

48 - Substitution**Part 1 - Method**

Substitution is an algebraic method of solving systems of equations.

Process:

1. Solve one equation for one variable.
2. Substitute that expression into the other equation.
3. Solve for the other variable.

Part 1 - Algebraic and Graphical Questions

Q1: Find the solution to the system of equations both *algebraically* and *graphically*.

$$y = x + 1 \quad y = -2x - 7$$

$$y = x + 1 \quad y = -2x - 7$$

$$y = y$$

$$x + 1 = -2x - 7$$

$$\text{so } x + 1 = -2x - 7$$

$$\begin{array}{r} +2x \\ 3x + 1 = -7 \end{array}$$

$$\begin{array}{r} -1 \\ -1 \\ 3x = -8 \end{array}$$

$$3x = -8$$

$$\begin{array}{r} \div 3 \\ \div 3 \end{array}$$

$$x = -\frac{8}{3}$$

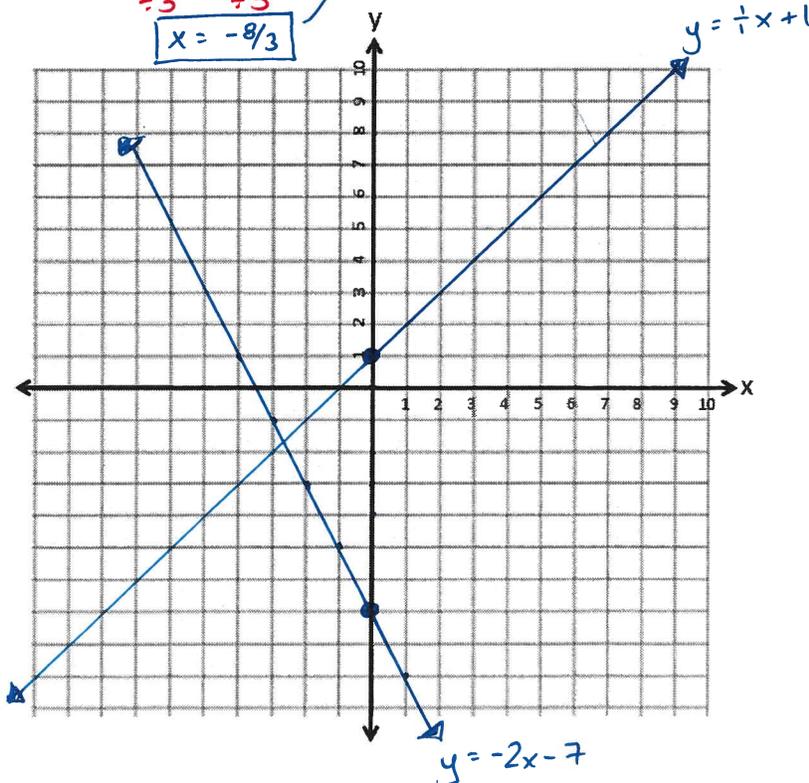
Plug into either equation.

$$y = x + 1$$

$$y = \left(-\frac{8}{3}\right) + 1$$

$$y = -\frac{5}{3}$$

so soln is $\left(-\frac{8}{3}, -\frac{5}{3}\right)$



Q2: Find the solution to the system of equations.

$$y = \frac{1}{2}x + 7$$

$$y = \frac{1}{2}x + 7$$

$$y = 2x + 2$$

$$y = 2x + 2$$

$$y = y$$

$$\frac{1}{2}x + 7 = 2x + 2$$

so $\frac{1}{2}x + 7 = 2x + 2$

$$-\frac{1}{2}x \quad -\frac{1}{2}x$$

$$7 = \frac{3}{2}x + 2$$

$$-2 \quad -2$$

$$5 = \frac{3}{2}x$$

$$5 = \frac{3}{2}x$$

$$\div \frac{3}{2} \quad \div \frac{3}{2}$$

$$\boxed{\frac{10}{3} = x}$$

Plug into either eqn.

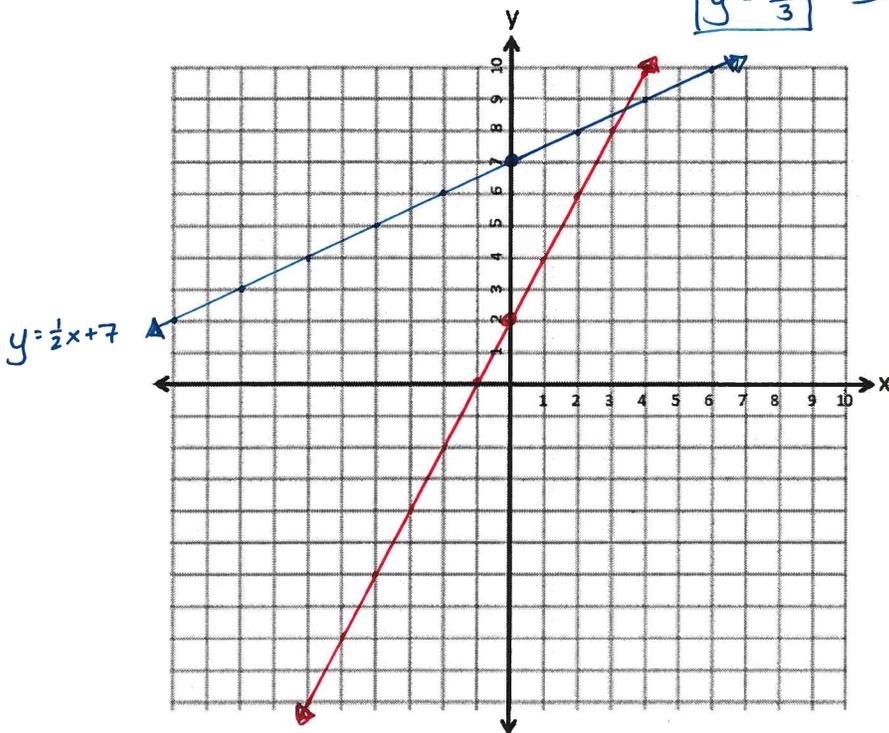
$$y = 2x + 2$$

$$y = 2\left(\frac{10}{3}\right) + 2$$

$$y = \frac{20}{3} + 2$$

$$\boxed{y = \frac{26}{3}}$$

Soln is $\left(\frac{10}{3}, \frac{26}{3}\right)$



Part 2 – Easy Algebraic Questions

Q3: Find the solution to the system of equations.

$$y = -4x + 22$$

$$y = -2x + 12$$

$$y = -4x + 22$$

$$y = -2x + 12$$

$$y = y$$

$$-4x + 22 = -2x + 12$$

so $-4x + 22 = -2x + 12$

$$+4x \quad +4x$$

$$22 = 2x + 12$$

$$-12 \quad -12$$

$$10 = 2x$$

$$\div 2 \quad \div 2$$

$$\boxed{5 = x}$$

Plug into either equation.

$$y = -2x + 12$$

$$y = -2(5) + 12$$

$$y = -10 + 12$$

$$\boxed{y = 2}$$

Soln is $(5, 2)$

Part 3 – Medium Algebraic Questions

Q4: Find the solution to the system of equations.

$$y = -5x + 21$$

$$x - 2y - 2 = 0$$

We have 4 options.
(Some are easier than others)Part 1: Solving $y = -5x + 21$ for y .

$$y = -5x + 21$$

$$x - 2y - 2 = 0$$

$$x - 2(-5x + 21) - 2 = 0$$

$$x + 10x - 42 - 2 = 0$$

$$11x - 44 = 0$$

$$+44 \quad +44$$

$$11x = 44$$

$$\div 11 \quad \div 11$$

$$\boxed{x = 4}$$

Plug into either equation.

$$y = -5x + 21$$

$$y = -5(4) + 21$$

$$y = -20 + 21$$

$$\boxed{y = 1}$$

Soln is (4, 1)

Part 2: Solving $y = -5x + 21$ for x .

$$y = -5x + 21$$

$$-21 \quad -21$$

$$y - 21 = -5x$$

$$+5 \quad \div -5 \quad \div -5$$

$$-\frac{1}{5}y + \frac{21}{5} = x$$

$$x - 2y - 2 = 0$$

$$\left(-\frac{1}{5}y + \frac{21}{5}\right) - 2y - 2 = 0$$

$$-\frac{11}{5}y + \frac{11}{5} = 0$$

$$-\frac{11}{5} \quad -\frac{11}{5}$$

$$-\frac{11}{5}y = -\frac{11}{5}$$

$$\div \left(-\frac{11}{5}\right) \quad \div \left(-\frac{11}{5}\right)$$

$$\boxed{y = 1}$$

Plug into either eqn.

$$y = -5x + 21$$

$$(1) = -5x + 21$$

$$-21 \quad -21$$

$$-20 = -5x$$

$$\div (-5) \quad \div (-5)$$

$$\boxed{4 = x}$$

Soln is (4, 1)

Part 3: Solving $x - 2y - 2 = 0$ for y .

$$x - 2y - 2 = 0$$

$$+2y \quad +2y$$

$$x - 2 = 2y$$

$$\div 2 \quad \div 2 \quad \div 2$$

$$\frac{1}{2}x - 1 = y$$

$$y = -5x + 21$$

$$\frac{1}{2}x - 1 = -5x + 21$$

$$+5x \quad +5x$$

$$\frac{11}{2}x - 1 = 21$$

$$+1 \quad +1$$

$$\frac{11}{2}x = 22$$

$$\div \left(\frac{11}{2}\right) \quad \div \left(\frac{11}{2}\right)$$

$$\boxed{x = 4}$$

Plug into either eqn.

$$y = -5x + 21$$

$$y = -5(4) + 21$$

$$y = -20 + 21$$

$$\boxed{y = 1}$$

Soln is (4, 1)

Part 4: Solving $x - 2y - 2 = 0$ for x .

$$x - 2y - 2 = 0$$

$$+2y \quad +2y$$

$$x - 2 = 2y$$

$$+2 \quad +2$$

$$x = 2y + 2$$

$$y = -5x + 21$$

$$y = -5(2y + 2) + 21$$

$$y = -10y - 10 + 21$$

$$+10y \quad +10y$$

$$11y = 11$$

$$\div 11 \quad \div 11$$

$$\boxed{y = 1}$$

Plug into either eqn.

$$x - 2y - 2 = 0$$

$$x - 2(1) - 2 = 0$$

$$x - 2 - 2 = 0$$

$$x - 4 = 0$$

$$+4 \quad +4$$

$$\boxed{x = 4}$$

Soln is (4, 1)

Q5: Find the solution to the system of equations.

$$y = 4x + 16$$

$$2x - 3y + 8 = 0$$

↓
Solve for y.
 $y = 4x + 16$
Plug into other eqn.

$$\begin{aligned} 2x - 3y + 8 &= 0 \\ 2x - 3(4x + 16) + 8 &= 0 \\ 2x - 12x - 48 + 8 &= 0 \\ -10x - 40 &= 0 \\ &\quad +40 \quad +40 \\ -10x &= 40 \\ \div(-10) \quad \div(-10) & \\ \boxed{x = -4} & \end{aligned}$$

Plug into either eqn.

$$\begin{aligned} y &= 4x + 16 \\ y &= 4(-4) + 16 \\ y &= -16 + 16 \\ \boxed{y = 0} \end{aligned}$$

soln is $(-4, 0)$

Q6: Find the solution to the system of equations.

$$y = 4x + 16$$

$$5x + 2y + 13 = 0$$

↓
Solve for y.
 $y = 4x + 16$

$$\begin{aligned} 5x + 2y + 13 &= 0 \\ 5x + 2(4x + 16) + 13 &= 0 \\ 5x + 8x + 32 + 13 &= 0 \\ 13x + 45 &= 0 \\ &\quad -45 \quad -45 \\ 13x &= -45 \\ \div 13 \quad \div 13 & \\ \boxed{x = \frac{-45}{13}} & \end{aligned}$$

Plug into either eqn.

$$\begin{aligned} y &= 4x + 16 \\ y &= 4\left(\frac{-45}{13}\right) + 16 \\ y &= \frac{-180}{13} + 16 \\ y &= \frac{-180}{13} + \frac{208}{13} \\ \boxed{y = \frac{28}{13}} \end{aligned}$$

soln is $\left(\frac{-45}{13}, \frac{28}{13}\right)$

Part 3 – Harder Algebraic Questions

Q7: Find the solution to the system of equations.

$$x + 4y + 18 = 0$$

$$2x - 3y - 8 = 0$$

↓
Solve for x
 $x + 4y + 18 = 0$
 $\quad -4y \quad -4y$
 $x + 18 = -4y$
 $\quad -18 \quad -18$
 $x = -4y - 18$

$$\begin{aligned} 2x - 3y - 8 &= 0 \\ 2(-4y - 18) - 3y - 8 &= 0 \\ -8y - 36 - 3y - 8 &= 0 \\ -11y - 44 &= 0 \\ &\quad +44 \quad +44 \\ -11y &= 44 \\ \div(-11) \quad \div(-11) & \\ \boxed{y = -4} & \end{aligned}$$

Plug into either eqn

$$\begin{aligned} x + 4y + 18 &= 0 \\ x + 4(-4) + 18 &= 0 \\ x - 16 + 18 &= 0 \\ x + 2 &= 0 \\ &\quad -2 \quad -2 \\ \boxed{x = -2} \end{aligned}$$

soln is $(-2, -4)$

Q8: Find the solution to the system of equations.

$$3x + 4y + 10 = 0$$

$$2x - 3y - 7 = 0$$

↓
Solve for x

$$\begin{aligned} 2x - 3y - 7 &= 0 \\ +3y & \quad +3y \\ 2x - 7 &= 3y \\ +7 & \quad +7 \\ 2x &= 3y + 7 \\ \div 2 & \quad \div 2 \quad \div 2 \\ x &= \frac{3}{2}y + \frac{7}{2} \end{aligned}$$

Plug into other eqn.

$$\begin{aligned} 3x + 4y + 10 &= 0 \\ 3\left(\frac{3}{2}y + \frac{7}{2}\right) + 4y + 10 &= 0 \\ \frac{9}{2}y + \frac{21}{2} + 4y + 10 &= 0 \\ \frac{17}{2}y + \frac{41}{2} &= 0 \\ -\frac{41}{2} & \quad -\frac{41}{2} \\ \frac{17}{2}y &= -\frac{41}{2} \\ \div \frac{17}{2} & \quad \div \frac{17}{2} \\ \boxed{y = -\frac{41}{17}} \end{aligned}$$

Plug into any eqn.

I'll use " $x = \frac{3}{2}y + \frac{7}{2}$ " since it is already solved for x.

$$\begin{aligned} x &= \frac{3}{2}y + \frac{7}{2} \\ x &= \frac{3}{2}\left(-\frac{41}{17}\right) + \frac{7}{2} \\ x &= -\frac{123}{34} + \frac{7}{2} \\ x &= -\frac{123}{34} + \frac{119}{34} \\ \boxed{x = -\frac{4}{34}} \end{aligned}$$

soln is $\left(-\frac{4}{34}, -\frac{41}{17}\right)$

Q9: Find the solution to the system of equations.

$$y = \frac{1}{3}x + 7$$

$$4x - 7y - 16 = 0$$

↓
Solve for y
 $y = \frac{1}{3}x + 7$

$$\begin{aligned} 4x - 7y - 16 &= 0 \\ 4x - 7\left(\frac{1}{3}x + 7\right) - 16 &= 0 \\ 4x - \frac{7}{3}x - 49 - 16 &= 0 \\ \frac{5}{3}x - 65 &= 0 \\ +65 & \quad +65 \\ \frac{5}{3}x &= 65 \\ \div \frac{5}{3} & \quad \div \frac{5}{3} \\ \boxed{x = 39} \end{aligned}$$

Plug into any eqn.

$$\begin{aligned} y &= \frac{1}{3}x + 7 \\ y &= \frac{1}{3}(39) + 7 \\ y &= 13 + 7 \\ \boxed{y = 20} \end{aligned}$$

soln is $(39, 20)$

↓

This one would be hard to graph because of how far from origin this point is.