KEY

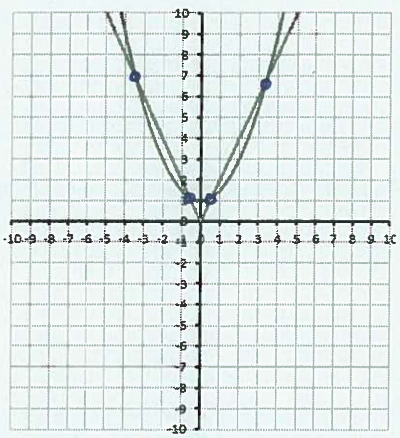
Math 20-1

Lxx – Worksheet – 7.3 Absolute Value Equations

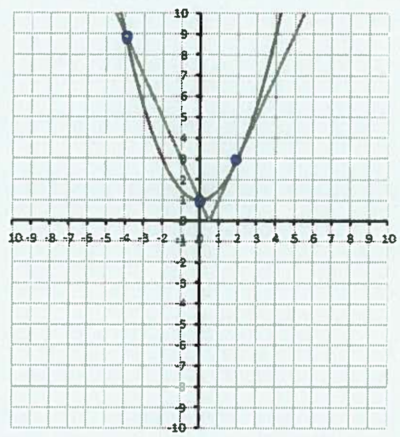
Part 1 – Overview

Be aware that even if your quadratic has multiple solutions, <u>you need to verify all of them</u>.

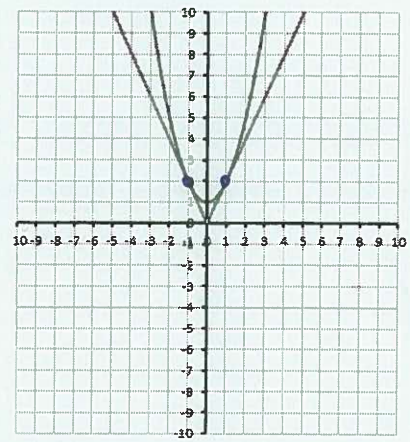
$|2x| = 0.5x^2 + 1$
has 4 solutions.



$|2x - 1| = 0.5x^2 + 1$
has 3 solutions.



$|2x| = x^2 + 1$
has 2 solutions.



Part – Overview

Q1: Given $x^2 + 12x + 24 = |2x|$

a. Solve the equation algebraically.

$x^2+12x+24=|2x|$

$x^2+12x+24=+(2x)$ $\quad -2x \quad -2x$

$x^2+10x+24=0$

$(x+6)(x+4)=0$

$x=-6$ Doesn't verify

$x=-4$ Doesn't verify.

$x^2+12x+24=-(2x)$ $\quad +2x \quad +2x$

$x^2+14x+24=0$

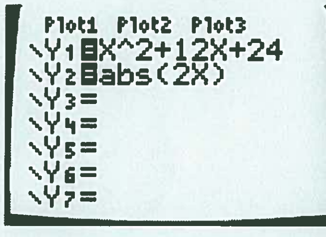
$(x+12)(x+2)=0$

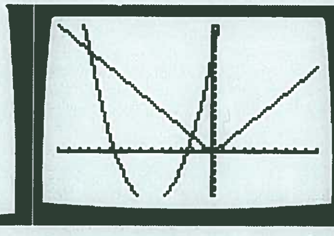
$x=-12$ works.

$x=-2$ Works.

So $x=-12$ or -2.

b. Using technology, solve the equation graphically.





X: -15 → 10
Y: -10 → 30

(-2,4)
(-12,24)

So $x=-2$ or -12.

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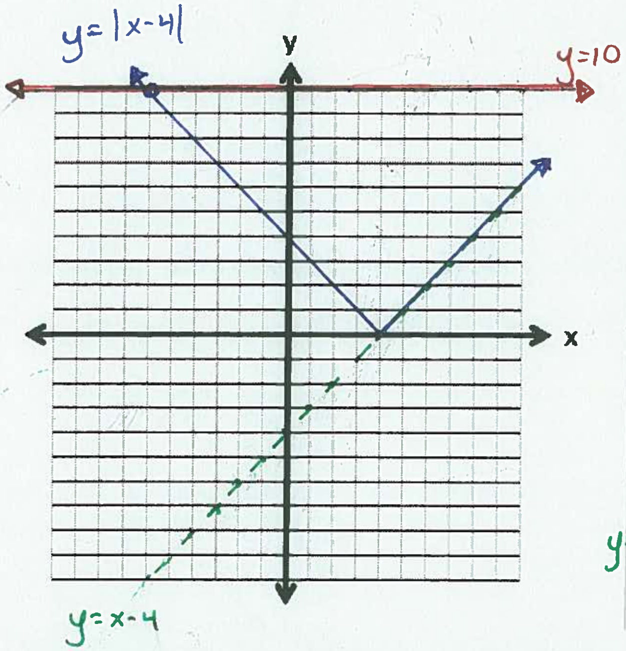
Part – Textbook Questions

Pg 389 #2ab: Solve each absolute value equation by graphing.

$|x - 4| = 10$

Solns are $(-6,10)$ and $(14,10)$

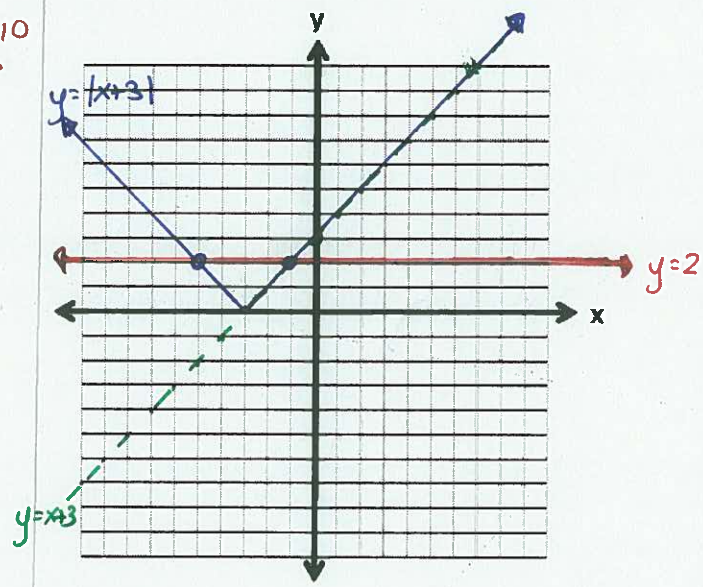
So $x = -6$ or 14.



$|x + 3| = 2$

Solns are $(-5,2)$ and $(-1,2)$

So $x = -5$ or -1.

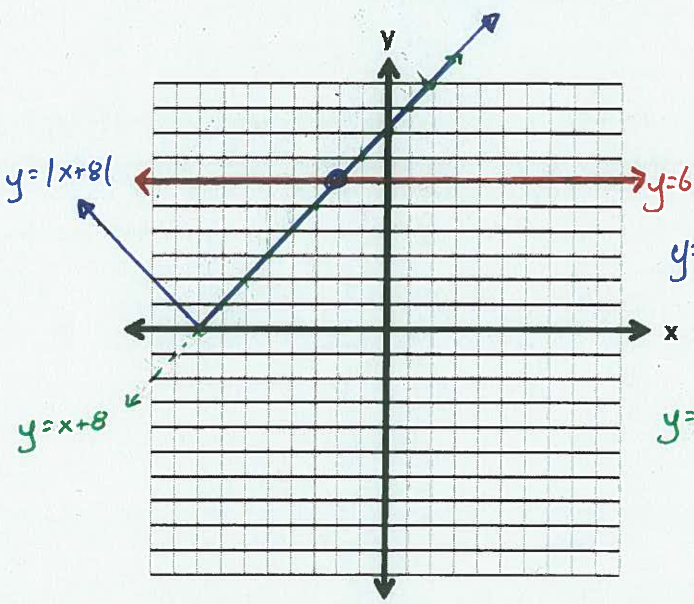


Pg 389 #2cd: Solve each absolute value equation by graphing.

$6 = |x + 8|$

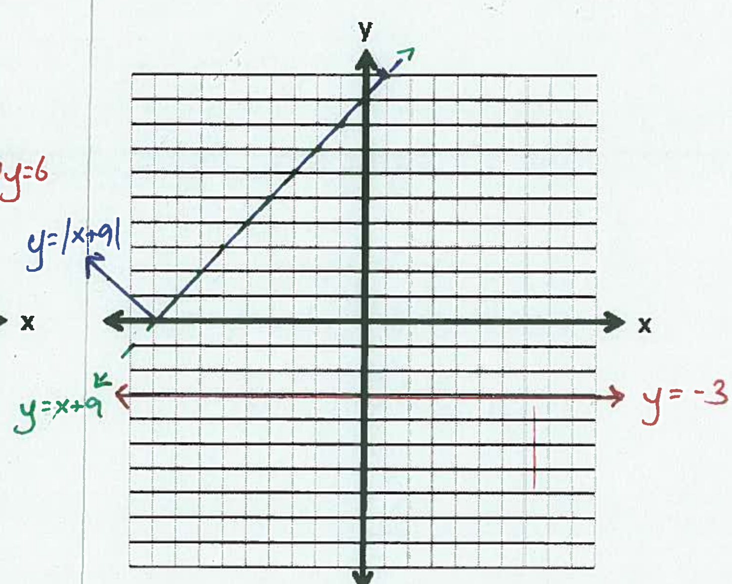
Solns are $(-2,6)$ and $(-14,6)$

So $x = -2$ and -14.



$|x + 9| = -3$

No solutions.



PRE-WORK FOR Pg 389 # 3ab

$$|x+a| = b$$

$+(x_1+a) = b$ $\qquad$ $-(x_2+a) = b$

$x_1 + a = b$ $\qquad$ $-x_2 - a = b$

$x_1 = b - a$ $\qquad$ $-b - a = x_2$

Difference between x's is $|x_2 - x_1|$

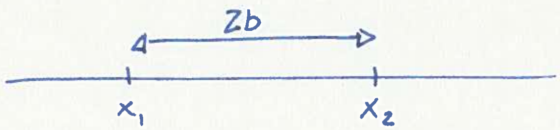
or $|(-b-a) - (b-a)|$

$= |-b-a-b+a|$

$= |-2b|$

so $\Delta x = 2b$

When we solve it, and plot our solutions on a number line,



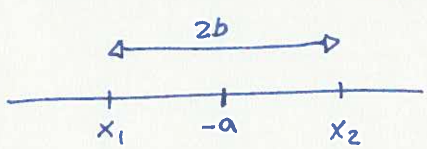
Halfway between or x-values is $\frac{x_2+x_1}{2}$ or $\frac{(-b-a)+(b-a)}{2}$

$= \frac{-b-a+b-a}{2}$

$= \frac{-2a}{2}$

$= -a$

So when we solve $|x+a| = b$ and plot our solutions, it looks like.



We'll use this diagram to answer Pg 389 #3ab.

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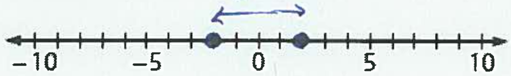
Pg 389 #3ab: Determine an absolute value equation in the form $|ax+b| = c$ given its solutions on the number line.

a)

$2b = 4 \Rightarrow b = 2$



$-a = 0 \Rightarrow a = 0$

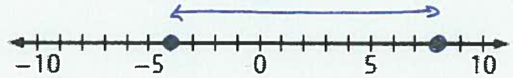
So $|x+a| = b$

is $|x+0| = 2$

or $|x| = 2$

b)

$2b = 12 \Rightarrow b = 6$



$-a = \frac{(8)+(-4)}{2}$

$-a = 2$

$a = -2$

So $|x+a| = b$

is $|x-2| = 6$

Pg 389 #4ab: Solve each absolute value equation algebraically. Verify your solutions.

$|x+7| = 12$

$+(x+7) = 12$

$x+7 = 12$

$x = 5$

$-(x+7) = 12$

$-x-7 = 12$

$-x = 19$

$x = -19$

So $x = -19$ or 5.

$|3x-4| + 5 = 7$

$|3x-4| = 2$

$+(3x-4) = 2$

$3x-4 = 2$

$3x = 6$

$x = 2$

$-(3x-4) = 2$

$-3x+4 = 2$

$-3x = -2$

$x = 2/3$

So $x = 2/3$ or 2.

Pg 389 #4cd: Solve each absolute value equation algebraically. Verify your solutions.

$2|x+6| + 12 = -4$

$2|x+6| = -16$

$|x+6| = -8$

$+(x+6) = -8$

$x+6 = -8$

$x = -14$

Doesn't verify.

$-(x+6) = -8$

$-x-6 = -8$

$-x = -2$

$x = 2$

Doesn't verify.

No solution.

$-6|2x-14| = -42$

$|2x-14| = 7$

$+(2x-14) = 7$

$2x-14 = 7$

$2x = 21$

$x = 21/2$

$-(2x-14) = 7$

$-2x+14 = 7$

$-2x = -7$

$x = 7/2$

So $x = 7/2$ or $21/2$

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Pg 389 #5ab: Solve each equation.

$$|2a + 7| = a - 4$$

$+(2a+7) = a-4$

$2a+7 = a-4$

$a+7 = -4$

$a = -11$

Doesn't verify

$-(2a+7) = a-4$

$-2a-7 = a-4$

$-7 = 3a-4$

$-3 = 3a$

$a = -1$

Doesn't verify

No solution.

$$|7 + 3x| = 11 - x$$

$+(7+3x) = 11-x$

$7+3x = 11-x$

$7+4x = 11$

$4x = 4$

$x = 1$

$-(7+3x) = 11-x$

$-7-3x = 11-x$

$-7 = 11+2x$

$-18 = 2x$

$x = -9$

$x = -9$ or 1.

Pg 389 #5cd: Solve each equation.

$$|1 - 2m| = m + 2$$

$+(1-2m) = m+2$

$1-2m = m+2$

$1 = 3m+2$

$-1 = 3m$

$-\frac{1}{3} = m$

$-(1-2m) = m+2$

$-1+2m = m+2$

$-1+m = 2$

$m = 3$

So $m = -\frac{1}{3}$ or 3.

$$|3x + 3| = 2x - 5$$

$+(3x+3) = 2x-5$

$3x+3 = 2x-5$

$x+3 = -5$

$x = -8$

Doesn't verify.

$-(3x+3) = 2x-5$

$-3x-3 = 2x-5$

$-3 = 5x-5$

$-2 = 5x$

$x = -\frac{2}{5}$

Doesn't verify.

No solution.

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Pg 389 #6ab: Solve each equation and verify your solutions graphically. Use your Calculator.

$|x| = x^2 + x - 3$

$+(x) = x^2+x-3$

$0 = x^2-3$

$3 = x^2$

$x = \pm\sqrt{3}$

$x_1 = +\sqrt{3}$ $\quad x_2 = -\sqrt{3}$

Doesn't verify.

$-(x) = x^2+x-3$

$0 = x^2+2x-3$

$0 = (x+3)(x-1)$

$x_3 = -3$ $\quad x_4 = 1$

Doesn't verify.

So $x = -3$ or $+\sqrt{3}$.

$|x^2 - 2x + 2| = 3x - 4$

$+(x^2-2x+2) = 3x-4$

$x^2-2x+2 = 3x-4$

$x^2-5x+6 = 0$

$(x-2)(x-3) = 0$

$x_1 = 2$ $\quad x_2 = 3$

$-(x^2-2x+2) = 3x-4$

$-x^2+2x-2 = 3x-4$

$0 = x^2+x-2$

$0 = (x+2)(x-1)$

$x_3 = -2$ $\quad x_4 = 1$

Doesn't verify. Doesn't verify.

Pg 389 #6cd: Solve each equation and verify your solutions graphically. Use your calculator.

$|x^2 - 9| = x^2 - 9$

$+(x^2-9) = x^2-9$

$x^2-9 = x^2-9$

$0 = 0$

$-(x^2-9) = x^2-9$

$-x^2+9 = x^2-9$

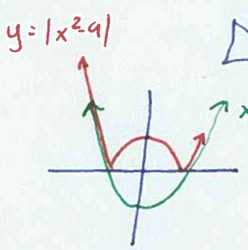
$0 = 2x^2-18$

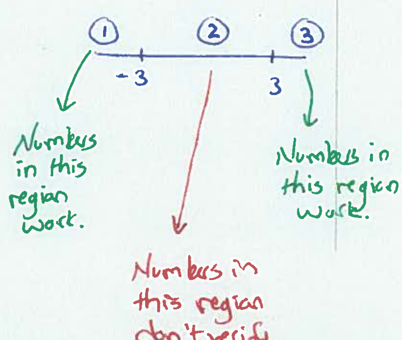
$18 = 2x^2$

$9 = x^2$

$x = \pm 3$

Examine with graphing or case analysis.





So $x \leq -3$ and $x \geq 3$.

$|x^2 - 1| = x$

$+(x^2-1) = x$

$x^2-1 = x$

$x^2-x-1 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{1 \pm \sqrt{1-4(1)(-1)}}{2(1)}$

$x = \frac{1 \pm \sqrt{5}}{2}$

$x_1 = \frac{1+\sqrt{5}}{2}$ $\quad x_2 = \frac{1-\sqrt{5}}{2}$

Doesn't verify.

$-(x^2-1) = x$

$-x^2+1 = x$

$0 = x^2+x-1$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{1-4(1)(-1)}}{2(1)}$

$x = \frac{-1 \pm \sqrt{5}}{2}$

$x_3 = \frac{-1+\sqrt{5}}{2}$ $\quad x_4 = \frac{-1-\sqrt{5}}{2}$

Doesn't verify.

So $x = \frac{1+\sqrt{5}}{2}$ and $x = \frac{-1+\sqrt{5}}{2}$

KEY

Pg 389 #8: One experiment measured the speed of light as 299,792,456.2 m/s with a measurement uncertainty of 1.1 m/s.

a. Write an absolute value equation in the form $|c - a| = b$ to describe the measured speed of light, c, in meters per second, where a and b are real numbers.
b. Solve the absolute value equation to find the maximum and minimum values for the speed of light for this experiment.

(A) Speed of light = 299,792,456.2 m/s ± 1.1 m/s
Speed of light − 299,792,456.2 m/s = ± 1.1 m/s
|Speed of light − 299,792,456.2| = 1.1

(B) $|c - 299{,}792{,}456.2| = 1.1$

$c_1 = 299{,}792{,}455.1$ m/s $c_2 = 299{,}792{,}457.3$ m/s

Pg 389 #10: Consider the statement $x = 7 \pm 4.8$.

a. Describe the values of x.
b. Translate the statement into an equation involving absolute value.

(A) x can be between 2.2 and 11.8,
or $2.2 \leq x \leq 11.8$. (Textbook says only 2.2 and 11.8)

(B) $x = 7 \pm 4.8$
$x - 7 = \pm 4.8$
$|x - 7| = 4.8$

Pg 389 #14: Solve each equation for *x*, where *a, b, c* ∈*R*.

$|ax| - b = c$

$|ax| = b + c$

$+(ax) = b + c$
$ax = b + c$
$x_1 = \dfrac{b+c}{a}$

$-(ax) = b + c$
$-ax = b + c$
$x_2 = \dfrac{b+c}{-a}$

$|x - b| = c$

$+(x - b) = c$
$x - b = c$
$x_1 = b + c$

$-(x - b) = c$
$-x + b = c$
$-x = -b + c$
$x_2 = b - c$