 + 13(6) - 40 < 0$
$-36 + 78 - 40 < 0$
$2 < 0$
FALSE

Try $x = 10$
$-(10)^2 + 13(10) - 40 < 0$
$-100 + 130 - 40 < 0$
$-10 < 0$
TRUE

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Part 4 – Solving a Quadratic Inequality in One Variable using Quadratic Formula and Sketching

Q11: Determine the solutions to the inequality $x^2 + 4x + 2 \leq 0$.

$$y = x^2 + 4x + 2$$
$$0 = x^2 + 4x + 2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 8}}{2}$$
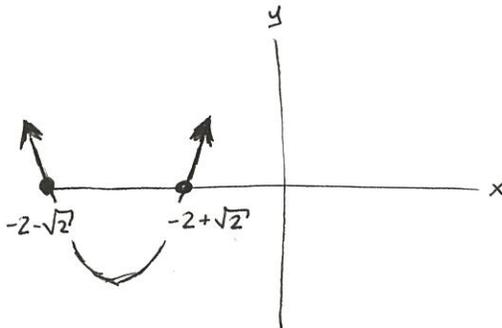
$$x = \frac{-4 \pm \sqrt{8}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{2}}{2}$$

$$x = -2 \pm \sqrt{2}$$

$$\sqrt{8} = \sqrt{2^2 \cdot 2}$$
$$= 2\sqrt{2}$$

$$\boxed{x_1 = -2 + \sqrt{2}} \qquad \boxed{x_2 = -2 - \sqrt{2}}$$
$$\approx -0.59 \qquad\qquad \approx -3.41$$

$$\{x \mid -2 - \sqrt{2} \leq x \leq -2 + \sqrt{2}, x \in \mathbb{R}\}$$