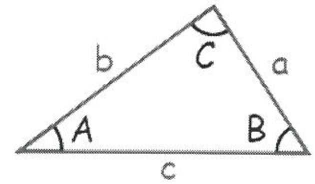
Math 20-1 KEY

Lxx – Worksheet – 2.3 The Sine Law

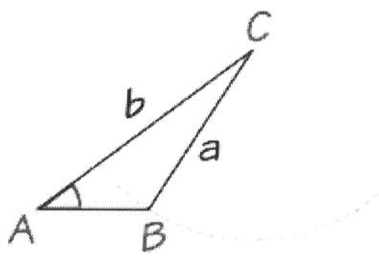
Part 1 – Sine Law Overview

Sine Law:

$$\frac{SinA}{a} = \frac{Sin\,B}{b} = \frac{Sin\,C}{c} \quad \text{or} \quad \frac{a}{Sin\,A} = \frac{b}{Sin\,B} = \frac{c}{Sin\,C}$$



"**A**mbiguou**S** ca**S**e" – This only happens in the "**A**ngle-**S**ide-**S**ide" case, where we know two sides and an angle ***not*** between them. For example, in the diagram below, if we know ∠A , *b*, and *a*, this is a case of "**A**ngle-**S**ide-**S**ide" resulting in an **A**mbiguou**S** ca**S**e with two possible solutions.



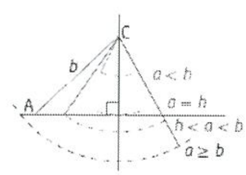
The solutions to the **A**mbiguou**S** ca**S**e for ∠*B* will be:

- $\angle B_1 = \theta$ *(the value you solved for)*
- $\angle B_2 = 180^0 - \theta$

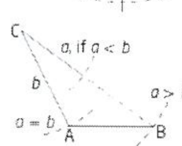
} Don't do the "180 − θ" thing for the angle between the two known sides.

Key Ideas

- You can use the sine law to find the measures of sides and angles in a triangle.
- For △ABC, state the sine law as $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.
- Use the sine law to solve a triangle when you are given the measures of
  - two angles and one side
  - two sides and an angle that is opposite one of the given sides
- The ambiguous case of the sine law may occur when you are given two sides and an angle opposite one of the sides.
- For the ambiguous case in △ABC, when ∠A is an acute angle:
  - $a \geq b$ one solution
  - $a = h$ one solution $h = b \sin A$
  - $a < h$ no solution
  - $b \sin A < a < b$ two solutions



- For the ambiguous case in △ABC, when ∠A is an obtuse angle:
  - $a \leq b$ no solution
  - $a > b$ one solution



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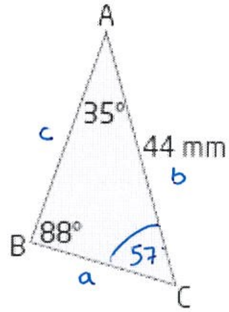
Part 2 – Textbook Questions

Pg 108 #2: Determine the length of AB in each.

a)

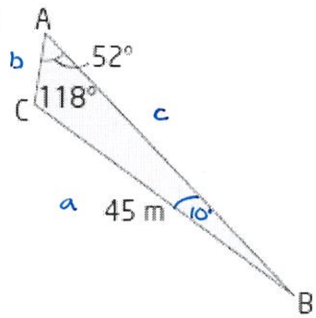


$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 88}{44} = \frac{\sin 57}{c}$

$c = 36.9$ mm

b)



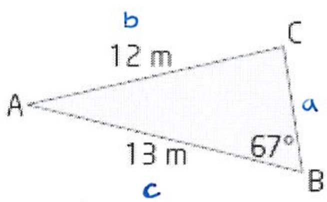
$\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin 118}{c} = \frac{\sin 52}{45}$

$c = 50.4$ m

Pg 108 #4a: Determining the lengths of all three sides and the measure of all three angles is called solving a triangle. Solve each triangle.

a)



Angle-Side-Side
Ambiguous case

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 67}{12} = \frac{\sin C}{13}$

$\angle C = 85.7°$ or $\angle C = 180 - 85.7$
$= 94.3°$

$\angle A = 27.3°$

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 27.3}{a} = \frac{\sin 67}{12}$

$a = 6.0$ m

$\angle A = 18.7°$

$\frac{\sin A}{a} = \frac{\sin B}{b}$

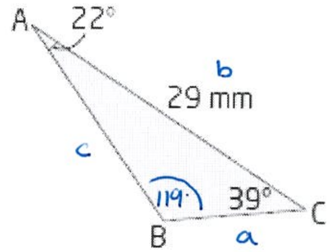
$\frac{\sin 18.7}{a} = \frac{\sin 67}{12}$

$a = 4.2$ m

KEY

Pg 108 #4cd: Determining the lengths of all three sides and the measure of all three angles is called solving a triangle. Solve each triangle.

c)



$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 119}{29} = \frac{\sin 22}{a}$$

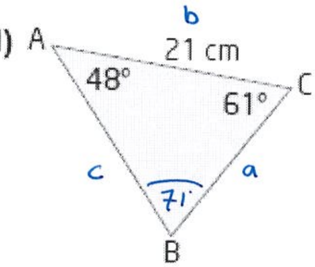
$a = 12.4$ mm

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 119}{29} = \frac{\sin 39}{c}$$

$c = 20.9$ mm

d)



$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 71}{21} = \frac{\sin 48}{a}$$

$a = 16.5$ cm

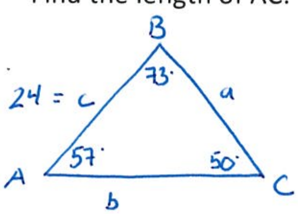
$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 71}{21} = \frac{\sin 61}{c}$$

$c = 19.4$ cm

Pg 108 #5ab: Sketch each triangle. Determine the measure of the indicated side.

In ΔABC, $\angle A = 57^0$, $\angle B = 73^0$, and AB = 24cm. Find the length of AC.

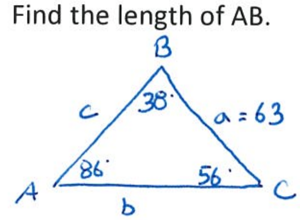


$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 73}{b} = \frac{\sin 50}{24}$$

$b = 30.0$ cm

In ΔABC, $\angle B = 38^0$, $\angle C = 56^0$, and BC = 63cm. Find the length of AB.



$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 86}{63} = \frac{\sin 56}{c}$$

$c = 52.4$ cm

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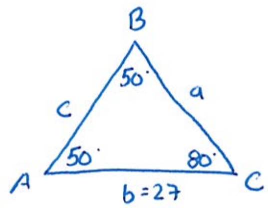
Pg 108 #5cd: Sketch each triangle. Determine the measure of the indicated side.

In ΔABC, ∠A = 50⁰, ∠B = 50⁰, and AC = 27cm. Find the length of AB.

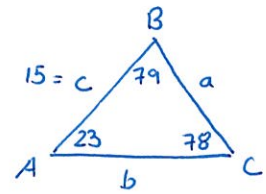


$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 50}{27} = \frac{\sin 80}{c}$$

$c = 34.7$ cm

In ΔABC, ∠A = 23⁰, ∠C = 78⁰, and AB = 15cm. Find the length of BC.



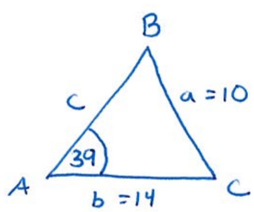
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 23}{a} = \frac{\sin 78}{15}$$

$a = 6.0$ cm

Pg 108 #6ab: For each triangle, determine whether there is no solution, one solution, or two solutions.

In ΔABC, ∠A = 39⁰, a = 10cm, and b = 14cm.



Angle-Side-Side
Ambiguous case

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

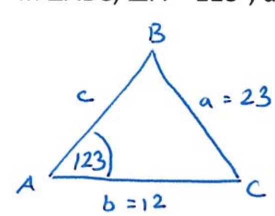
$$\frac{\sin 39}{10} = \frac{\sin B}{14}$$

$\sin B = 0.881$

$\angle B = 61.8°$ or $118.2°$

Two solutions.

In ΔABC, ∠A = 123⁰, a = 23cm, and b = 12cm.



Angle-Side-Side
Ambiguous case

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 123}{23} = \frac{\sin B}{12}$$

$\sin B = 0.4376$

$\angle B = 25.9°$ or $154.1°$

Nope. Only 180° in a triangle.

One solution.

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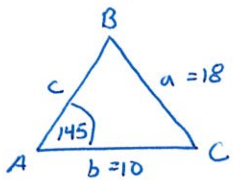
Pg 108 #6cd: For each triangle, determine whether there is no solution, one solution, or two solutions.

In ΔABC, ∠A = 145⁰, a = 18cm, and b = 10cm.



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 145}{18} = \frac{\sin B}{10}$$

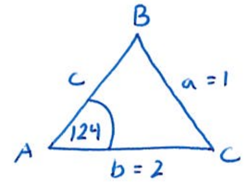
$\sin B = 0.3187$

$\angle B = 18.6^\circ$ or 161.4°

Nope. Only 180° in a triangle.

One solution.

In ΔABC, ∠A = 124⁰, a = 1cm, and b = 2cm.



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

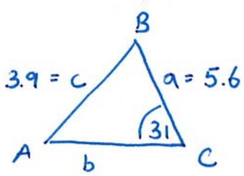
$$\frac{\sin 124}{1} = \frac{\sin B}{2}$$

$\sin B = 1.658$

No solutions.

Pg 108 #8ab: Determine the unknown side and angles in each triangle. If two solutions are possible, give both.

In ΔABC, ∠C = 31⁰, a = 5.6cm, and c = 3.9cm.



Angle - Side - Side
Ambiguous case

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 31}{3.9} = \frac{\sin A}{5.6}$$

$\sin A = 0.7395$

$\angle A = 47.7^\circ$ or 132.3°

$\angle B = 101.3^\circ$

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin 31}{3.9} = \frac{\sin 101.3}{b}$$

$b = 7.4$cm

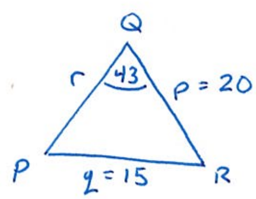
$\angle B = 16.7^\circ$

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin 31}{3.9} = \frac{\sin 16.7}{b}$$

$b = 2.2$ cm

In ΔPQR, ∠Q = 43⁰, p = 20cm, and q = 15cm.



$$\frac{\sin Q}{q} = \frac{\sin P}{p}$$

$$\frac{\sin 43}{15} = \frac{\sin P}{20}$$

$\sin P = 0.9093$

$\angle P = 65.4^\circ$ or 114.6°

$\angle R = 71.6^\circ$

$$\frac{\sin Q}{q} = \frac{\sin R}{r}$$

$$\frac{\sin 43}{15} = \frac{\sin 71.6}{r}$$

$r = 20.9$cm

$\angle R = 22.4^\circ$

$$\frac{\sin Q}{q} = \frac{\sin R}{r}$$

$$\frac{\sin 43}{15} = \frac{\sin 22.4}{r}$$

$r = 8.4$cm

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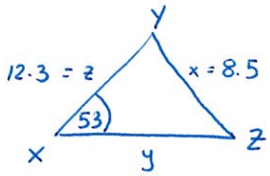
Pg 108 #8c: Determine the unknown side and angles in each triangle. If two solutions are possible, give both.

In ΔXYZ, $\angle X = 53^0$, x = 8.5cm, and z = 12.3cm.



Angle - Side - Side
Ambiguous case

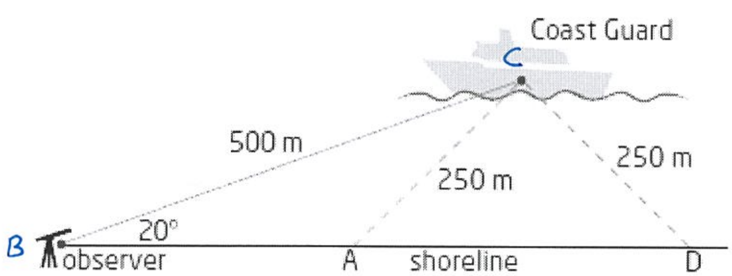
$$\frac{\sin X}{x} = \frac{\sin Z}{z}$$

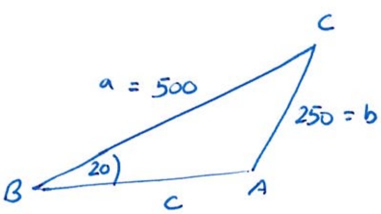
$$\frac{\sin 53}{8.5} = \frac{\sin Z}{12.3}$$

$\sin Z = 1.16$

No Solutions.

Pg 108 #11: The Canadian Coast Guard Pacific Region is responsible for more than 27,000 km of coastline. The rotating spotlight from the Coast Guard ship can illuminate up to a distance of 250 m. An observer on the shore is 500 m from the ship. His line of sight to the ship makes an angle of 20^0 with the shoreline. What length of shoreline is illuminated by the spotlight?



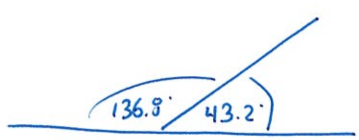


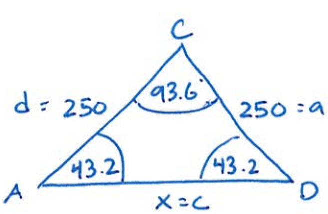
$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 20}{250} = \frac{\sin A}{500}$$

$\sin A = 0.684$

$\angle A = 43.2°$ or $136.8°$





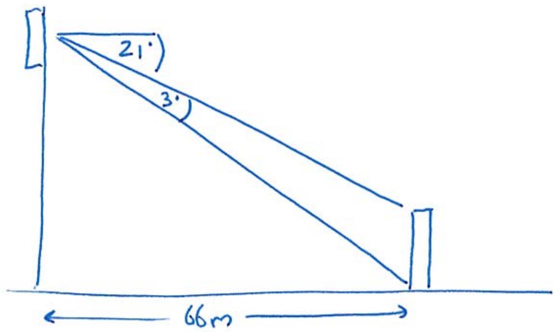
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 43.2}{250} = \frac{\sin 93.6}{x}$$

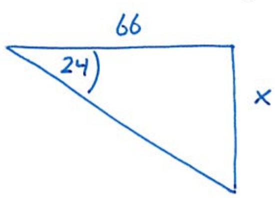
x = 364.5m

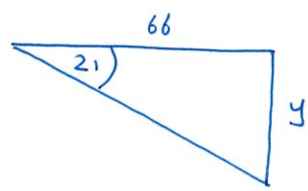
Pg 108 #14: From the window of his hotel in Saskatoon *(well above the statue)*, Max can see statues of Chief Whitecap of the Whitecap First nation and John Lake, leader of the Temperance Colonists, who founded Saskatoon. The angle formed by Max's lines of sight to the top and to the foot of the statue of Chief Whitecap is 3^0. The angle of depression of Max's line of sight to the top of the statue is 21^0. The horizontal distance between Max and the front of the statue is 66m.

a. Sketch a diagram to represent this problem.



b. Determine the height of the statue of Chief Whitecap.





$\tan\theta = \frac{o}{a}$

$\tan 24 = \frac{x}{66}$

$x = 29.4\,m$

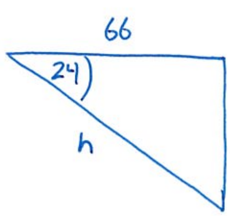
$\tan\theta = \frac{o}{a}$

$\tan 21 = \frac{y}{66}$

$y = 25.3\,m$

Height $= x - y$
$= 4.1\,m$

c. Determine the line-of-sight distance from where Max is standing at the window to the foot of the statue.



$\cos\theta = \frac{a}{h}$

$\cos 24 = \frac{66}{h}$

$h = \frac{66}{\cos 24}$

$h = 72.2\,m$

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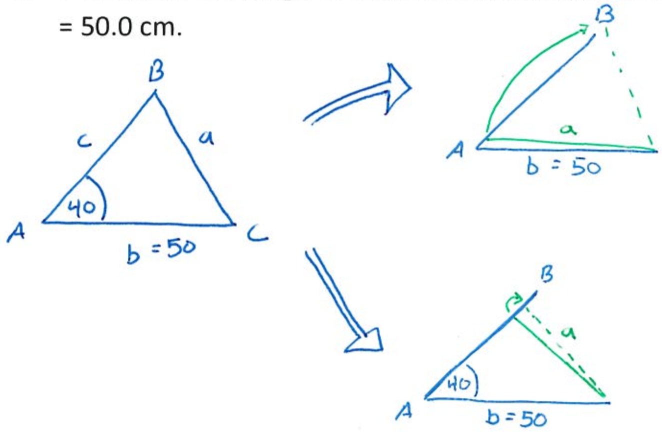
Pg 108 #22: For each of the following, include a diagram in your solution.

a. Determine the range of values side a can have so that ΔABC has two solutions if $\angle A = 40^0$ and $b = 50.0$ cm.



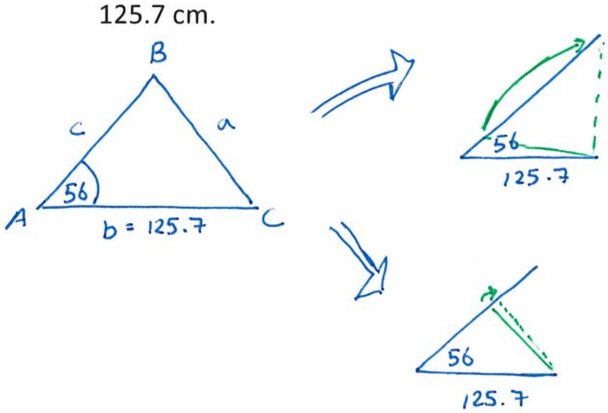
$a < 50$

$\sin 40 = \frac{a}{50}$

$a > 32.1$

$\boxed{\text{So } 32.1 < a < 50 \text{ cm for 2 solutions.}}$

b. Determine the range of values side a can have so that ΔABC has no solutions if $\angle A = 56^0$ and $b = 125.7$ cm.



$a < 125.7$

If $104.2 < a < 125.7$ then 2 solutions.

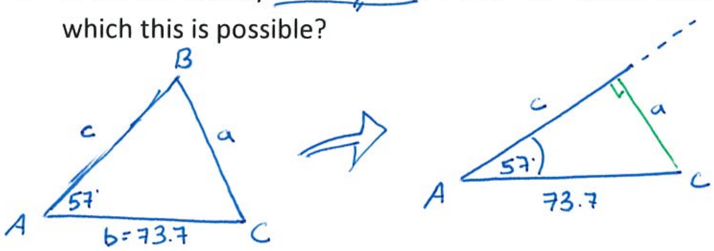
$\sin 56 = \frac{a}{125.7}$

$a = 104.2$

If $a = 104.2$ then 1 solution.

$\boxed{\text{If } a < 104.2 \text{ then no solutions.}}$

c. ΔABC has exactly one solution. If $\angle A = 57^0$ and $b = 73.7$ cm, what are the values of side a for which this is possible?



$\sin 57 = \frac{a}{73.7}$

$\boxed{a = 61.8 \text{ cm}}$