KEY

Math 20-1

Lxx – Worksheet – 7.4 Reciprocal Functions

Part 1 – Textbook Questions

Pg 403 #1: Given the function $y = f(x)$, write the corresponding reciprocal function.

$y = -x + 2$	$y = 3x - 5$	$y = x^2 - 9$	$y = x^2 - 7x + 10$
$y = \frac{1}{-x+2}$	$y = \frac{1}{3x-5}$	$y = \frac{1}{x^2-9}$ or $y = \frac{1}{(x+3)(x-3)}$	$y = \frac{1}{x^2-7x+10}$ $y = \frac{1}{(x-5)(x-2)}$

Pg 403 #2: Complete the table:

	State the zeroes. x-intercept, when $y=0$	Write the reciprocal function.	State the non-permissible values fo the corresponding rational expression.	Explain how the zeroes of the original function are related to the non-permissible values of the reciprocal function.	State the equation(s) of the vertical asymptote(s).
$f(x) = x + 5$	$0 = x+5$ $x = -5$	$y = \frac{1}{x+5}$	$x \neq -5$	Same	$x = -5$
$g(x) = 2x + 1$	$0 = 2x+1$ $-1 = 2x$ $x = -1/2$	$y = \frac{1}{2x+1}$	$x \neq -\frac{1}{2}$	Same	$x = -1/2$
$h(x) = x^2 - 16$	$0 = (x+4)(x-4)$ $x+4=0$ $x-4=0$ $x = -4, +4$	$y = \frac{1}{(x+4)(x-4)}$	$x \neq -4, +4$	Same	$x = -4, 4$
$t(x) = x^2 + x - 12$	$0 = (x+4)(x-3)$ $x+4=0$ $x-3=0$ $x=-4$ $x=3$	$y = \frac{1}{(x+4)(x-3)}$	$x \neq 3, -4$	Same	$x = -4, 3$

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Pg 403 #3ab: State the equation(s) of the vertical asymptote(s) for each function.

$$f(x) = \frac{1}{5x-10}$$

$5x-10 \neq 0$

$5x \neq 10$

$x \neq 2$

$\boxed{x=2}$

$$f(x) = \frac{1}{3x+7}$$

$3x+7 \neq 0$

$3x \neq -7$

$x \neq -7/3$

$\boxed{x=-7/3}$

Pg 403 #3cd: State the equation(s) of the vertical asymptote(s) for each function.

$$f(x) = \frac{1}{(x-2)(x+4)}$$

$x-2 \neq 0$ $\quad$ $x+4 \neq 0$

$x \neq 2$ $\quad$ $x \neq -4$

$\boxed{x=2}$ $\quad$ $\boxed{x=-4}$

$$f(x) = \frac{1}{x^2-9x+20} = \frac{1}{(x-4)(x-5)}$$

$x-4 \neq 0$ $\quad$ $x-5 \neq 0$

$x \neq 4$ $\quad$ $x \neq 5$

$\boxed{x=4}$ $\quad$ $\boxed{x=5}$

Pg 403 #5ab: What are they x-intercept(s) and the y-intercept of each function?

$$f(x) = \frac{1}{x+5}$$

x-int (set y = 0)

$0 = \frac{1}{x+5}$

No solution.

No x-intercept.

y-int (set x = 0)

$y = \frac{1}{0+5}$

$y = \frac{1}{5}$

$$f(x) = \frac{1}{3x-4}$$

x-int (set y = 0)

$0 = \frac{1}{3x-4}$

No solution

No x-intercept.

y-int (set x = 0)

$y = \frac{1}{3(0)-4}$

$y = -1/4$

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Pg 403 #5ab: What are they x-intercept(s) and the y-intercept of each function?

$f(x) = \frac{1}{x^2-9}$

x-int (set y=0)

$0 = \frac{1}{x^2-9}$

No solution

No x-intercept.

y-int (set x=0)

$y = \frac{1}{(0)^2-9}$

$y = -1/9$

$f(x) = \frac{1}{x^2+7x+12}$

x-int (set y=0)

$0 = \frac{1}{x^2+7x+12}$

No solution

No x-intercept.

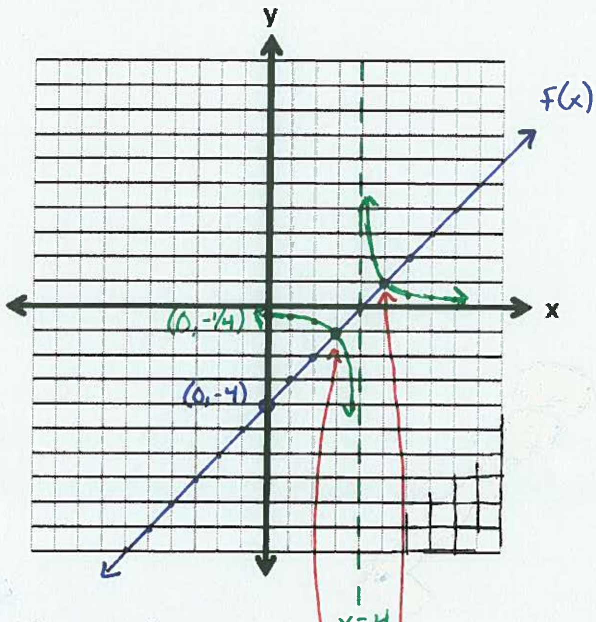
y-int (set x=0)

$y = \frac{1}{(0)^2+7(0)+12}$

$y = 1/12$

Pg 403 #7ab: Sketch the graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$ on the same set of axes. Label the asymptotes, the invariant points, and the intercepts.

$f(x) = x - 4$



x	f(x)	1/f(x)
0	-4	-0.25
1	-3	-0.33
2	-2	-0.50
3	-1	-1.00
4	0	Undefined
5	1	1.00
6	2	0.50
7	3	0.33
8	4	0.25
9	5	0.20
10	6	0.17

These values don't change.

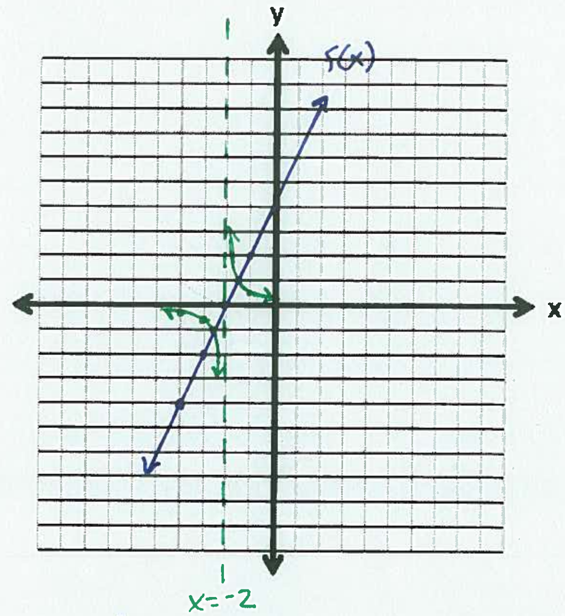
Asymptote: x=4

Invariant Pts:

(3,-1) and (5,1)

y-int: On graph

$f(x) = 2x + 4$



x	f(x)	1/f(x)
-4	-4	-0.25
-3	-2	-0.50
-2	0	Undefined
-1	2	+0.50
0	4	+0.25

Asymptote: x=-2

y-int:

f(x) → (0,4)

1/f(x) → (0, 1/4)

Invariant Points when $y = \pm 1$

$1 = 2x+4$

$-3 = 2x$

$x = -3/2$

$(-3/2, 1)$

$-1 = 2x+4$

$-5 = 2x$

$x = -5/2$

$(-5/2, -1)$

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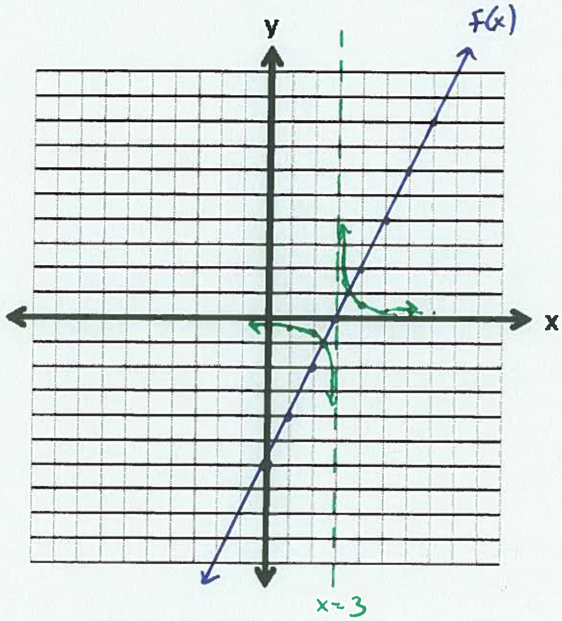
Pg 403 #7cd: Sketch the graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$ on the same set of axes. Label the asymptotes, the invariant points, and the intercepts.

$f(x) = 2x - 6$



x	f(x)	1/f(x)
0	-6	-0.167
1	-4	-0.250
2	-2	-0.500
3	0	Undefined
4	2	+0.500
5	4	+0.250
6	6	+0.167
7	8	+0.125

Asymptote: $x = 3$

y-intercepts:

$f(x) \rightarrow (0, -6)$

$1/f(x) \rightarrow (0, -1/6)$

Invariant points when $y = +1, -1$

$1 = 2x - 6$

$7 = 2x$

$x = 7/2$

so $(7/2, 1)$

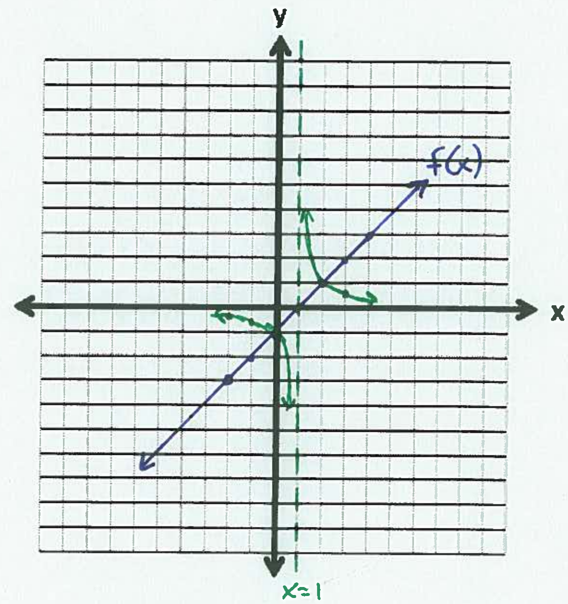
$-1 = 2x - 6$

$5 = 2x$

$x = 5/2$

so $(5/2, -1)$

$f(x) = x - 1$



x	f(x)	1/f(x)
-2	-3	-0.33
-1	-2	-0.50
0	-1	-1.00
1	0	Undefined
2	1	+1.00
3	2	+0.50
4	3	+0.33

Asymptote: $x = 1$

y-intercepts:

$f(x) \rightarrow (0, -1)$

$1/f(x) \rightarrow (0, -1)$

Invariant points when $y = +1, -1$

so $(0, -1)$ and $(2, 1)$

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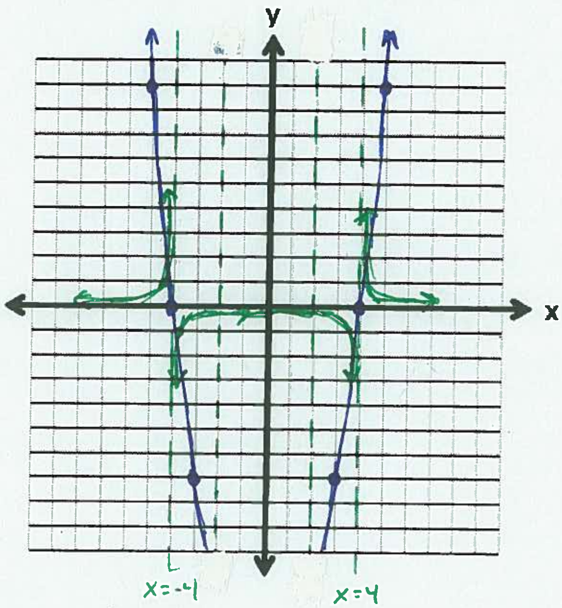
Pg 403 #8ab: Sketch the graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$ on the same set of axes. Label the asymptotes, the invariant points, and the intercepts.

$f(x) = x^2 - 16 = (x+4)(x-4)$



x	f(x)	1/f(x)
-5	9	0.111
-4	0	Undefined
-3	-7	-0.143
0	-16	-0.0625
3	-7	-0.143
4	0	Undefined
5	9	0.111

Asymptote: $x = -4, 4$

y-intercepts:

$f(x) \rightarrow (0, -16)$

$1/f(x) \rightarrow (0, -1/16)$

Invariant points when $y = +1, -1$

$1 = x^2 - 16$

$17 = x^2$

$x = \pm$

$(+4.12, 1)$ and $(-4.12, -1)$

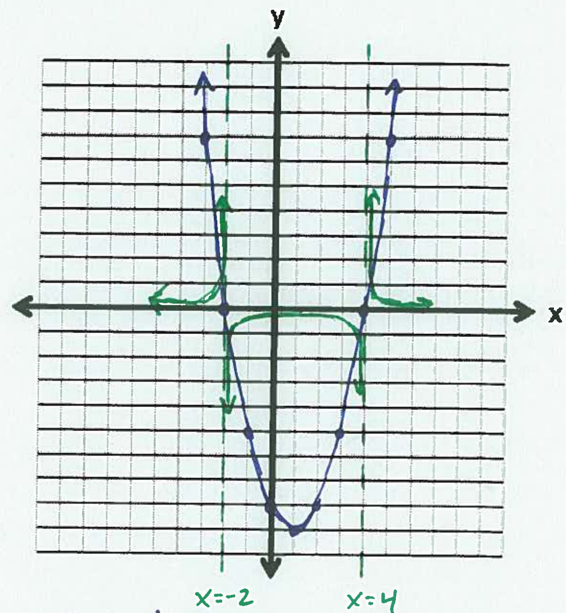
$-1 = x^2 - 16$

$15 = x^2$

$x = \pm 3.87$

$(+3.87, -1)$ and $(-3.87, -1)$

$f(x) = x^2 - 2x - 8 = (x-4)(x+2)$



x	f(x)	1/f(x)
-3	7	0.143
-2	0	Undefined
-1	-5	-0.200
0	-8	-0.125
1	-9	-0.111
2	-8	-0.125
3	-5	-0.200
4	0	Undefined
5	7	0.143

Asymptote: $x = -2, +4$

y-intercepts:

$f(x) \rightarrow (0, -8)$

$1/f(x) \rightarrow (0, -1/8)$

Invariant Points when $y = +1, -1$

$1 = x^2 - 2x - 8$

$0 = x^2 - 2x - 9$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = -2.16, +4.16$

$(-2.16, 1)$ and $(4.16, 1)$

$-1 = x^2 - 2x - 8$

$0 = x^2 - 2x - 7$

$x = \frac{-b \pm \sqrt{b^2 - 4ab}}{2a}$

$x = -1.83, +3.83$

$(-1.83, -1)$ and $(3.83, -1$

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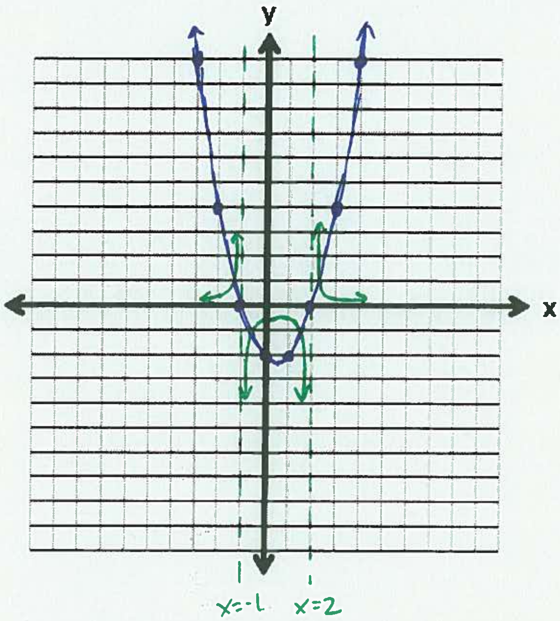
Pg 403 #8cd: Sketch the graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$ on the same set of axes. Label the asymptotes, the invariant points, and the intercepts.

$f(x) = x^2 - x - 2 = (x-2)(x+1)$



x	f(x)	1/f(x)
-3	10	0.10
-2	4	0.25
-1	0	Undefined
0	-2	-0.50
1	-2	-0.50
2	0	Undefined
3	4	0.25
4	10	0.10

Asymptotes: $x = -1,\ x = 2$

y-intercepts:

$f(x) \rightarrow (0, -2)$

$1/f(x) \rightarrow (0, -1/2)$

Invariant Points when $y = -1, +1$

$-1 = x^2 - x - 2$

$0 = x^2 - x - 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = -0.62,\ +1.62$

So $(-0.62, -1)$ and $(+1.62, -1)$

$+1 = x^2 - x - 2$

$0 = x^2 - x - 3$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = -1.30,\ +2.30$

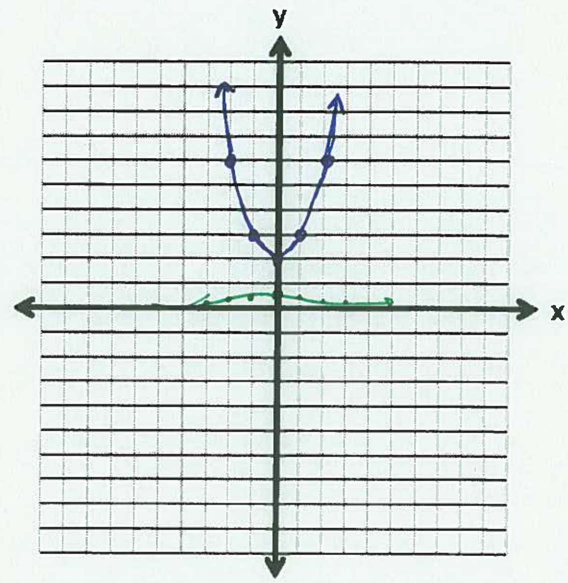
So $(-1.30, +1)$ and $(+2.30, +1)$

$f(x) = x^2 + 2$

$0 = x^2 + 2$

$-2 = x^2$

No zeroes.



x	f(x)	1/f(x)
-3	11	0.091
-2	6	0.167
-1	3	0.333
0	2	0.500
1	3	0.333
2	6	0.167
3	11	0.091

Asymptotes: None

y-intercepts:

$f(x) \rightarrow (0, 2)$

$1/f(x) \rightarrow (0, 1/2)$

Invariant Points → None.

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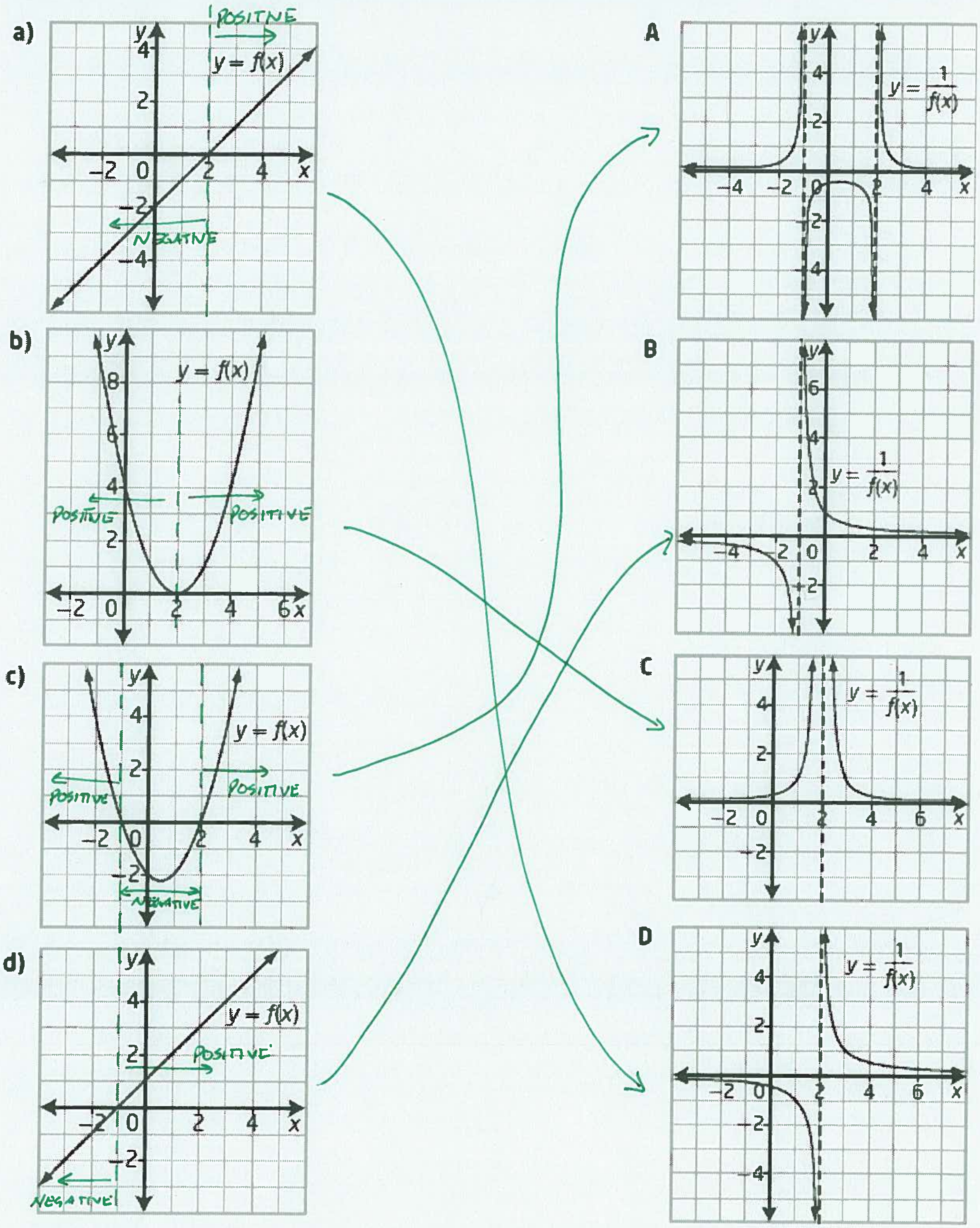
Pg 403 #9: Match the graph of the function with the graph of its reciprocal.



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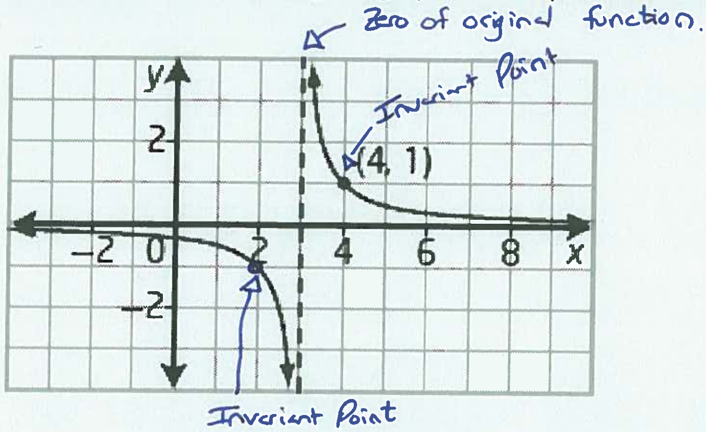
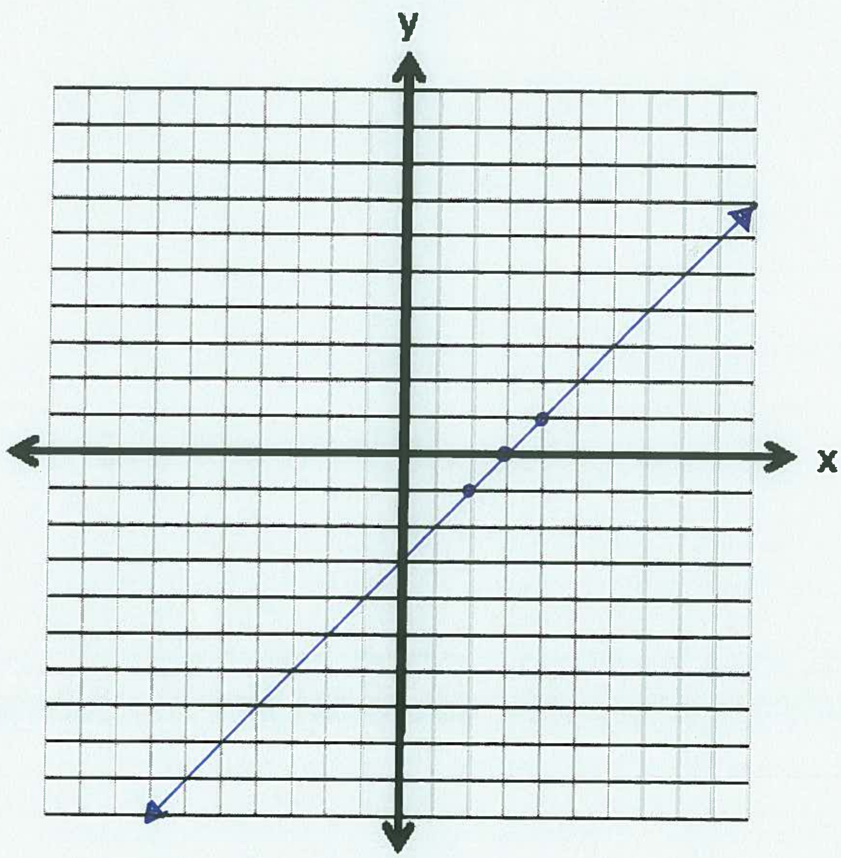
Pg 403 #10a: The following is a graph of a reciprocal function, $y = \frac{1}{f(x)}$.



i. Sketch the graph of the original function, $y = f(x)$.



ii. Explain the strategies you used.

The invariant points don't change.
The asymptote occurs at the zero.
Connect the dots.

iii. What is the original function, $y = f(x)$?

$y = x - 3$

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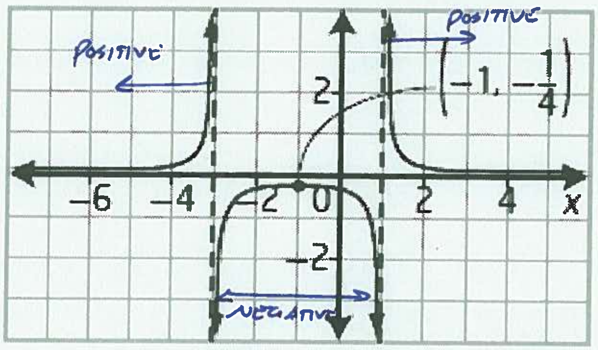
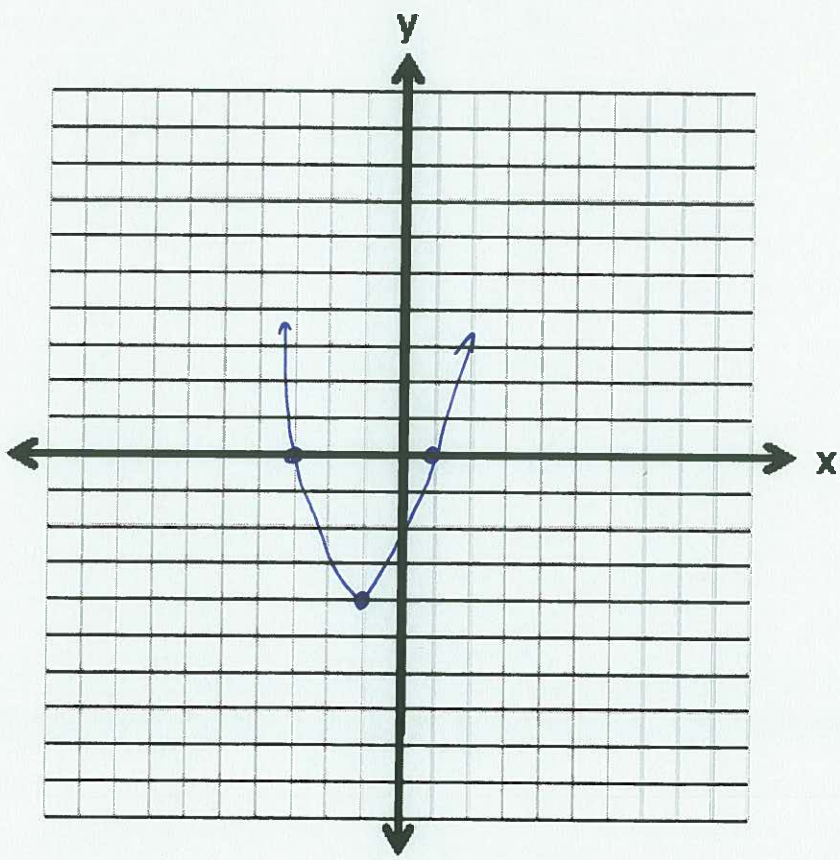
Pg 403 #10b: The following is a graph of a reciprocal function, $y = \frac{1}{f(x)}$.



$1 \div (-\frac{1}{4}) = -4$

i. Sketch the graph of the original function, $y = f(x)$.



ii. Explain the strategies you used.

Plot the zeroes.
Inverse the y-value of the given coordinate.

iii. What is the original function, $y = f(x)$?

$y = a(x+1)^2 - 4$ Use $(1,0)$

$0 = a(1+1)^2 - 4$

$0 = a(2)^2 - 4$

$4 = a(4)$

$a = 1$

$\boxed{y = (x+1)^2 - 4}$ or $\boxed{y = x^2 + 2x - 3}$

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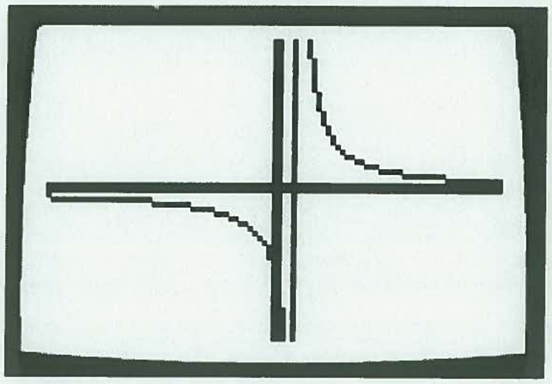
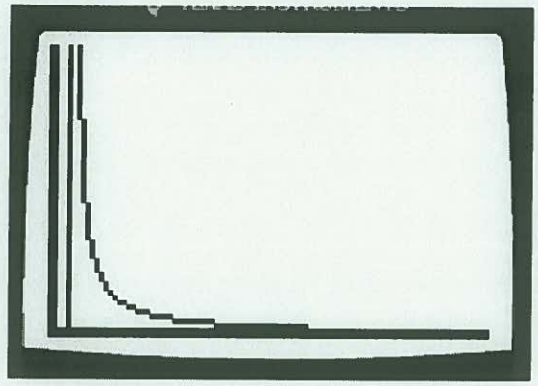
Pg 403 #12: The greatest amount of time, *t*, in minutes, that a scuba diver can take to rise toward the water surface without stopping for decompression is defined by the function $t = \frac{525}{d-10}$, where *d* is the depth, in meters, of the diver.

a. Graph the function using graphing technology.



X: -100 → 100
Y: -100 → 100



X: 0 → 200
Y: 0 → 200

b. Determine a suitable domain which represents this application.

Domain $(10, \infty)$

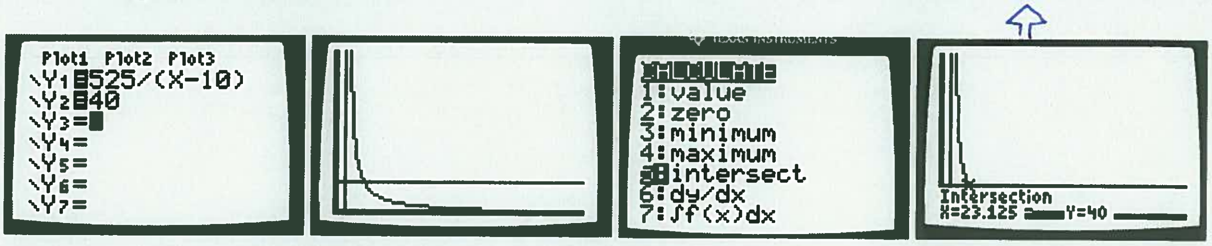
c. Determine the maximum time without stopping for a scuba diver who is 40m deep.

$t = \frac{525}{40-10} = \frac{525}{30} = 17.5$ minutes

d. Graph a second function, *t = 40*. Find the intersection point of the two graphs. Interpret this point in terms of the scuba diver rising to the surface. Check this result algebraically with the original function.

$(d,t) = (23.125, 40)$

At a depth of 23.125 m, it takes 40 min.



e. Does this graph have a horizontal asymptote? What does this mean with respect to the diver?

Yes, at $y = 0$.

So for large depths (x-values), the maximum time a diver can rise without stopping for decompression is almost zero... basically they need to be constantly stopping to decompress.