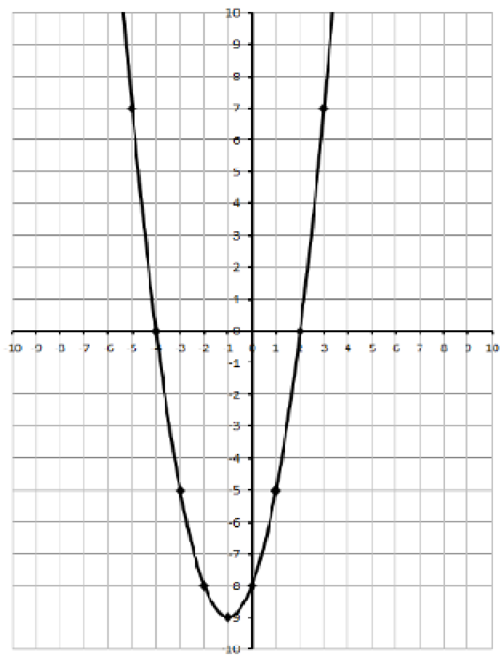
Lxx – Worksheet – 9.2 Quadratic Inequalities in One Variable

Part 1 – Basic Concept

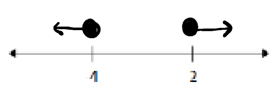
Graphed below is the function $y = x^2 + 2x - 8$.



Q1: Use your knowledge of this graph to determine the x-values that satisfy the following inequalities:

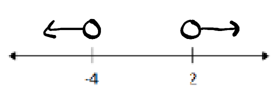
a. $x^2 + 2x - 8 \geq 0$

$y \geq 0$



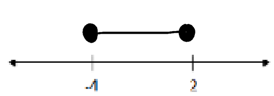
$\{x \mid x \leq -4 \text{ or } x \geq 2,\ x \in \mathbb{R}\}$

b. $x^2 + 2x - 8 > 0$



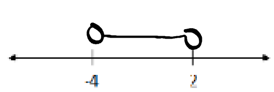
$\{x \mid x < -4 \text{ or } x > 2,\ x \in \mathbb{R}\}$

c. $x^2 + 2x - 8 \leq 0$



$\{x \mid -4 \leq x \leq 2,\ x \in \mathbb{R}\}$

d. $x^2 + 2x - 8 < 0$



$\{x \mid -4 < x < 2,\ x \in \mathbb{R}\}$

Part 2 – Easy Questions

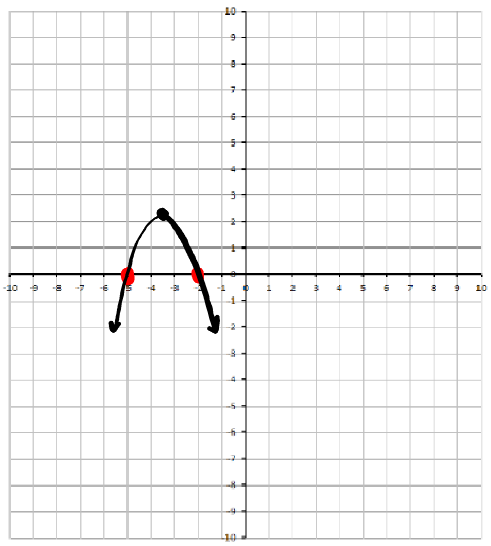
Q2: Solve the inequality $-x^2 - 7x \geq 10$

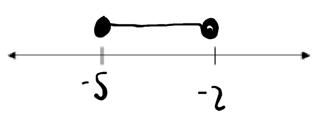
$\ldots solved\ as - x^2 - 7x - 10 \geq 0$

$-1(x^2+7x+10) \geq 0$

$-1(x+2)(x+5) \geq 0$

$x=-2$ $x=-5$



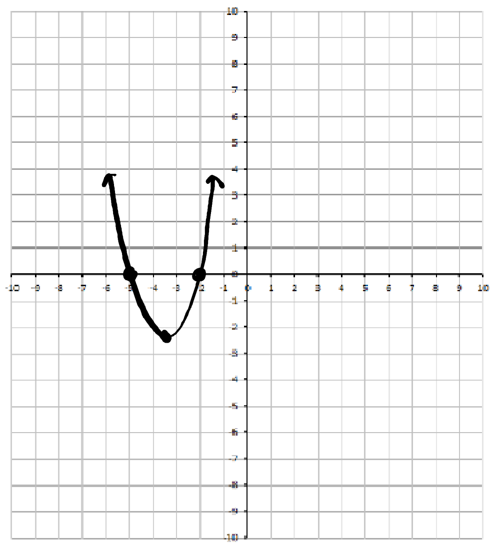


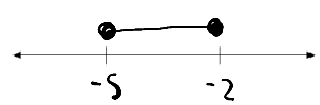
$\{x \mid -5 \leq x \leq -2, x \in \mathbb{R}\}$

$\ldots solved\ as\ 0 \geq x^2 + 7x + 10$

$\ldots or\ x^2 + 7x + 10 \leq 0$

$(x+2)(x+5) \leq 0$

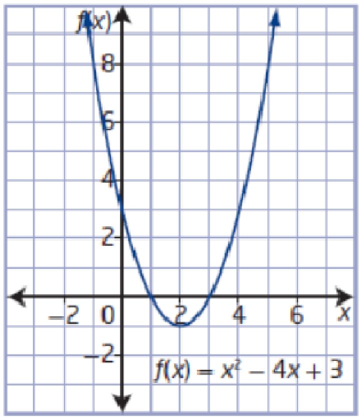




$\{x \mid -5 \leq x \leq -2, x \in \mathbb{R}\}$

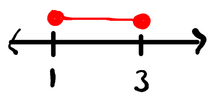
Part 3 – Textbook Questions

Pg 484 #1: Consider the graph of the quadratic function $(x) = x^2 - 4x + 3$.



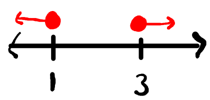
What is the solution to

a. $x^2 - 4x + 3 \le 0$



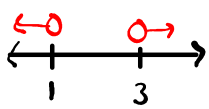
$\{x \mid 1 \le x \le 3, x \in \mathbb{R}\}$

b. $x^2 - 4x + 3 \ge 0$



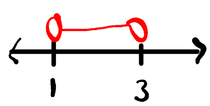
$\{x \mid x \le 1$ or $x \ge 3, x \in \mathbb{R}\}$

c. $x^2 - 4x + 3 > 0$



$\{x \mid x < 1$ or $x > 3, x \in \mathbb{R}\}$

d. $x^2 - 4x + 3 < 0$



$\{x \mid 1 < x < 3, x \in \mathbb{R}\}$

Pg 484 #3ab: Is the value of *x* a solution to the given inequality?

$x = 4$ *for* $x^2 - 3x - 10 > 0$

$(4)^2 - 3(4) - 10 > 0$

$16 - 12 - 10 > 0$

$-6 > 0$

Nope

$x = 1$ *for* $x^2 + 3x - 4 \ge 0$

$(1)^2 + 3(1) - 4 \ge 0$

$1 + 3 - 4 \ge 0$

$0 \ge 0$

Yep!

Pg 484 #3cd: Is the value of x a solution to the given inequality?

$x = -2 \; for \; x^2 + 4x + 3 < 0$

$(-2)^2 + 4(-2) + 3 < 0$

$4 - 8 + 3 < 0$

$-1 < 0$

Yep!

$x = -3 \; for \; -x^2 - 5x - 4 \leq 0$

$-(-3)^2 - 5(-3) - 4 \leq 0$

$-9 + 15 - 4 \leq 0$

$2 \leq 0$

Nope

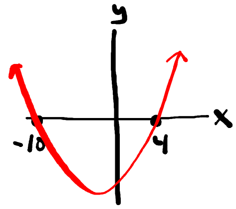
Pg 484 #4bd: Use <u>roots and test points</u> to determine the solution to each inequality.

a. $x(x+6) \geq 40$

$x^2 + 6x \geq 40$

$x^2 + 6x - 40 \geq 0$

$(x+10)(x-4) \geq 0$



Test point $x = 0$

$(0)(0+6) \geq 40$

$0 \geq 40$

Nope

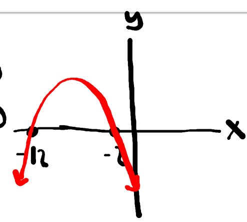
So point between zeroes doesn't work. Must be outside region.

$\{x \mid x \leq -10 \text{ or } x \geq 4, x \in \mathbb{R}\}$

b. $-x^2 - 14x - 24 < 0$

$-1(x^2 + 14x + 24) < 0$

$-1(x+2)(x+12) < 0$



Test point $x = -4$

$-(-4)^2 - 14(-4) - 24 < 0$

$-16 + 56 - 24 < 0$

$16 < 0$

Nope

So point between zeroes doesn't work. Must be outside region.

$\{x \mid x < -12 \text{ or } x > -2, x \in \mathbb{R}\}$

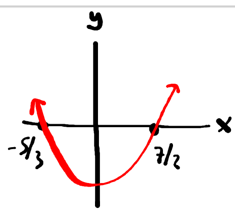
c. $6x^2 > 11x + 35$

$6x^2 - 11x - 35 > 0$

$6x^2 + 10x - 21x - 35 > 0$

$(3x+5)(2x-7) > 0$

$x = -\frac{5}{3}$ $\quad$ $x = \frac{7}{2}$



Test point $x = 0$

$6(0)^2 > 11(0) + 35$

$0 > 35$

Nope

So point between zeroes doesn't work. Must be outside region

$\{x \mid x < -\frac{5}{3} \text{ or } x > \frac{7}{2}, x \in \mathbb{R}\}$

d. $8x + 5 \leq -2x^2$

$2x^2 + 8x + 5 \leq 0$

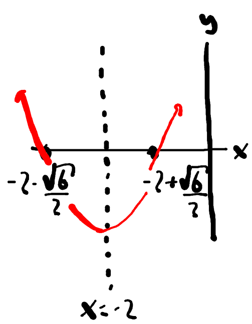
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-8 \pm \sqrt{64 - 40}}{4}$

$= -2 \pm \frac{\sqrt{24}}{4}$

$= -2 \pm \frac{2\sqrt{6}}{4}$

$= -2 \pm \frac{\sqrt{6}}{2}$



Test point $x = -2$

$8(-2) + 5 \leq -2(-2)^2$

$-16 + 5 \leq -8$

$-11 \leq -8$

Yep!

So point in between zeroes works. This is our region.

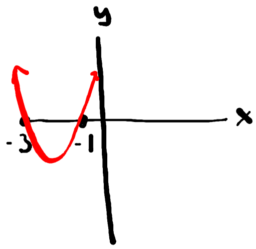
$\{x \mid -2 - \frac{\sqrt{6}}{2} \leq x \leq -2 + \frac{\sqrt{6}}{2}, x \in \mathbb{R}\}$

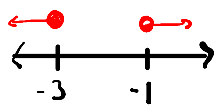
Pg 484 #5bd: Determine the solution to each inequality.

$x^2 + 3 \ge -4x$

$x^2+4x+3\ge 0$

$(x+3)(x+1)\ge 0$





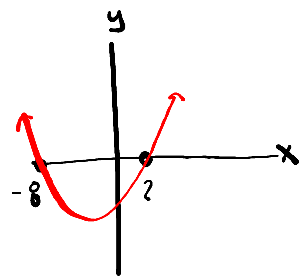
$\{x \mid x\le -3 \text{ or } x\ge -1, x\in\mathbb{R}\}$

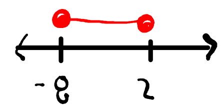
$-6x \ge x^2 - 16$

$0\ge x^2+6x-16$

$x^2+6x-16\le 0$

$(x+8)(x-2)\le 0$





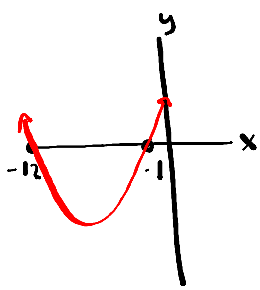
$\{x \mid -8\le x\le 2, x\in\mathbb{R}\}$

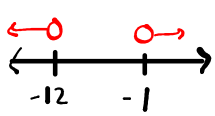
Pg 484 #6bd: Determine the solution to each inequality.

$x^2 + 13x > -12$

$x^2+13x+12>0$

$(x+12)(x+1)>0$





$\{x \mid x< -12 \text{ or } x> -1, x\in\mathbb{R}\}$

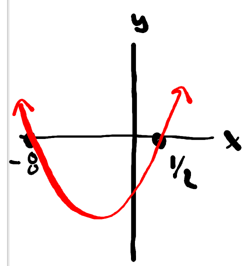
$2x^2 \ge 8 - 15x$

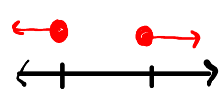
$2x^2+15x-8\ge 0$

$2x^2+16x-1x-8\ge 0$

$(2x-1)(x+8)\ge 0$

$x=\frac{1}{2}$ $\quad$ $x=-8$





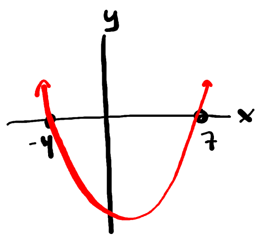
$\{x \mid x\le -8 \text{ or } x\ge \frac{1}{2}, x\in\mathbb{R}\}$

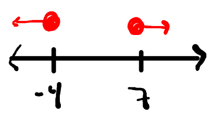
Pg 484 #7bd: Determine the solution to each inequality.

$x^2 \geq 3x + 28$

$x^2 - 3x - 28 \geq 0$

$(x+4)(x-7) \geq 0$





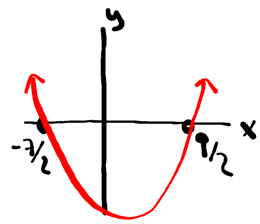
$\{x \mid x \leq -4 \text{ or } x \geq 7, x \in \mathbb{R}\}$

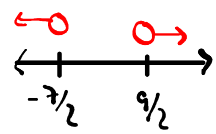
$4x(x-1) > 63$

$4x^2 - 4x - 63 > 0$

$(2x-9)(2x+7) > 0$

$x = \frac{9}{2}$ $\quad x = -\frac{7}{2}$





$\{x \mid x < -\frac{7}{2} \text{ or } x > \frac{9}{2}, x \in \mathbb{R}\}$

Pg 484 #9ab: Solve each inequality.

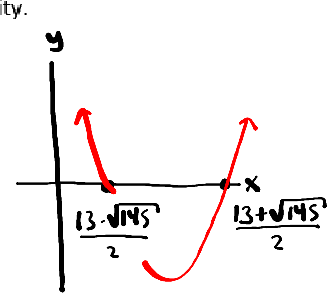
a. $x^2 - 3x + 6 \leq 10x$

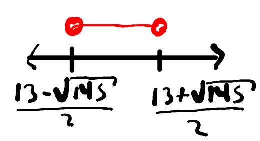
$x^2 - 13x + 6 \leq 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{13 \pm \sqrt{169 - 24}}{2}$

$= \frac{13 \pm \sqrt{145}}{2}$



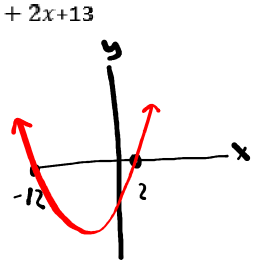


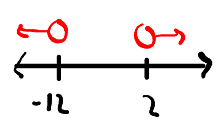
$\{x \mid \frac{13-\sqrt{145}}{2} \leq x \leq \frac{13+\sqrt{145}}{2}, x \in \mathbb{R}\}$

b. $2x^2 + 12x - 11 > x^2 + 2x + 13$

$x^2 + 10x - 24 > 0$

$(x+12)(x-2) > 0$





$\{x \mid x < -12 \text{ or } x > 2, x \in \mathbb{R}\}$

Pg 484 #9cd: Solve each inequality.

c. $x^2 - 5x < 3x^2 - 18x + 20$

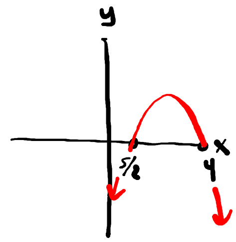
$-2x^2+13x-20<0$

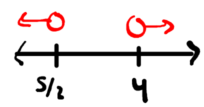
$-1(2x^2-13x+20)<0$

$-1(2x^2-5x-8x+20)<0$

$-1(2x-5)(x-4)<0$

$x=5/2$ $x=4$





$\{x \mid x<\frac{5}{2} \text{ or } x>4, x\in \mathbb{R}\}$

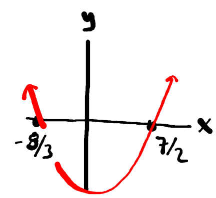
d. $-3(x^2 + 4) \le 3x^2 - 5x - 68$

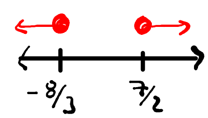
$-3x^2-12\le 3x^2-5x-68$

$0\le 6x^2-5x-56$

$0\le (2x-7)(3x+8)$

$x=7/2$ $x=-8/3$





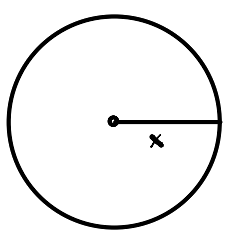
$\{x \mid x\le -8/3 \text{ or } x\ge 7/2, x\in \mathbb{R}\}$

Pg 484 #11: Many farmers in Southern Alberta irrigate their crops. A center-pivot irrigation system spreads water in a circular pattern over a crop.

a. Suppose that Murray has acquired rights to irrigate up to 63 ha (hectares) of his land. Write an inequality to model the maximum circular area, in square meters, that he can irrigate.

b. What are the possible radii of circles that Murray can irrigate? Express your answer as an exact value.

c. Express your answer in part *B)* to the nearest hundredth of a meter.



(A) 1 hectare = 10,000 m^2

63 hectares = 630,000 m^2

0 ≤ Area of Circle ≤ Max allowed Area

$0 \le \boxed{\pi x^2 \le 630{,}000}$

(B) $x^2 \le \frac{630{,}000}{\pi}$

$x \le \sqrt{\frac{630{,}000}{\pi}}$

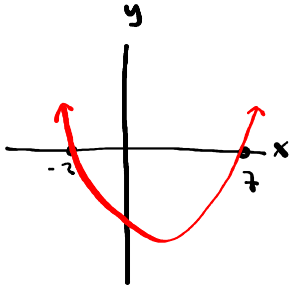
So $\boxed{0 \le x \le \sqrt{\frac{630{,}000}{\pi}}}$

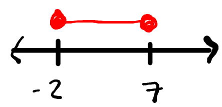
(C) $0 \le x \le 447.811599108$

$\boxed{0 \le x \le 447.81\,m}$

Pg 484 #15abc: For each of the following, give an inequality that has the given solution.

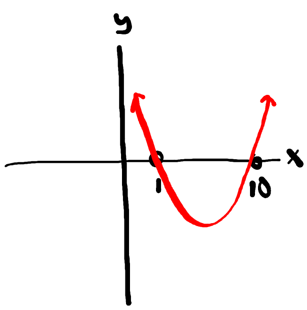
a. $-2 \le x \le 7$

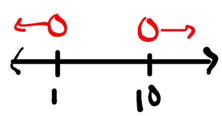




$(x-2)(x+7) \le 0$

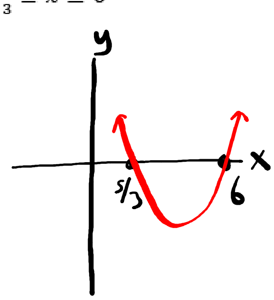
b. $x < 1 \; or \; x > 10$

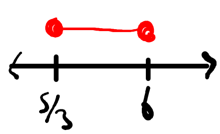




$(x-1)(x-10) > 0$

c. $\frac{5}{3} \le x \le 6$





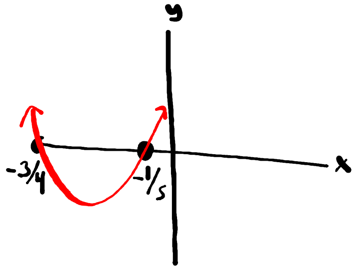
$a(x-5/3)(x-6) \le 0$

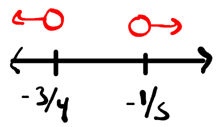
$3(x-5/3)(x-6) \le 0$

$(3x-5)(x-6) \le 0$

Pg 484 #15defg: For each of the following, give an inequality that has the given solution.

d. $x < -\frac{3}{4}$ *or* $x > -\frac{1}{5}$



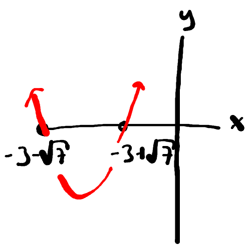


$a(x+3/4)(x+1/5) > 0$

$(4)(x+3/4)(5)(x+1/5) > 0$

$(4x+3)(5x+1) > 0$

e. $x \le -3-\sqrt{7}$ *or* $x \ge -3+\sqrt{7}$



$[x-(-3-\sqrt{7})][x-(-3+\sqrt{7})] \ge 0$

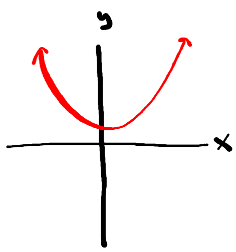
$(x+3+\sqrt{7})(x+3-\sqrt{7}) \ge 0$

$x^2+3x+\sqrt{7}x+3x+9+3\sqrt{7}-\sqrt{7}x-3\sqrt{7}-7 \ge 0$

$x^2+6x+2 \ge 0$

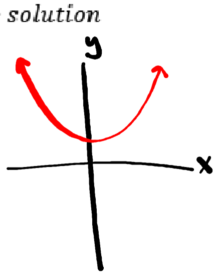
	x	$+3$	$+\sqrt{7}$
x	x^2	$+3x$	$+\sqrt{7}x$
$+3$	$+3x$	$+9$	$+3\sqrt{7}$
$-\sqrt{7}$	$-\sqrt{7}x$	$-3\sqrt{7}$	-7

f. $x \in R$



$x^2+1 > 0$

g. *No solution*



$x^2+1 < 0$