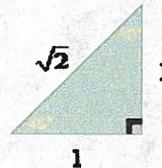
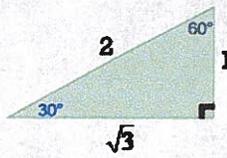


L49 - 2.2 Trigonometric Ratios of Any Angle

Key Ideas

The 30°-60°-90° and 45°-45°-90° special triangles will be used again in this lesson.

Evaluating Functions of a 30°, 45°, or 60° Angle



$\sin 30^\circ = \frac{1}{2}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\sin 45^\circ = \frac{\sqrt{2}}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\cos 60^\circ = \frac{1}{2}$

$\sin 45^\circ = \frac{\sqrt{2}}{2}$

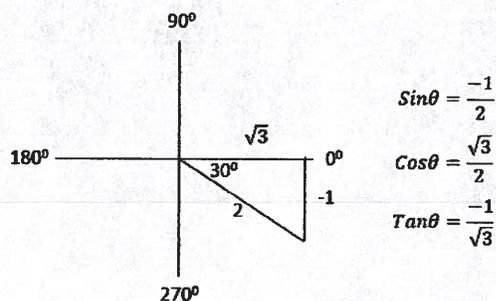
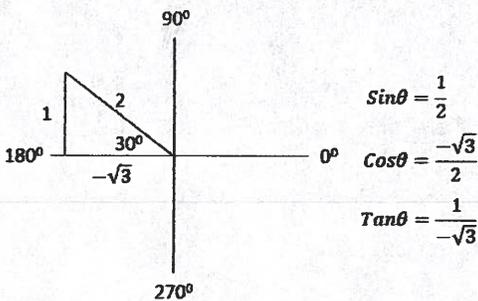
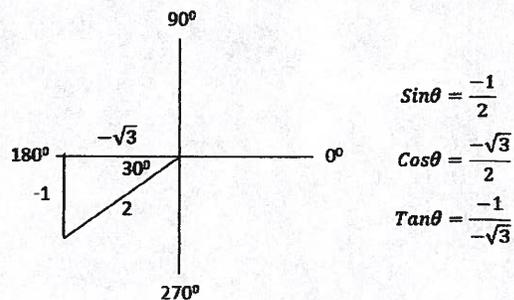
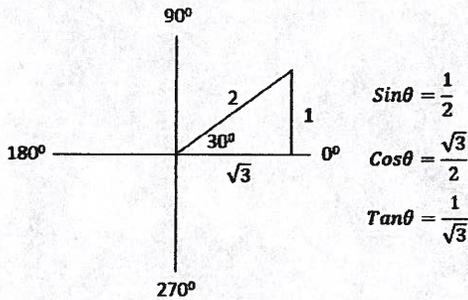
$\tan 30^\circ = \frac{\sqrt{3}}{3}$

$\tan 60^\circ = \sqrt{3}$

$\sin 45^\circ = 1$

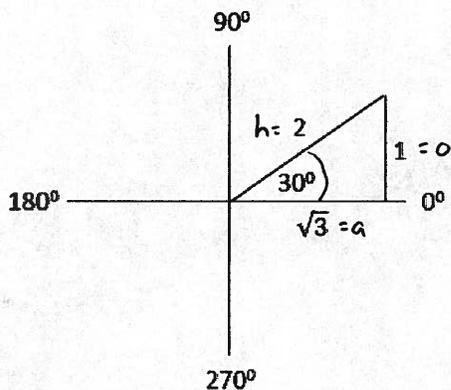
This time, however, the orientation of the triangle matters.

- Terminal arms to the right of the origin will have positive x-values (i.e. "adjacent sides")
- Terminal arms to the left of the origin will have negative x-values (i.e. "adjacent sides")
- Terminal arms above the origin will have positive y-values (i.e. "opposite sides")
- Terminal arms below the origin will have negative y-values (i.e. "opposite sides")



Part 1 – Determine Exact Value of Trig Ratios (Quadrant 1 Only)

Q1: Determine the trigonometric ratios for each triangle.



$$\sin \theta = \frac{b}{h}$$

$$\cos \theta = \frac{a}{h}$$

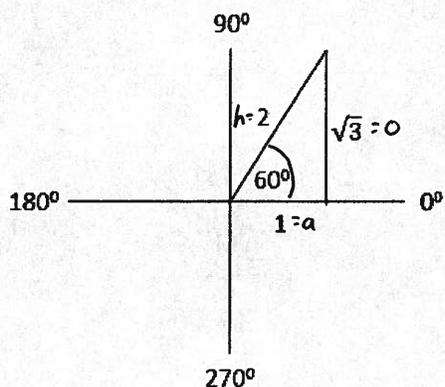
$$\tan \theta = \frac{b}{a}$$

$$\sin 30 = \frac{1}{2}$$

$$\cos 30 = \frac{\sqrt{3}}{2}$$

$$\tan 30 = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\tan 30 = \frac{\sqrt{3}}{3}$$



$$\sin \theta = \frac{b}{h}$$

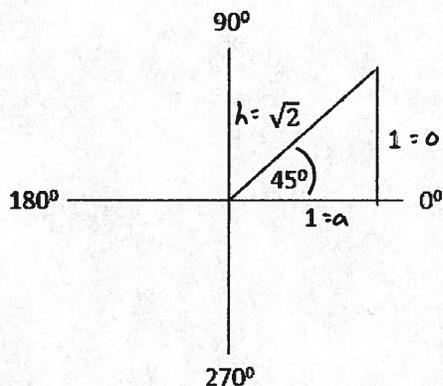
$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{b}{a}$$

$$\sin 60 = \frac{\sqrt{3}}{2}$$

$$\cos 60 = \frac{1}{2}$$

$$\tan 60 = \frac{\sqrt{3}}{1}$$



$$\sin \theta = \frac{b}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{b}{a}$$

$$\sin 45 = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\cos 45 = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

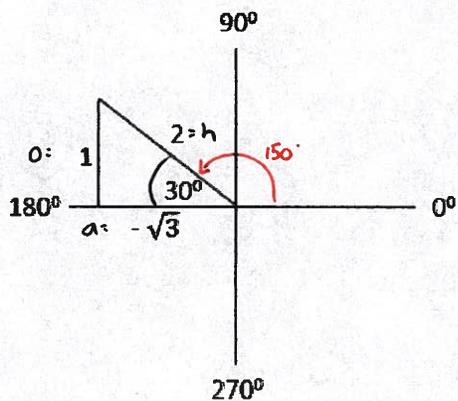
$$\tan 45 = \frac{1}{1}$$

$$\sin 45 = \frac{\sqrt{2}}{2}$$

$$\cos 45 = \frac{\sqrt{2}}{2}$$

Part 2 – Determine Exact Value of Trig Ratios (All Quadrants)

Q2: Determine the trigonometric ratios for each triangle.



$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

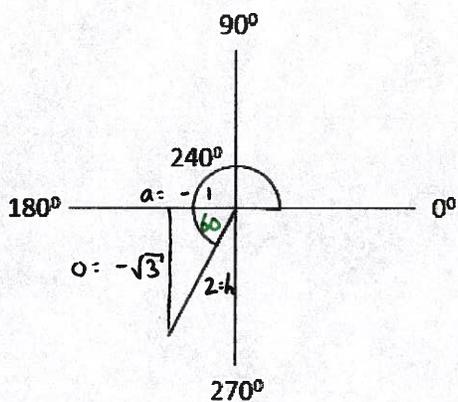
$$\tan \theta = \frac{o}{a}$$

$$\sin 150^\circ = \frac{1}{2}$$

$$\cos 150^\circ = \frac{-\sqrt{3}}{2}$$

$$\tan 150^\circ = \frac{1}{-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\tan 150^\circ = \frac{-\sqrt{3}}{3}$$



$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

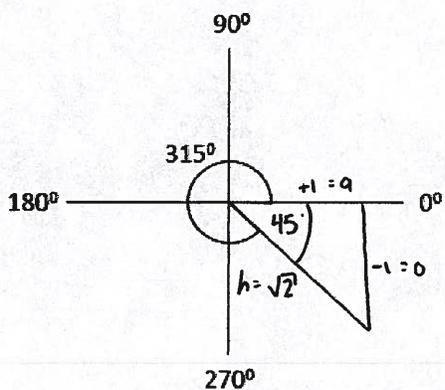
$$\tan \theta = \frac{o}{a}$$

$$\sin 240^\circ = \frac{-\sqrt{3}}{2}$$

$$\cos 240^\circ = \frac{-1}{2}$$

$$\tan 240^\circ = \frac{-\sqrt{3}}{-1}$$

$$\tan 240^\circ = \sqrt{3}$$



$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

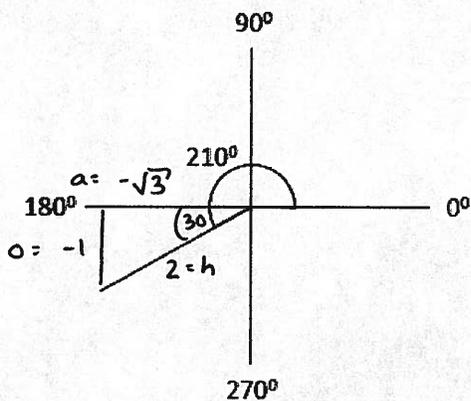
$$\sin 315^\circ = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\cos 315^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\tan 315^\circ = \frac{-1}{1}$$

$$\sin 315^\circ = \frac{-\sqrt{2}}{2}$$

$$\cos 315^\circ = \frac{\sqrt{2}}{2}$$



$$\sin \theta = \frac{b}{h}$$

$$\sin 210 = \frac{-1}{2}$$

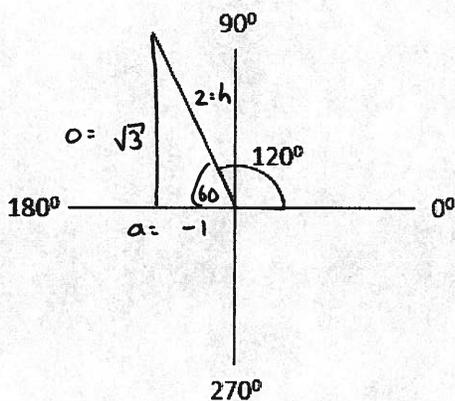
$$\cos \theta = \frac{a}{h}$$

$$\cos 210 = \frac{-\sqrt{3}}{2}$$

$$\tan \theta = \frac{b}{a}$$

$$\tan 210 = \frac{-1}{-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\tan 210 = \frac{\sqrt{3}}{3}$$



$$\sin \theta = \frac{b}{h}$$

$$\sin 120 = \frac{\sqrt{3}}{2}$$

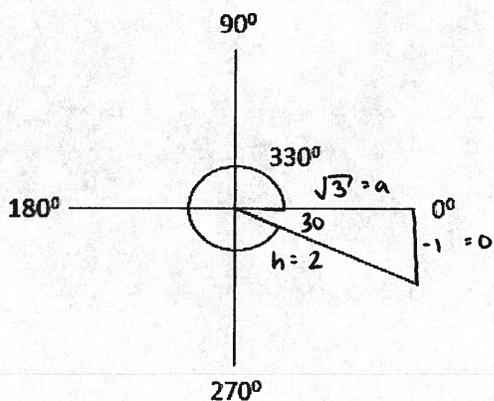
$$\cos \theta = \frac{a}{h}$$

$$\cos 120 = \frac{-1}{2}$$

$$\tan \theta = \frac{b}{a}$$

$$\tan 120 = \frac{\sqrt{3}}{-1}$$

$$\tan 120 = -\sqrt{3}$$



$$\sin \theta = \frac{b}{h}$$

$$\sin 330 = \frac{-1}{2}$$

$$\cos \theta = \frac{a}{h}$$

$$\cos 330 = \frac{\sqrt{3}}{2}$$

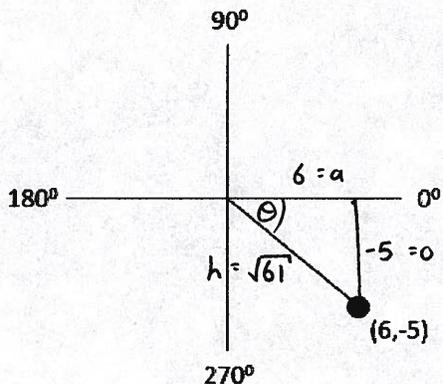
$$\tan \theta = \frac{b}{a}$$

$$\tan 330 = \frac{-1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\tan 330 = \frac{-\sqrt{3}}{3}$$

Part 3 – Determine Exact Value of Trig Ratios (non-Special Triangles)

Determine the exact values of $\sin \theta$, $\cos \theta$, and $\tan \theta$ if the terminal arm of an angle in standard position passes through the given point.



$$a^2 + b^2 = c^2$$

$$(6)^2 + (-5)^2 = c^2$$

$$c = \sqrt{61}$$

$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

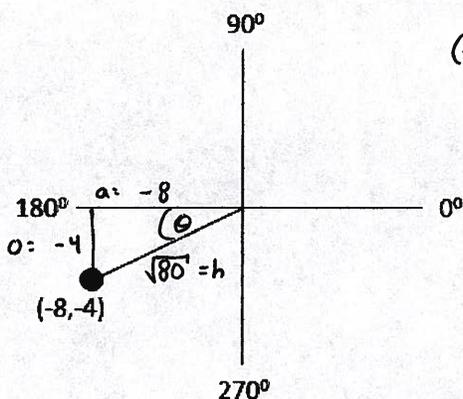
$$\sin \theta = \frac{-5}{\sqrt{61}} \cdot \frac{\sqrt{61}}{\sqrt{61}}$$

$$\cos \theta = \frac{6}{\sqrt{61}} \cdot \frac{\sqrt{61}}{\sqrt{61}}$$

$$\tan \theta = \frac{-5}{6}$$

$$\sin \theta = \frac{-5\sqrt{61}}{61}$$

$$\cos \theta = \frac{6\sqrt{61}}{61}$$



$$a^2 + b^2 = c^2$$

$$(-8)^2 + (-4)^2 = c^2$$

$$c = \sqrt{80}$$

$$\begin{matrix} 80 & & 10 \\ \wedge & & \wedge \\ 4 & & 5 \\ \wedge & & \wedge \\ 2 & & 2 \end{matrix}$$

$$\sqrt{80} = \sqrt{2^2 \cdot 2^2 \cdot 5}$$

$$= 2 \cdot 2 \cdot \sqrt{5}$$

$$= 4\sqrt{5}$$

$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{-4}{4\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

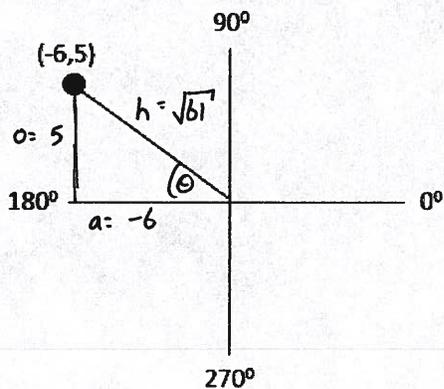
$$\cos \theta = \frac{-8}{4\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$\tan \theta = \frac{-4}{-8}$$

$$\sin \theta = \frac{-\sqrt{5}}{5}$$

$$\cos \theta = \frac{-2\sqrt{5}}{5}$$

$$\tan \theta = \frac{1}{2}$$



$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

$$\sin \theta = \frac{5}{\sqrt{61}} \cdot \frac{\sqrt{61}}{\sqrt{61}}$$

$$\cos \theta = \frac{-6}{\sqrt{61}} \cdot \frac{\sqrt{61}}{\sqrt{61}}$$

$$\tan \theta = \frac{5}{-6}$$

$$\sin \theta = \frac{5\sqrt{61}}{61}$$

$$\cos \theta = \frac{-6\sqrt{61}}{61}$$

Part 4 – Solving a Trig Ratio

Solve each equation, for $0 \leq \theta < 360 \text{ deg}$, using a diagram involving a special right triangle.

	Sketch of Angle in Standard Position	Sketch of Special Right Triangle	Solution to Equation
$\tan \theta = \frac{+b}{+a}$ $\tan \theta = \frac{-b}{-a}$ $\tan \theta = \frac{1}{\sqrt{3}}$			$\theta = 30^\circ$ $\theta = 210^\circ$
$\cos \theta = \frac{-a}{h}$ + or - $\cos \theta = -\frac{1}{2}$			$\theta = 120^\circ$ $\theta = 240^\circ$
$\sin \theta = \frac{+b}{h}$ + or - $\sin \theta = \frac{1}{2}$			$\theta = 30^\circ$ $\theta = 150^\circ$
$\cos \theta = \frac{-a}{h}$ + or - $\cos \theta = -\frac{1}{\sqrt{2}}$			$\theta = 135^\circ$ $\theta = 225^\circ$
$\tan \theta = \frac{-b}{+a}$ $\tan \theta = \frac{+b}{-a}$ $\tan \theta = -1$			$\theta = 135^\circ$ $\theta = 315^\circ$