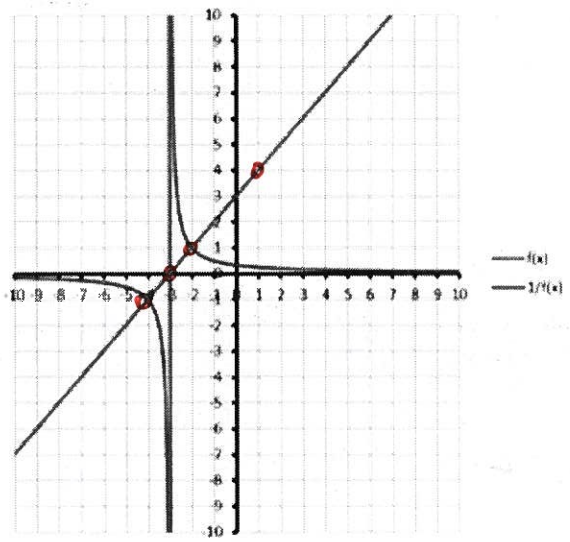
L45 – 7.4 Reciprocal Functions

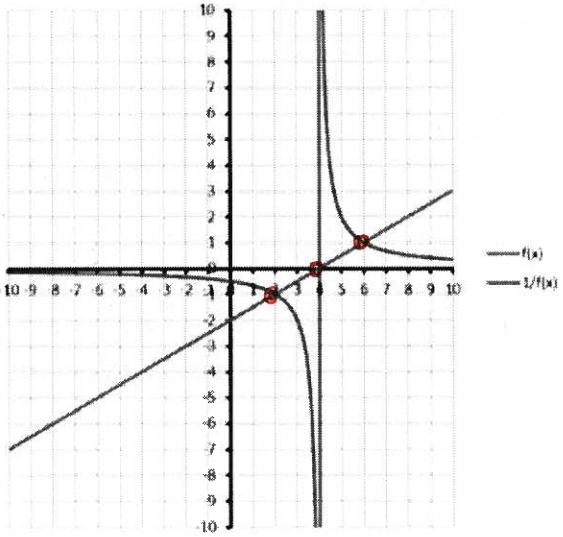
Key Ideas

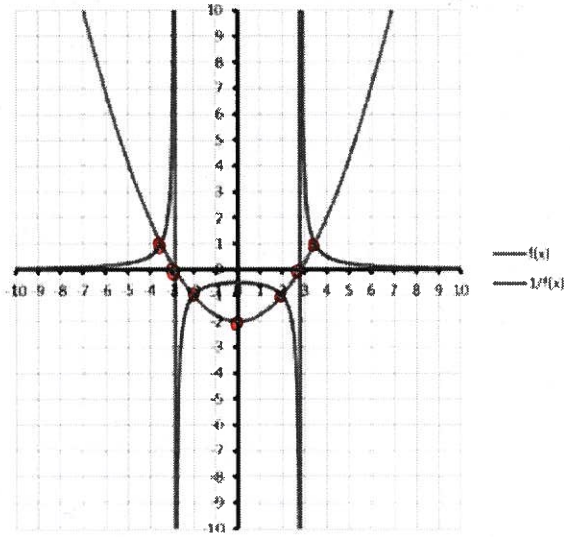
If $y = f(x)$, then the inverse function is $y = \frac{1}{f(x)}$.

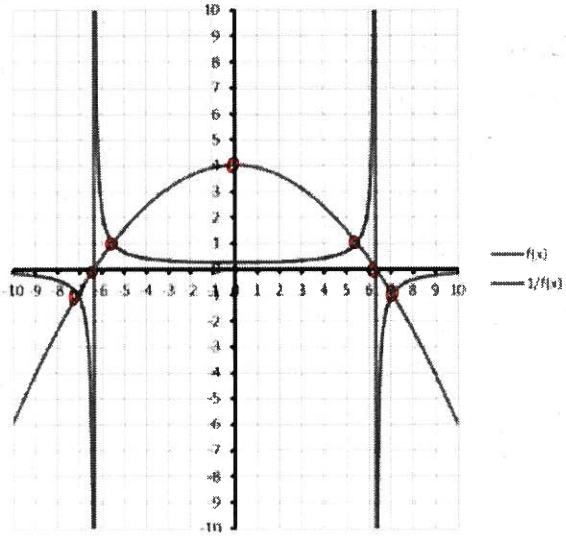
Important Points:

- Where the original function has $y = 0$, the new function has a "***Non-Permissible Value***", and there is a vertical asymptote.
- Where the original function has $y = \pm 1$, the new function has $y = \pm 1$. This is referred to as a "***Invariant Point***".









Math 20-1

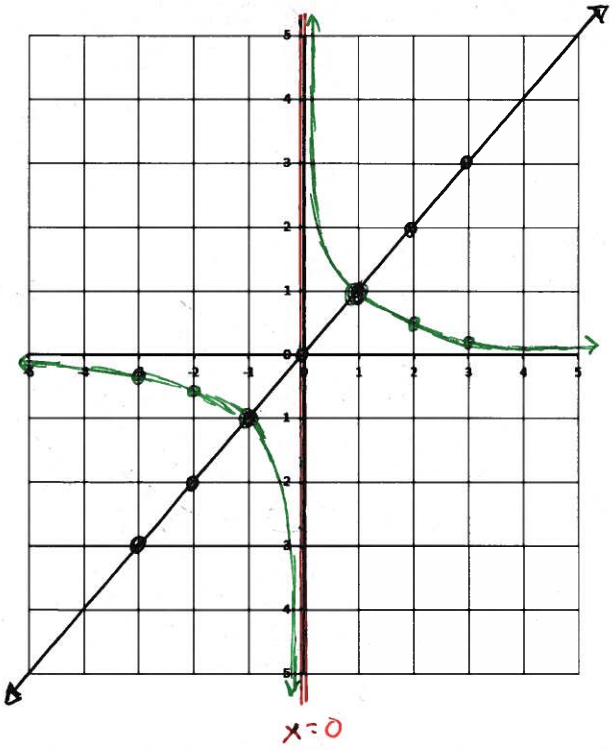
Part 1 – Reciprocal Functions of Linear Functions

Q1: Given the function $f(x) = x$

a. Build a table for $y = f(x)$ and $y = \frac{1}{f(x)}$.

x	$y = f(x)$	$y = \frac{1}{f(x)}$
-3	-3	$\frac{1}{-3} = -0.\overline{3}$
-2	-2	$\frac{1}{-2} = -0.5$
-1	-1	$\frac{1}{-1} = -1$
0	0	$\frac{1}{0}$ NPV
1	1	$\frac{1}{1} = 1$
2	2	$\frac{1}{2} = 0.5$
3	3	$\frac{1}{3} = 0.\overline{3}$

b. Sketch both functions on the graph paper.



c. Identify (a) the NPV's for the reciprocal function, (b) the equation of the vertical asymptote of the reciprocal function, (c) the Invariant Point(s).

(A) NPV: $x \neq 0$

(B) $x = 0$

(C) Invariant Points: $(-1, -1)$ and $(1, 1)$

KEY

Math 20-1

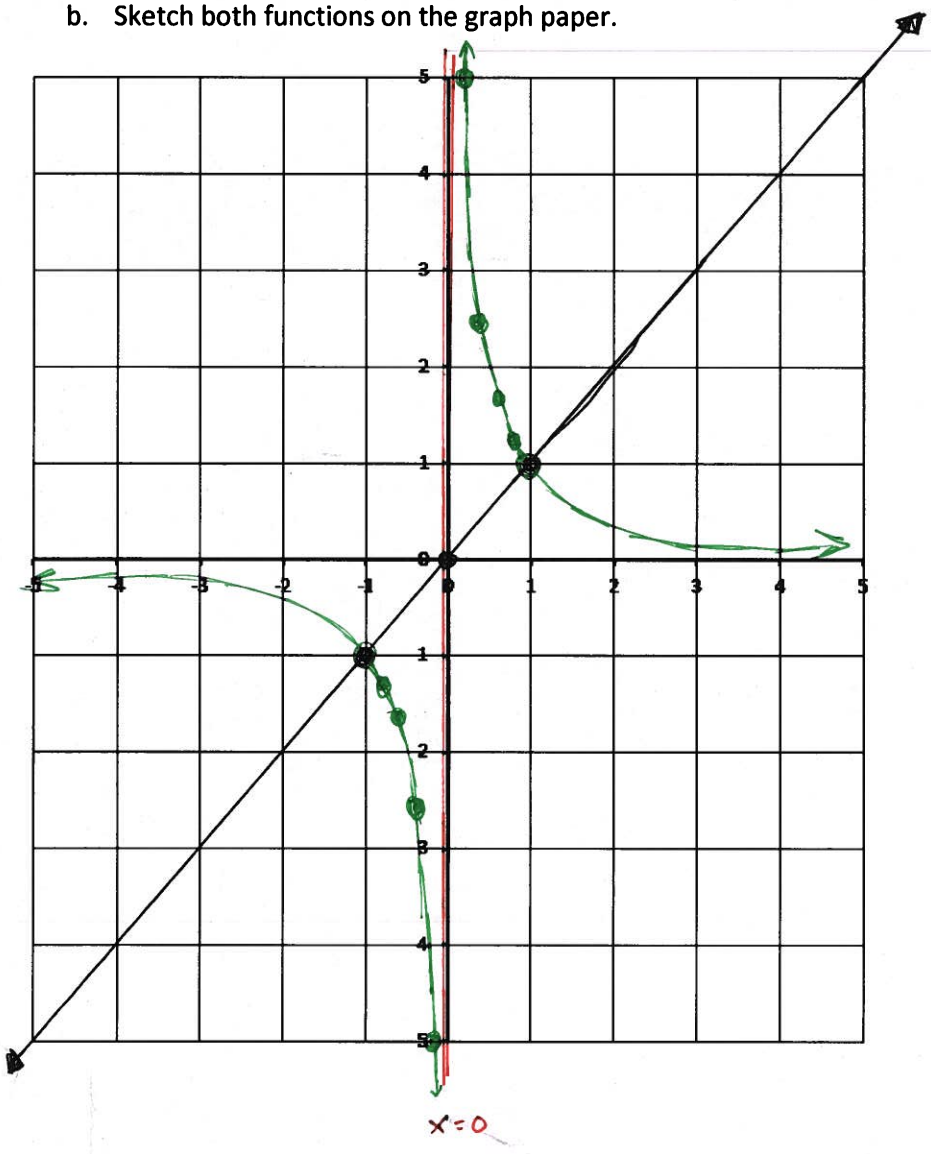
Q2: Given the function $f(x) = x$ (again)

a. Build a table for $y = f(x)$ and $y = \frac{1}{f(x)}$.

x	$y = f(x)$	$y = \frac{1}{f(x)}$
-1.0	-1.0	$\frac{1}{-1.0} = -1$
-0.8	-0.8	$\frac{1}{-0.8} = -1.25$
-0.6	-0.6	$\frac{1}{-0.6} = -1.\overline{6}$
-0.4	-0.4	$\frac{1}{-0.4} = -2.5$
-0.2	-0.2	$\frac{1}{-0.2} = -5$
0	0	$\frac{1}{0}$ NPV

x	$y = f(x)$	$y = \frac{1}{f(x)}$
0.2	0.2	$\frac{1}{0.2} = 5$
0.4	0.4	$\frac{1}{0.4} = 2.5$
0.6	0.6	$\frac{1}{0.6} = 1.\overline{6}$
0.8	0.8	$\frac{1}{0.8} = 1.25$
1.0	1.0	$\frac{1}{1.0} = 1.0$

b. Sketch both functions on the graph paper.



KEY

Math 20-1

Q3: Given the function $f(x) = 2x + 1$

a. Build a table for $y = f(x)$ and $y = \frac{1}{f(x)}$.

x	$y = f(x)$	$y = \frac{1}{f(x)}$
-3	-5	$\frac{1}{-5} = -0.2$
-2	-3	$\frac{1}{-3} = -0.\overline{3}$
-1	-1	$\frac{1}{-1} = -1$
0	1	$\frac{1}{1} = 1$
1	3	$\frac{1}{3} = 0.\overline{3}$
2	5	$\frac{1}{5} = 0.2$
3	7	$\frac{1}{7} = 0.\overline{142857}$

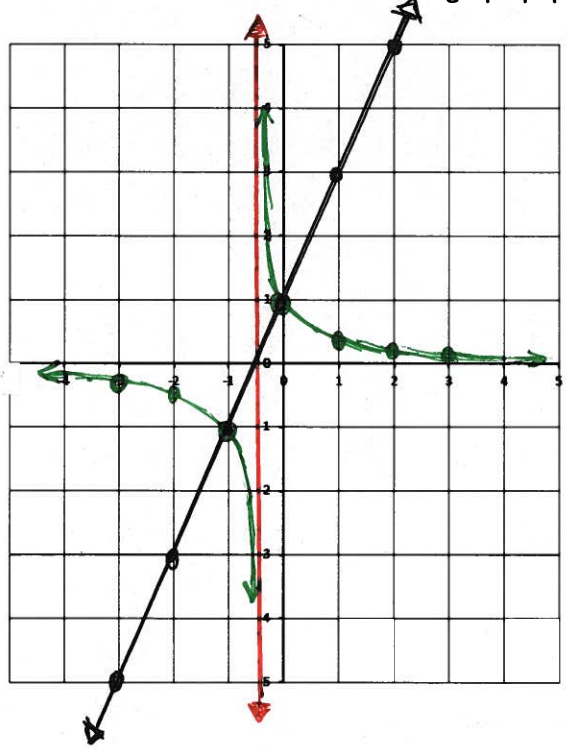
$y = \frac{1}{2x+1}$

$2x+1 \neq 0$

$\boxed{x \neq -1/2}$ Non-Permissible Value.

Asymptote at $x = -1/2$

b. Sketch both functions on the graph paper.



c. Identify (a) the NPV's for the reciprocal function, (b) the equation of the vertical asymptote of the reciprocal function, (c) the Invariant Point(s).

(A) NPV: $x \neq -\frac{1}{2}$

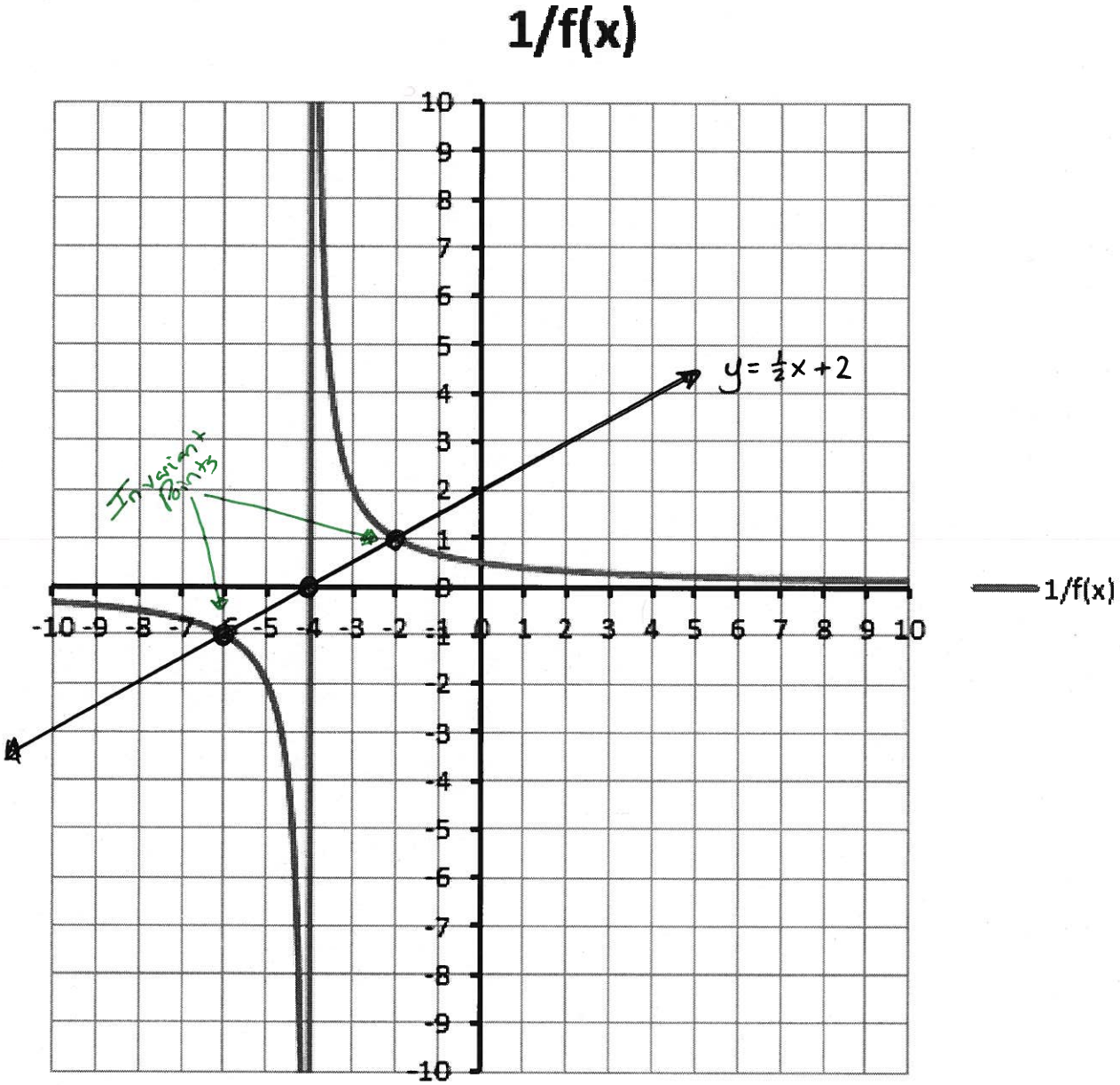
(B) $x = -1/2$

(C) Invariant Points $(-1,-1)$ and $(0,1)$

Math 20-1

KEY

Q4: Determine the equation of the function shown:



Function shown is reciprocal of $y = \frac{1}{2}x + 2$

So $\boxed{y = \frac{1}{\frac{1}{2}x+2}}$ or... if we want to get fancy

$$y = \frac{2}{2}\left(\frac{1}{\frac{1}{2}x+2}\right)$$

$$y = \frac{2}{2(\frac{1}{2}x+2)}$$

$$\boxed{y = \frac{2}{x+4}}$$

Math 20-1

KEY

Q5: Given the function $f(x) = x^2 + 2x - 3$

a. Write the Reciprocal Function
b. Determine the Non-Permissible Values
c. Determine the equation of the vertical asymptote(s).
d. Graph the Reciprocal Function

(A) $y = \frac{1}{x^2+2x-3} = \frac{1}{(x+3)(x-1)}$

(B) $x+3 \neq 0$ $\quad x-1 \neq 0$
$x \neq -3$ $\quad x \neq 1$

(C) $x = -3, \; x = 1$