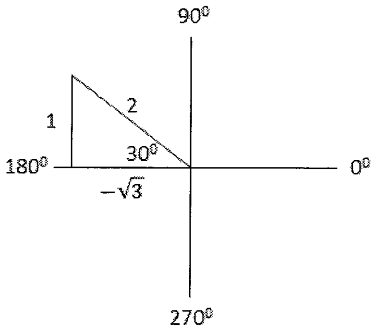
KEY

Math 20-1

Lxx – Worksheet – 2.2 Trigonometric Ratios of Any Angle

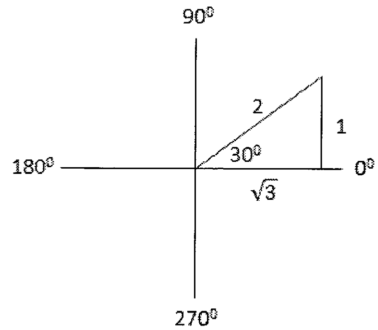
Part 1 – Quick Review of Concepts



$Sin\theta = \frac{1}{2}$

$Cos\theta = \frac{-\sqrt{3}}{2}$

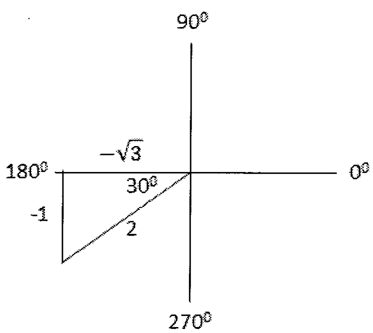
$Tan\theta = \frac{1}{-\sqrt{3}}$



$Sin\theta = \frac{1}{2}$

$Cos\theta = \frac{\sqrt{3}}{2}$

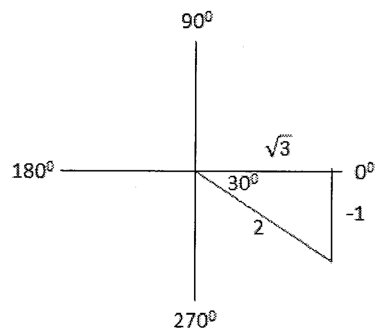
$Tan\theta = \frac{1}{\sqrt{3}}$



$Sin\theta = \frac{-1}{2}$

$Cos\theta = \frac{-\sqrt{3}}{2}$

$Tan\theta = \frac{-1}{-\sqrt{3}}$

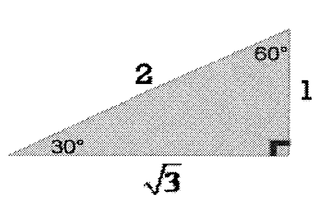


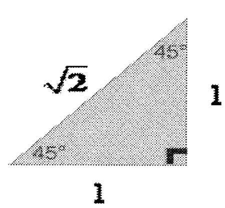
$Sin\theta = \frac{-1}{2}$

$Cos\theta = \frac{\sqrt{3}}{2}$

$Tan\theta = \frac{-1}{\sqrt{3}}$

Evaluating Functions of a 30°, 45°, or 60° Angle





$\sin 30° = \frac{1}{2}$	$\sin 60° = \frac{\sqrt{3}}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$
$\cos 30° = \frac{\sqrt{3}}{2}$	$\cos 60° = \frac{1}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$
$\tan 30° = \frac{\sqrt{3}}{3}$	$\tan 60° = \sqrt{3}$	$\sin 45° = 1$

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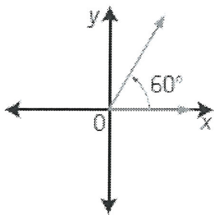
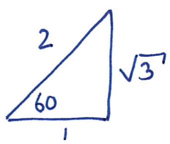
Part 2 – Textbook Questions

Pg 96 #2: Determine the exact values of the sine, cosine, and tangent ratios for each angle.

a)



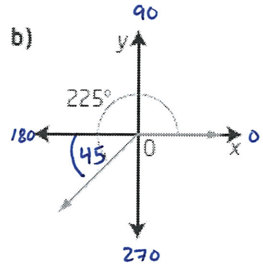


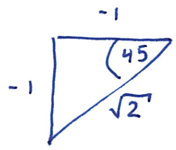
$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \frac{\sqrt{3}}{1}$

b)



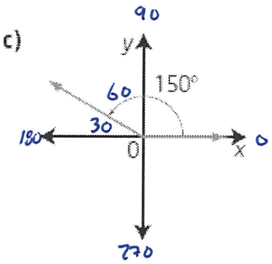


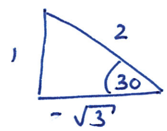
$\sin 225° = \frac{-1}{\sqrt{2}}$

$\cos 225° = \frac{-1}{\sqrt{2}}$

$\tan 225° = \frac{-1}{-1} = +1$

c)



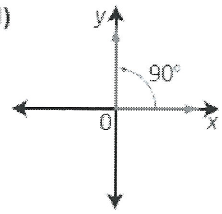


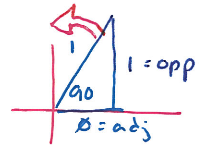
$\sin 150° = \frac{1}{2}$

$\cos 150° = \frac{-\sqrt{3}}{2}$

$\tan 150° = -\frac{1}{\sqrt{3}}$

d)





or Adjacent side is zero, so line is vertical.

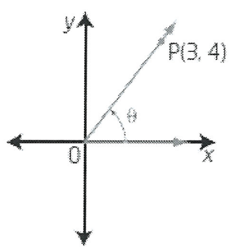
$\sin 90° = \frac{1}{1} = 1$

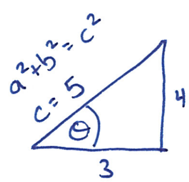
$\cos 90° = \frac{0}{1} = \varnothing$

$\tan 90 = \frac{1}{0}$ = Undefined

Pg 96 #3: The coordinate of a point P on the terminal arm of each angle are shown. Write the exact trigonometric ratios of sin θ, cos θ and tan θ for each.

a)



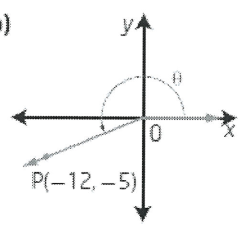


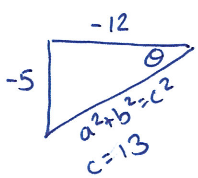
$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\tan\theta = \frac{4}{3}$

b)



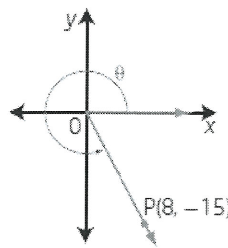


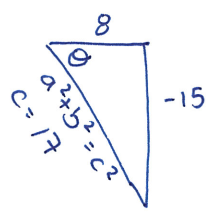
$\sin\theta = \frac{-5}{13}$

$\cos\theta = \frac{-12}{13}$

$\tan\theta = \frac{-5}{-12} = \frac{5}{12}$

c)



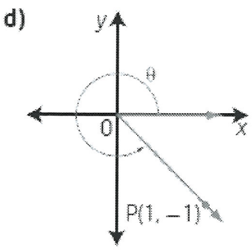


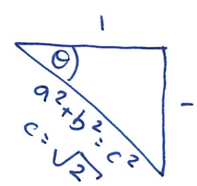
$\sin\theta = \frac{-15}{17}$

$\cos\theta = \frac{8}{17}$

$\tan\theta = \frac{-15}{8}$

d)





$\sin\theta = \frac{-1}{\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\tan\theta = \frac{-1}{1}$

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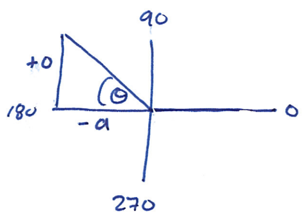
Pg 96 #4ab: For each description, in which quadrant does the terminal arm of the angle θ lie?

Cos θ < 0 and Sin θ > 0

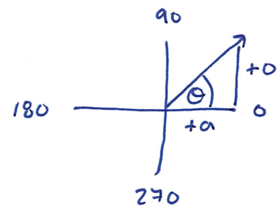
$\cos\theta = \frac{-a}{h}$ $\sin\theta = \frac{+o}{h}$



Cos θ > 0 and Tan θ > 0

$\cos\theta = \frac{+a}{h}$ $\tan\theta = \frac{+o}{+a}$ or $\frac{-o}{-a}$

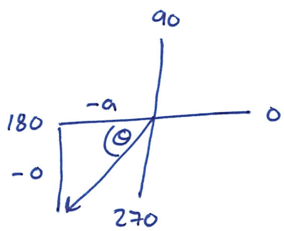
This one.



Pg 96 #4cd: For each description, in which quadrant does the terminal arm of the angle θ lie?

Sin θ < 0 and Cos θ < 0

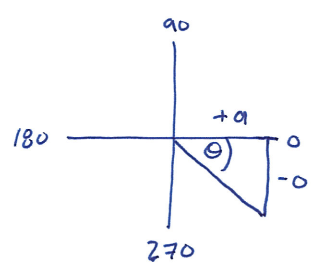
$\sin\theta = \frac{-o}{h}$ $\cos\theta = \frac{-a}{h}$



Tan θ < 0 and Cos θ > 0

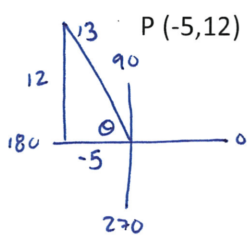
$\tan\theta = \frac{-o}{a}$ or $\frac{o}{-a}$ $\cos\theta = \frac{+a}{h}$

This one



Pg 96 #5ab: Determine the exact values of Sin θ, Cos θ, and Tan θ if the terminal arm of an angle in standard position passes through the given point.

P (-5,12)



$a^2+b^2=c^2$

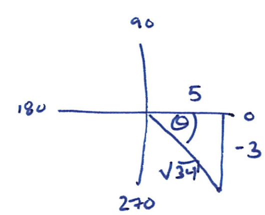
$c=13$

$\sin\theta = \frac{12}{13}$

$\cos\theta = \frac{-5}{13}$

$\tan\theta = \frac{12}{-5}$

P (5,-3)



$a^2+b^2=c^2$

$c=\sqrt{34}$

$\sin\theta = \frac{-3}{\sqrt{34}}$

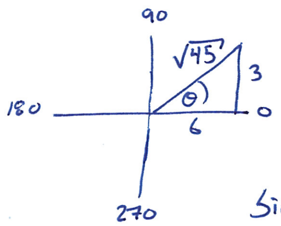
$\cos\theta = \frac{5}{\sqrt{34}}$

$\tan\theta = \frac{-3}{5}$

Math 20-1 KEY

Pg 96 #5cd: Determine the exact values of Sin θ, Cos θ, and Tan θ if the terminal arm of an angle in standard position passes through the given point.

P (6,3)



$a^2+b^2=c^2$

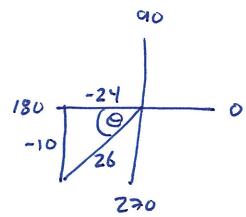
$c=\sqrt{45}$

$\sin\theta = \frac{3}{\sqrt{45}}$

$\cos\theta = \frac{6}{\sqrt{45}}$

$\tan\theta = \frac{3}{6} = \frac{1}{2}$

P (-24,-10)



$a^2+b^2=c^2$

$c=26$

$\sin\theta = \frac{-10}{26} = \frac{-5}{13}$

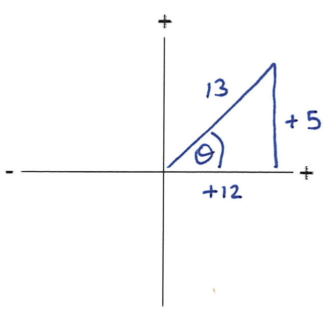
$\cos\theta = \frac{-24}{26} = \frac{-12}{13}$

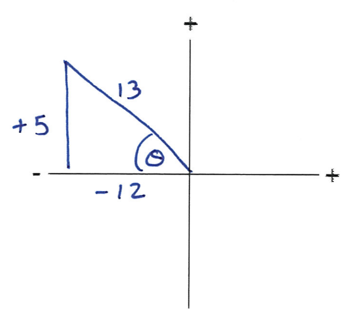
$\tan\theta = \frac{-10}{-24} = \frac{5}{12}$

Pg 96 #7: An angle is in standard position such that $Sin\ \theta = \frac{5}{13}$

$\sin\theta = \frac{+o}{h}$ w/ +a or $\sin\theta = \frac{+o}{h}$ w/ -a

a. Sketch a diagram to show the two possible positions of the angle.



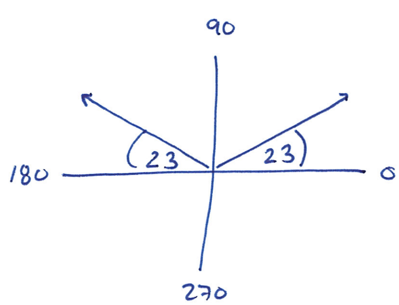


b. Determine the possible values of θ, to the nearest degree, if $0 \leq \theta < 360\ deg$.

$\sin\theta = \frac{5}{13}$

$\theta = \sin^{-1}\left(\frac{5}{13}\right)$

$\theta = 23°$



$\boxed{23° \text{ or } 157°}$

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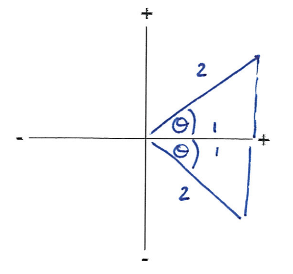
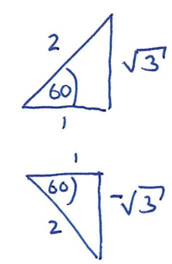
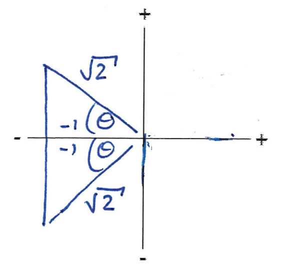
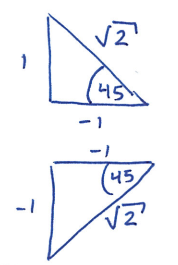
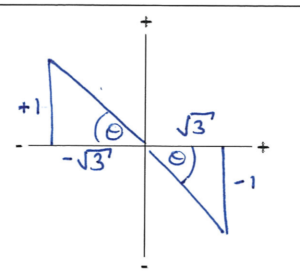
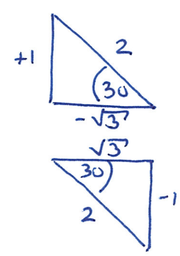
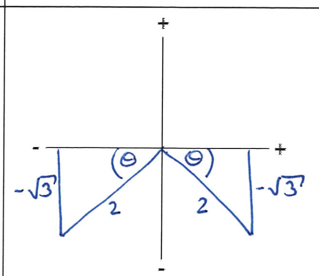
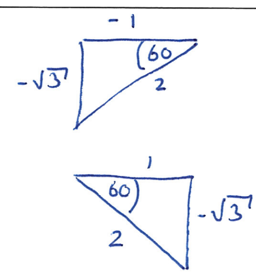
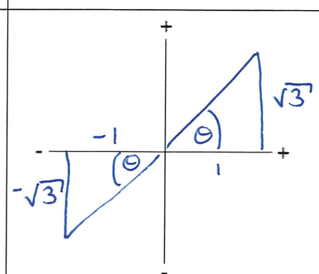
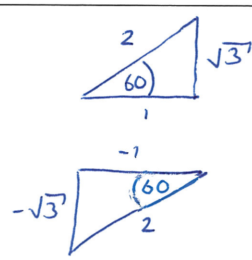
Pg 96 #9abcde: Solve each equation, for $0 \leq \theta < 360\ deg$, using a diagram involving a special right triangle.

	Sketch of Angle in Standard Position	Sketch of Special Right Triangle	Solution to Equation
$Cos\theta = \frac{+a}{h}$ $Cos\theta = \frac{1}{2}$ so +o, -o			θ = 60° θ = 300°
$Cos\theta = \frac{-a}{h}$ $Cos\theta = -\frac{1}{\sqrt{2}}$ so +o, -o			θ = 135° θ = 225°
$Tan\theta = \frac{-o}{a}$ $Tan\theta = \frac{o}{-a}$ $Tan\theta = -\frac{1}{\sqrt{3}}$			θ = 150° θ = 330°
$Sin\theta = \frac{-o}{h}$ $Sin\theta = -\frac{\sqrt{3}}{2}$ so +a, -a			θ = 240° θ = 300°
$Tan\theta = \frac{+o}{+a}$ $Tan\theta = \frac{-o}{-a}$ $Tan\theta = \frac{\sqrt{3}}{1}$			θ = 60° θ = 240°

KEY

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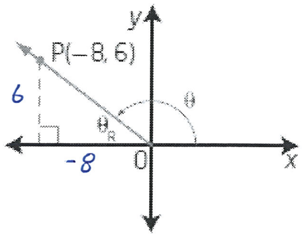
Pg 96 #11: Determine the values of *x, y, r,* sin θ, cos θ, and tan θ in each.

a)



$x = -8$

$y = +6$

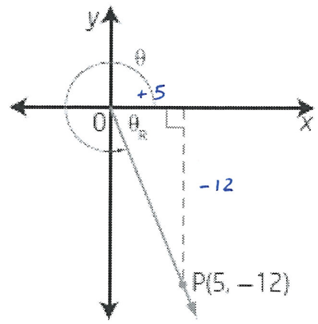
$r = \sqrt{(-8)^2 + (6)^2} = 10$

$\sin\theta = \frac{6}{10} = \frac{3}{5}$

$\cos\theta = \frac{-8}{10} = \frac{-4}{5}$

$\tan\theta = \frac{6}{-8} = -\frac{3}{4}$

b)



$x = +5$

$y = -12$

$r = \sqrt{(5)^2 + (-12)^2} = 13$

$\sin\theta = \frac{-12}{13}$

$\cos\theta = \frac{5}{13}$

$\tan\theta = \frac{-12}{5}$

Pg 96 #16: If $\cos\theta = \frac{1}{5}$ and $tan\theta = 2\sqrt{6}$, determine the exact value of sin θ.

$\cos\theta = \frac{+a}{h}$ $\tan\theta = \frac{+o}{+a}$ or $\frac{-o}{-a}$

This one.

So $\frac{a}{h} = \frac{1}{5}$ and $\frac{o}{a} = 2\sqrt{6} \Rightarrow o = 2\sqrt{6}a$

$h = 5a \Rightarrow h = 5a$

$\sin\theta = \frac{o}{h} = \frac{2\sqrt{6}a}{5a}$

$\sin\theta = \frac{2\sqrt{6}}{5}$

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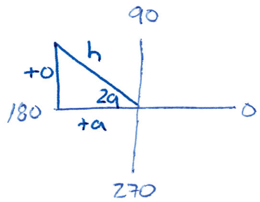
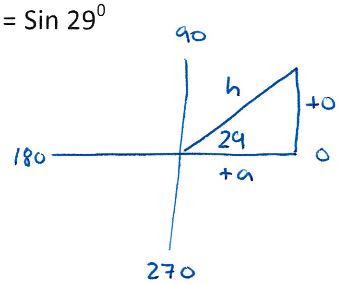
Pg 96 #18ab: Without using technology, determine whether each statement is true or false. Justify your answer.

Sin 151^0 = Sin 29^0



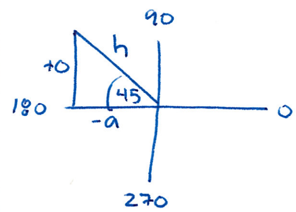


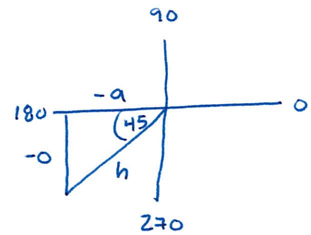
$\sin 151^\circ = \frac{+o}{h}$

$\sin 29^\circ = \frac{+o}{h}$

So "True"

Cos 135^0 = Sin 225^0





$\cos 135 = \frac{-a}{h}$

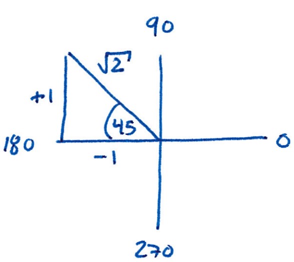
$\sin 225 = \frac{-o}{h}$

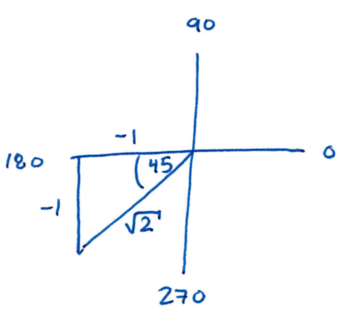
For a 45 deg triangle o and a are the same length.

So "True".

Pg 96 #18cd: Without using technology, determine whether each statement is true or false. Justify your answer.

Tan 135^0 = Tan 225^0



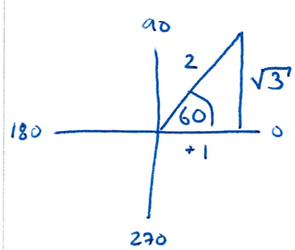


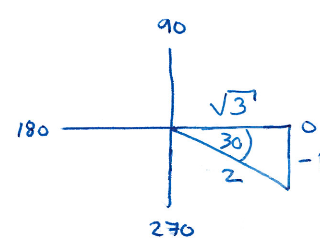
$\tan 135 = \frac{+1}{-1}$

$\tan 225 = \frac{-1}{-1}$

Not the same.
"False"

Sin 60^0 = Cos 330^0





$\sin 60 = \frac{\sqrt{3}}{2}$

$\cos 330 = \frac{\sqrt{3}}{2}$

Yeppers... those are the same.
"True"