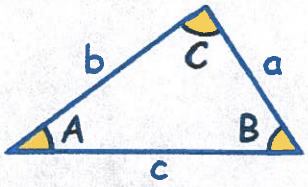
Math 20-1 KEY

L50 – 2.3 Sine Law

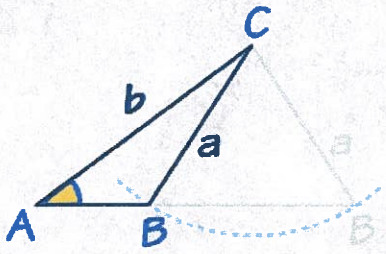
Key Ideas

Sine Law:

$$\frac{Sin A}{a} = \frac{Sin B}{b} = \frac{Sin C}{c} \quad \text{or} \quad \frac{a}{Sin A} = \frac{b}{Sin B} = \frac{c}{Sin C}$$



"AmbiguouS caSe" – This only happens in the **"Angle-Side-Side"** case, where we know two sides and an angle ***not*** between them. For example, in the diagram below, if we know ∠A, *b*, and *a*, this is a case of **"Angle-Side-Side"** resulting in an **AmbiguouS caSe** with two possible solutions.



The solutions to the **AmbiguouS caSe** for ∠C will be:

- $\angle C_1 = \theta$ *(the value you solved for)*
- $\angle C_2 = 180^0 - \theta$

Don't do the "180-θ" thing for the angle between the two known sides.

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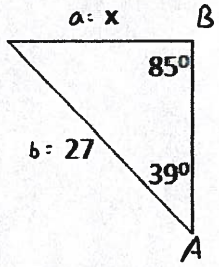
Part 1 – Solving for a Side

Q1: Find the length of *x*.



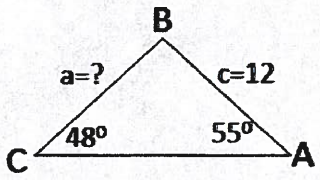
$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 39}{x} = \frac{\sin 85}{27}$

$\frac{0.629}{x} = \frac{0.996}{27}$

$x = 17.1$

Q2: Find the length of *a*.



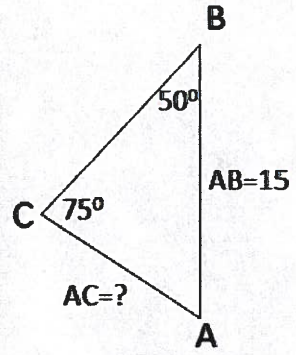
$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin 55}{a} = \frac{\sin 48}{12}$

$\frac{0.819}{a} = \frac{0.743}{12}$

$a = 13.2$

Q3: Find the length of *AC*



$\frac{\sin B}{b} = \frac{\sin C}{c}$

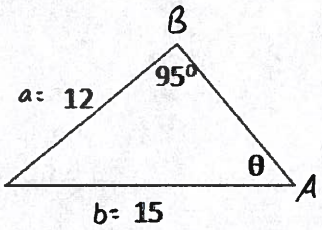
$\frac{\sin 50}{AC} = \frac{\sin 75}{15}$

$\frac{0.766}{AC} = \frac{0.966}{15}$

$AC = 11.9$

Part 2 – Solving for an Angle (Easy)

Q4: Solve for θ.



$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin \theta}{12} = \frac{\sin 95}{15}$

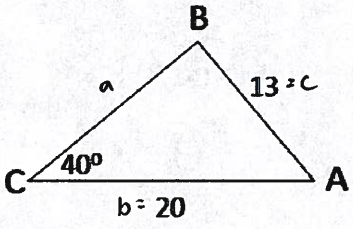
$\frac{\sin \theta}{12} = \frac{0.996}{15}$

$\sin \theta = 0.796\ldots$

$\theta = \sin^{-1}(0.796\ldots)$

$\theta = 52.8°$

Q5: Solve for ∠B.



$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin B}{20} = \frac{\sin 40}{13}$

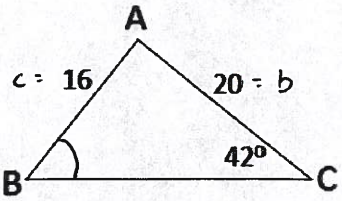
$\frac{\sin B}{20} = \frac{0.642\ldots}{13}$

$\sin B = 0.9889\ldots$

$B = \sin^{-1}(0.9889\ldots)$

$\angle B = 81.5°$

Q6: Solve for ∠ABC.



$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin B}{20} = \frac{\sin 42}{16}$

$\frac{\sin B}{20} = \frac{0.669\ldots}{16}$

$\sin B = 0.836\ldots$

$B = \sin^{-1}(0.836\ldots)$

$\angle B = 56.8°$

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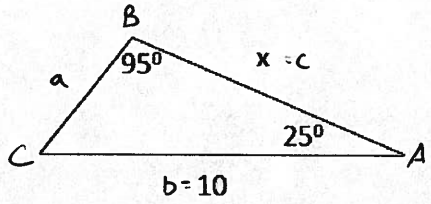
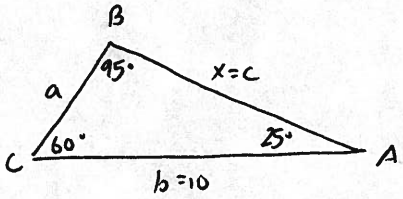
Part 3 – Solving for an Angle (Harder – 180deg in a Triangle)

Q7: Solve for x.



$95° + 25° + \angle C = 180°$

$\angle C = 60°$



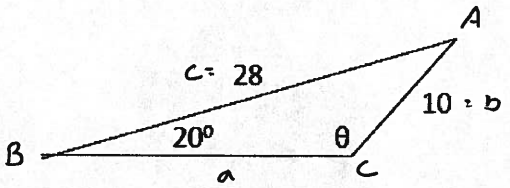
$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 95}{10} = \frac{\sin 60}{x}$

$\frac{0.996}{10} = \frac{0.866}{x}$

$\boxed{x = 8.69}$

Q8: Solve for $\angle A$.



$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 20}{10} = \frac{\sin C}{28}$

$\frac{0.342}{10} = \frac{\sin C}{28}$

$\sin C = 0.957...$

$\angle C = \sin^{-1}(0.957...)$

$\boxed{\angle C = 73.3°}$

$20° + 73.3° + \angle A = 180°$

$\boxed{\angle A = 86.7°}$

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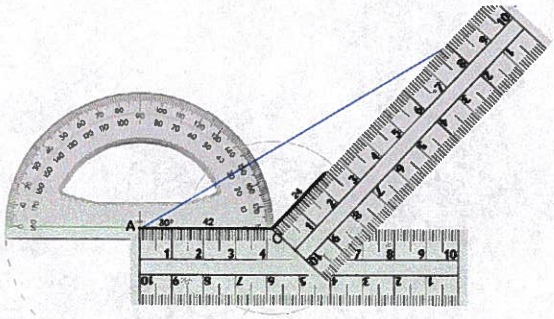
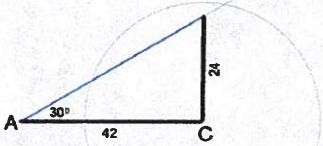
Part 4 – Solving for an Angle (Hardest – AmbiguouS caSe, Angle-Side-Side)

In ΔABC, <A = 30°, a = 24cm and b = 42cm.
Determine the measures of the other side and angles.

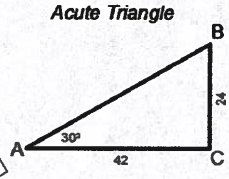


Option #1



Option #2

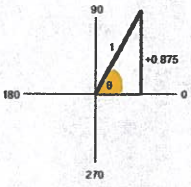
Acute Triangle



$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 30}{24} = \frac{\sin B}{42}$

$\sin B = 0.875$

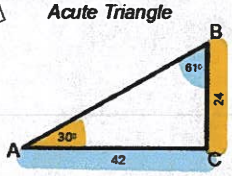


$\sin B = 0.875$

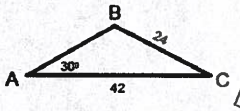
$\angle B = \sin^{-1}(0.875)$

$\angle B = 61°$

Acute Triangle



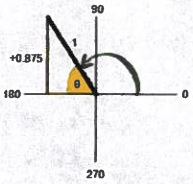
Obtuse Triangle



$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 30}{24} = \frac{\sin B}{42}$

$\sin B = 0.875$



$\sin B = 0.875$

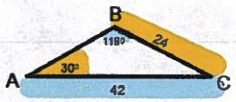
$\angle B = \sin^{-1}(0.875)$

$\angle B = 61°$

or... $180 - 61°$

$\angle B = 119°$

Obtuse Triangle



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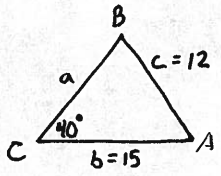
Use the following information to answer Q9:

In triangle ΔABC, side AB=12, side AC=15, and ∠C=40°.

Q9: Determine the value(s) of ∠B.



Oh noes! It's Angle-Side-Side... that means it's the Ambiguous case.

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin B}{15} = \frac{\sin 40}{12}$$

$$\frac{\sin B}{15} = \frac{0.6427\ldots}{12}$$

$$\sin B = 0.803\ldots$$

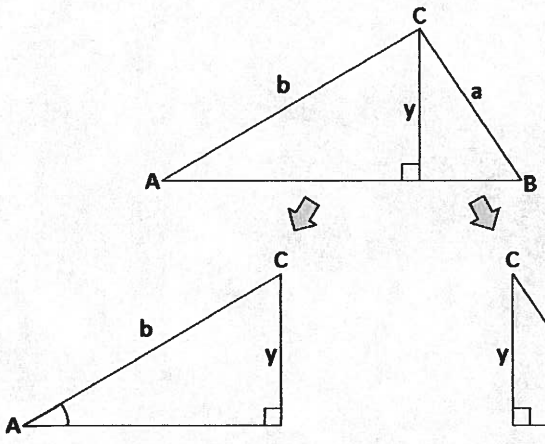
$$\angle B = \sin^{-1}(0.803\ldots)$$

$\boxed{\angle B = 53.5°}$ or $\angle B = 180 - 53.5°$

$\boxed{\angle B = 126.5°}$

Part 5 – Derivation of Sine Law

Q10: Derive the Sine Law.



$\sin A = \frac{y}{b}$

$y = b\sin A$

$\sin B = \frac{y}{a}$

$y = a\sin B$

$y = y$

$b\sin A = a\sin B$

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b}}$$