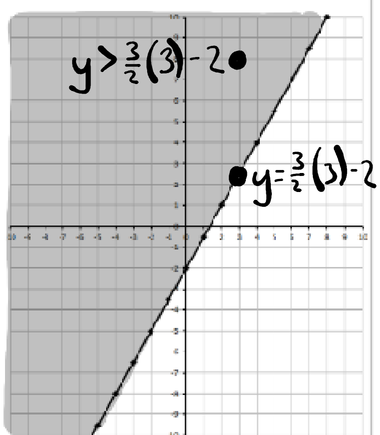
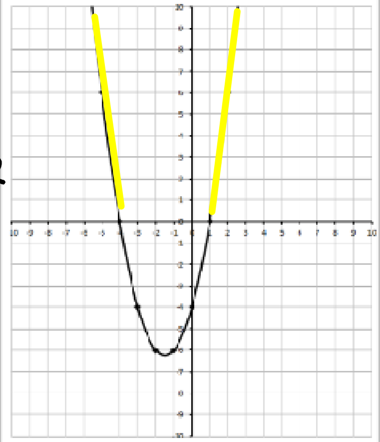
Lxx – Worksheet – 9.3 Quadratic Inequalities in Two Variables

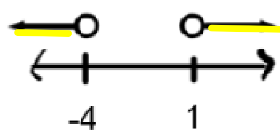
Part 1 – Three Basic Concepts *(9.1, 9,2 , 9.3)*

$y \geq \frac{3}{2}x - 2$

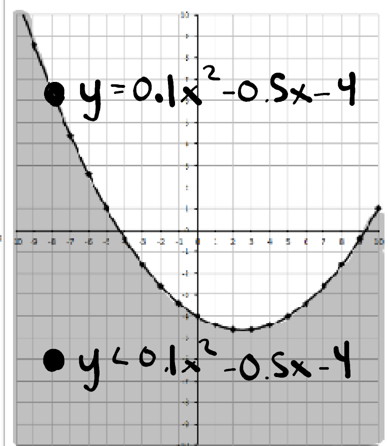


$x^2 + 3x - 4 > 0$





$y \leq 0.1x^2 - 0.5x - 4$



Part 2 – Textbook Questions

Pg 496 #1: Which of the ordered pairs are solutions to the inequality?

$y < x^2 + 3$

{ (2,6) , (4, 20) , (-1,3) , (-3,12) }

$6 < 2^2+3$	$20 < 4^2+3$	$3 < (-1)^2+3$	$12 < (-3)^2+3$
$6 < 4+3$	$20 < 16+3$	$3 < 1+3$	$12 < 9+3$
$6 < 7$	$20 < 19$	$3 < 4$	$12 < 12$
Yes!	Nope	Yes!	Nope

$y \leq -x^2 + 3x - 4$

{ (2,-2) , (4, -1) , (0,-6) , (-2,-15) }

$-2 \leq -(2)^2+3(2)-4$	$-1 \leq -(4)^2+3(4)-4$	$-6 \leq -(0)^2+3(0)-4$	$-15 \leq -(-2)^2+3(-2)-4$
$-2 \leq -4+6-4$	$-1 \leq -16+12-4$	$-6 \leq -4$	$-15 \leq -4-6-4$
$-2 \leq -2$	$-1 \leq -8$	Yes!	$-15 \leq -14$
Yes!	Nope		Yes!

$y > 2x^2 + 3x + 6$

{ (-3,5) , (0, -6) , (2,10) , (5, 40) }

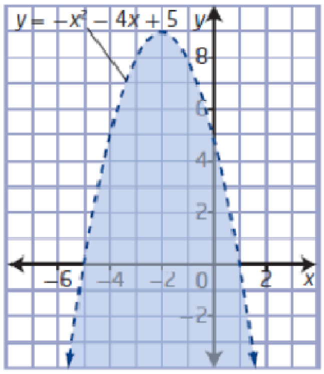
$5 > 2(-3)^2+3(-3)+6$	$-6 > 2(0)^2+3(0)+6$	$10 > 2(2)^2+3(2)+6$	$40 > 2(5)^2+3(5)+6$
$5 > 18-9+6$	$-6 > 6$	$10 > 8+6+6$	$40 > 50+15+6$
$5 > 15$	Nope	$10 > 20$	$40 > 71$
Nope		Nope	Nope

$y \geq -\frac{1}{2}x^2 - x + 5$

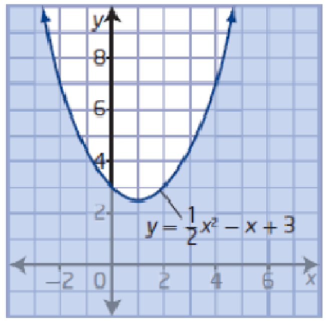
{ (-4,2) , (-1, 5) , (1,3.5) , (3 ,2.5) }

$2 \geq -\frac{1}{2}(-4)^2-(-4)+5$	$5 \geq -\frac{1}{2}(-1)^2-(-1)+5$	$3.5 \geq -\frac{1}{2}(1)^2-(1)+5$	$2.5 \geq -\frac{1}{2}(3)-(3)+5$
$2 \geq -8+4+5$	$5 \geq -0.5+1+5$	$3.5 \geq -0.5-1+5$	$2.5 \geq -1.5-3+5$
$2 \geq 1$	$5 \geq 5.5$	$3.5 \geq 3.5$	$2.5 \geq 0.5$
Yes!	Nope	Yes!	Yes!

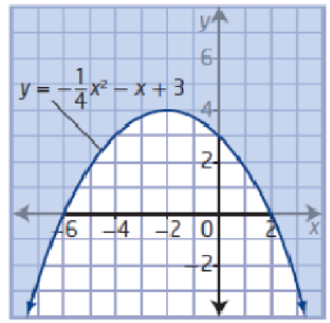
Pg 496 #3abc: Write an inequality to describe each graph given the function defining the boundary parabola.



$y < -x^2 - 4x + 5$



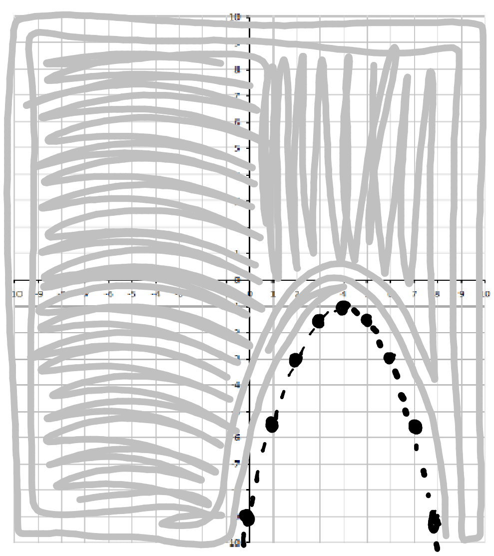
$y \leq \frac{1}{2}x^2 - x + 3$



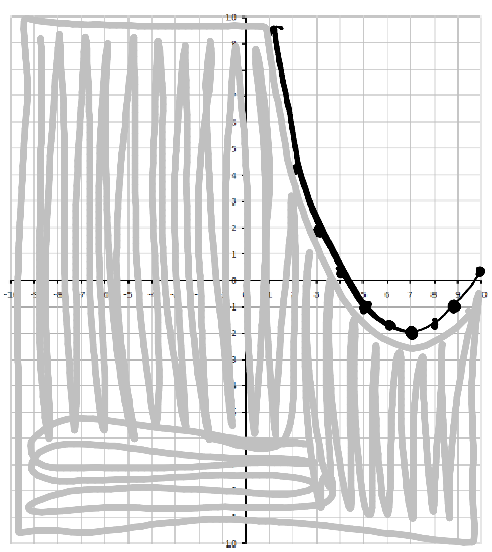
$y \geq -\frac{1}{4}x^2 - x + 3$

Pg 496 #4bd: Graph each quadratic inequality using transformations to sketch the boundary parabola.

$y > -\frac{1}{2}(x-4)^2 - 1$

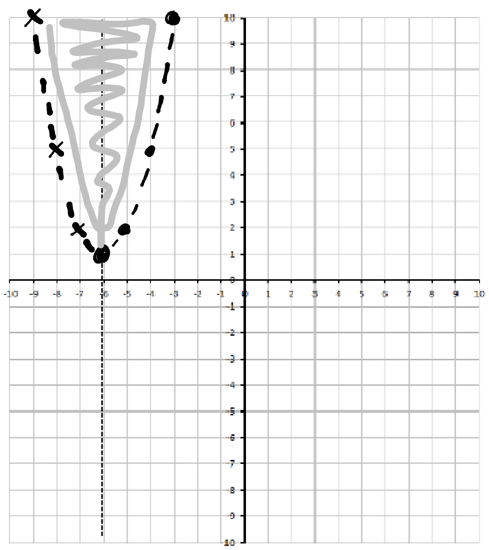


$y \leq \frac{1}{4}(x-7)^2 - 2$



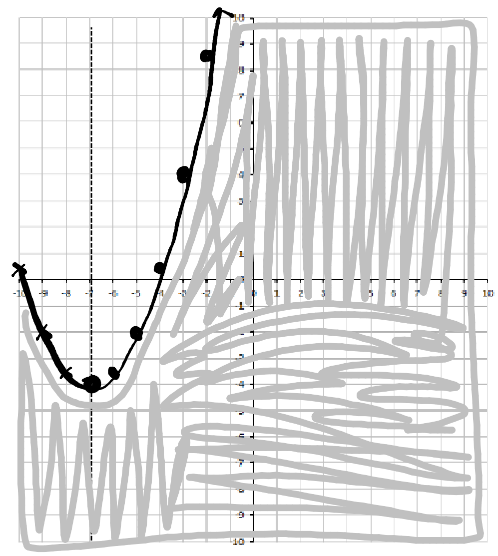
Pg 496 #5bd: Graph each quadratic inequality using points and symmetry to sketch the boundary parabola.

$y > (x+6)^2 + 1$



x	$(x+6)^2+1$
-6	1
-5	2
-4	5
-3	10

$y \leq \frac{1}{2}(x+7)^2 - 4$

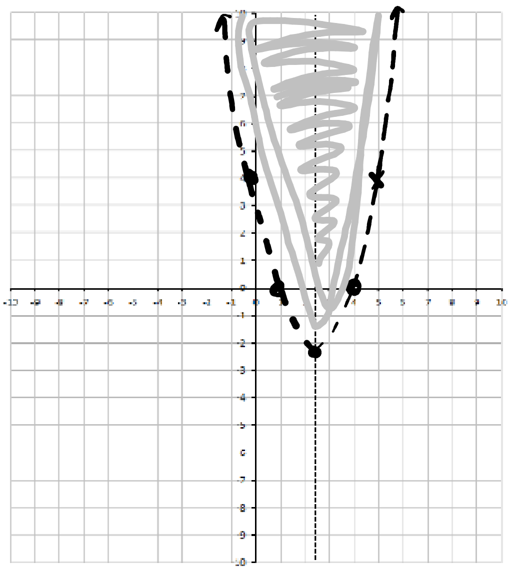


x	$\frac{1}{2}(x+7)^2-4$
-6	-3.5
-5	-2
-4	0.5
-3	4
-2	8.5
-1	14

Pg 496 #6bd: Graph each inequality.

$y > x^2 - 5x + 4$

y-intercept



$y = x^2 - 5x + 4$

$y = (x^2 - 5x) + 4$

$y = (x^2 - \frac{5}{2}x - \frac{5}{2}x) + 4$

$y = (x^2 - \frac{5}{2}x - \frac{5}{2}x + \frac{25}{4}) + 4 - \frac{25}{4}$

$y = (x - \frac{5}{2})^2 - \frac{9}{4}$

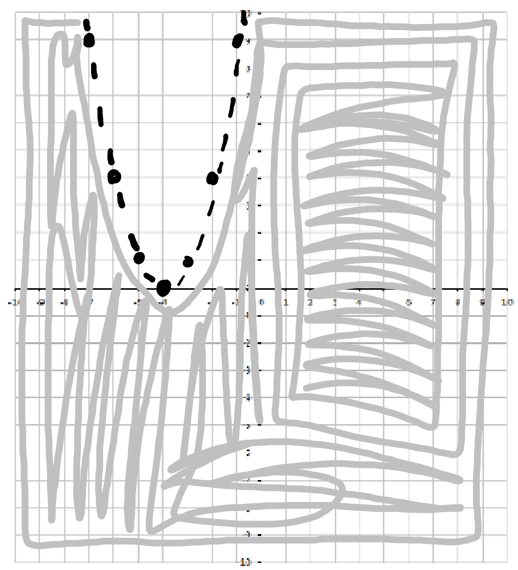
$y = x^2 - 5x + 4$

$0 = (x-4)(x-1)$

$x = 4$ $x = 1$

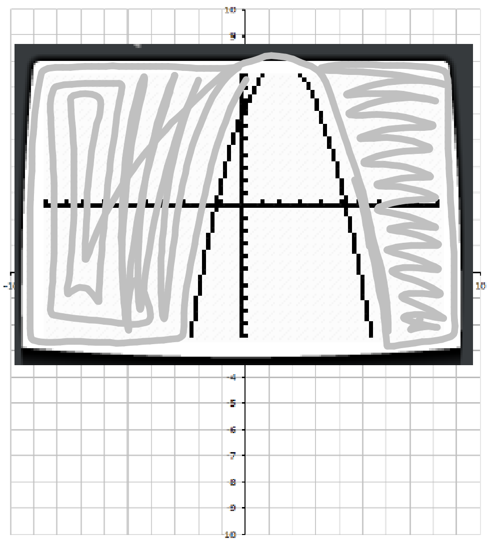
$y < x^2 + 8x + 16$

$y < (x+4)(x+4)$

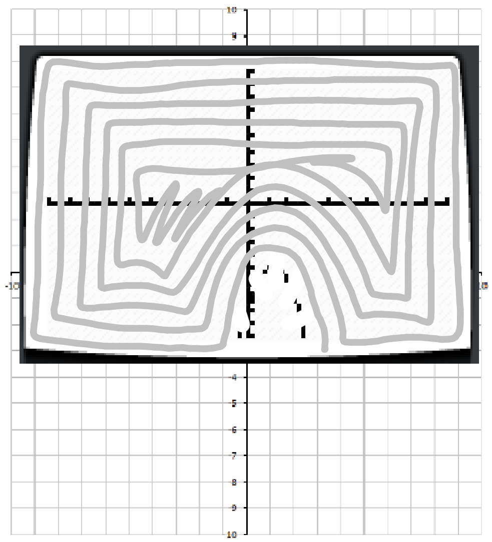


Pg 496 #7bd: Graph each inequality using graphing technology.

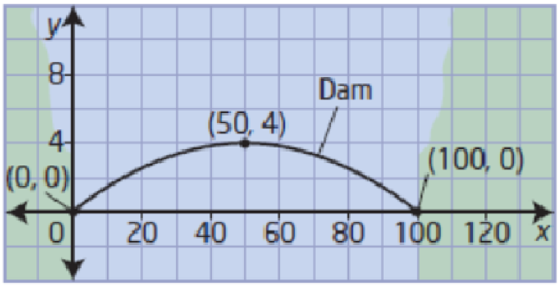
$y \geq -x^2 + 4x + 7$



$y > -2x^2 + 5x - 8$



Pg 496 #9: When a dam is built across a river, it is often constructed in the shape of a parabola. A parabola is used so that the force that the river exerts on the dam helps hold the dam together. Suppose a dam is to be built as shown in the diagram.



a. What is the quadratic function that models the parabolic arch of the dam?
b. Write the inequality that approximates the region below the parabolic arch of the dam.

A

$y = a(x-h)^2 + k$

$y = a(x-50)^2 + 4$

Use $(100, 0)$

$0 = a(100-50)^2 + 4$

$0 = a(50)^2 + 4$

$-4 = a(2500)$

$a = -0.0016$

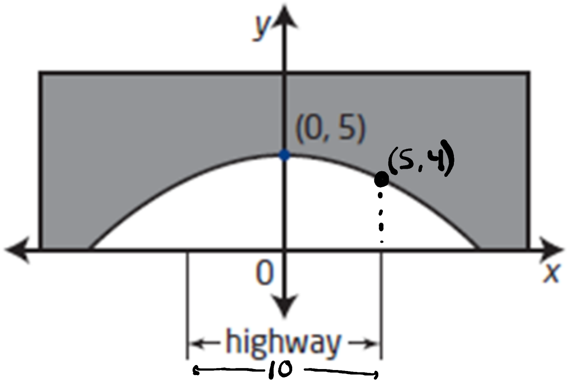
$\boxed{y = -0.0016(x-50)^2 + 4}$

B

$y < -0.0016(x-50)^2 + 4$

where $0 \le x \le 100$

Pg 496 #13: A highway goes under a bridge formed by a parabolic arch, as shown. The highest point of the arch is 5 m high. The road is 10 m wide, and the minimum height of the bridge over the road is 4 m.



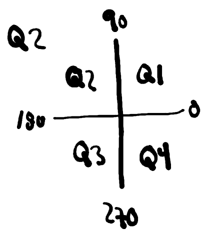
a. Determine the quadratic function that models the parabolic arch of the bridge.
b. What is the inequality that represents the space under the bridge in quadrants I and II?

A
$y = a(x-h)^2+k$
$y = a(x-0)^2+5$
Use $(5,4)$
$4 = a(5)^2+5$
$-1 = a(25)$
$a = -0.04$
$\boxed{y = -0.04x^2+5}$

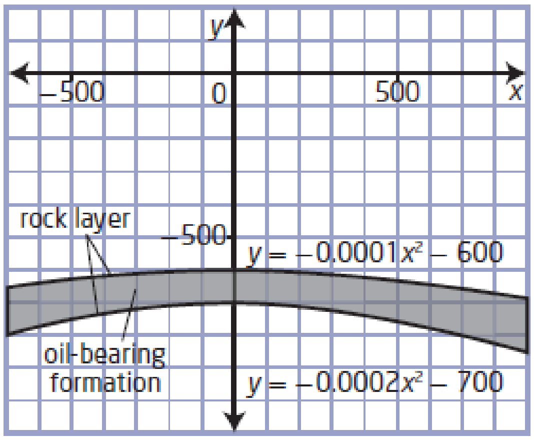
B
$\boxed{y < -0.04x^2+5}$ for Q1, Q2

or

$\boxed{0 < y < -0.04x^2+5}$



Pg 496 #15: Oil is often recovered from a formation bounded by layers of rock that form a parabolic shape. Suppose a geologist has discovered such an oil-bearing formation. The quadratic functions that model the rock layers are $y = -0.0001x^2 - 600$ and $= -0.0002x^2 - 700$, where *x* represents the horizontal distance from the center of the formation and *y* represents the depth below ground level, both in meters. Write the inequality that describes the oil-bearing formation.



Option #1:

$-0.0002x^2 - 700 \le y \le -0.0001x^2 - 600$

Option #2:

$y \ge -0.0002x^2 - 700$

$y \le -0.0001x^2 - 600$