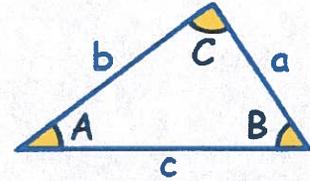
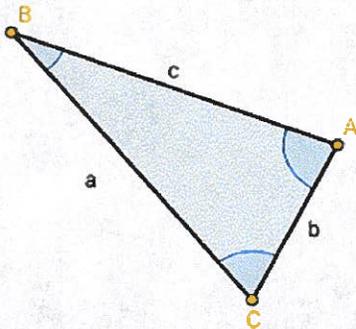


1.51 - 2.4 Cosine Law**Key Ideas****Sine Law:**

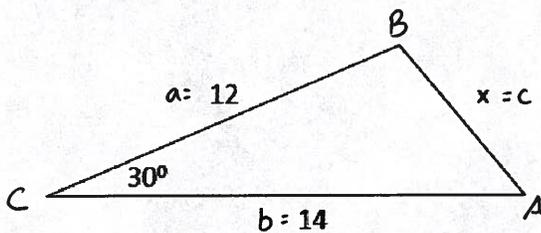
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

**Cosine Law:****Law of Cosines**

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ b^2 &= a^2 + c^2 - 2ac \cos B \\ c^2 &= a^2 + b^2 - 2ab \cos C \end{aligned}$$

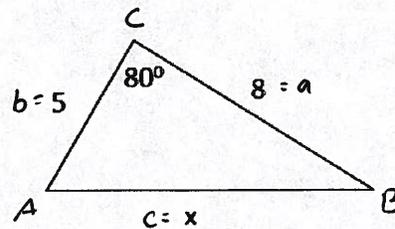
Part 1 - Solving for a Side

Q1: Solve for x.



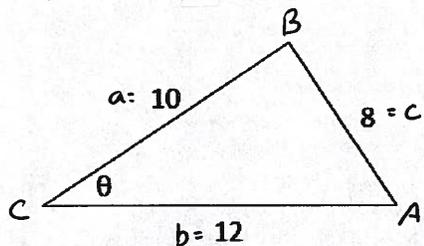
$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab \cos C \\ c^2 &= 12^2 + 14^2 - 2(12)(14) \cos 30^\circ \\ c^2 &= 144 + 196 - 290.98 \\ c^2 &= 49.0 \\ \boxed{c} &= \boxed{7.0} \end{aligned}$$

Q2: Solve for x.



$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab \cos C \\ c^2 &= 8^2 + 5^2 - 2(8)(5) \cos 80^\circ \\ c^2 &= 64 + 25 - 13.89 \\ c^2 &= 75.1 \\ \boxed{c} &= \boxed{8.7} \end{aligned}$$

Part 2 – Solving for an Angle

Q3: Solve for θ .

$$c^2 = a^2 + b^2 - 2ab \cos C$$

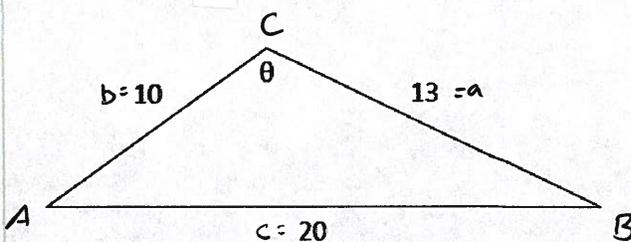
$$8^2 = 10^2 + 12^2 - 2(10)(12) \cos C$$

$$-180 = -240 \cos C$$

$$0.75 = \cos C$$

$$\angle C = 41.4^\circ$$

$$\boxed{\theta = 41.4^\circ}$$

Q4: Solve for θ .

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$20^2 = 13^2 + 10^2 - 2(13)(10) \cos \theta$$

$$400 = 169 + 100 - 260 \cos \theta$$

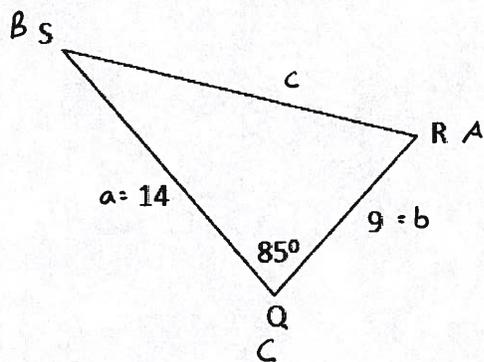
$$131 = -260 \cos \theta$$

$$-0.5038 = \cos \theta$$

$$\boxed{\theta = 120.3^\circ}$$

Part 3 – Solving a Triangle (Cosine Law, then Sine Law)

Q5: Solve the triangle.



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 14^2 + 9^2 - 2(14)(9) \cos 85$$

$$c^2 = 196 + 81 - 21.96$$

$$c^2 = 255.04$$

$$\boxed{c = 16.0} \Rightarrow \boxed{RS = 16.0}$$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 85}{16.0} = \frac{\sin A}{14}$$

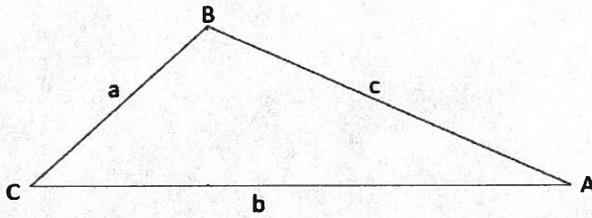
$$\boxed{\angle A = 60.7^\circ} \Rightarrow \boxed{\angle R = 60.7^\circ}$$

$$\angle A + \angle B + \angle C = 180^\circ$$

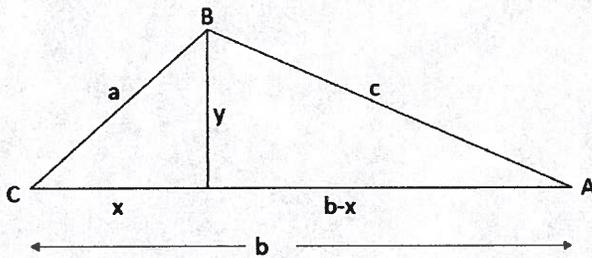
$$\boxed{\angle B = 34.3^\circ} \Rightarrow \boxed{\angle S = 34.3^\circ}$$

Part 4 – Derivation of Cosine Law

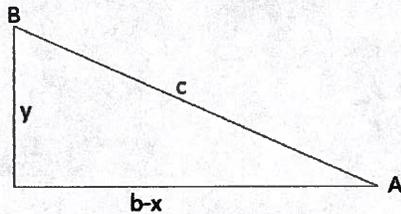
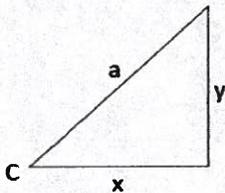
Q6: Derive the Cosine Law.



Step #1: Split the triangle into two.



Step #2: Perform *Pythagoras Theorem* on each triangle, solving for y^2 . Set up the Cosine Ratio for the left triangle.



$$x^2 + y^2 = a^2$$

$$y^2 = a^2 - x^2$$

$$(b-x)^2 + y^2 = c^2$$

$$y^2 = c^2 - (b-x)^2$$

$$\cos C = \frac{x}{a}$$

$$a \cos C = x$$

$$y^2 = y^2$$

$$a^2 - x^2 = c^2 - (b-x)^2$$

$$a^2 - x^2 = c^2 - (b^2 - 2bx + x^2)$$

$$a^2 - x^2 = c^2 - b^2 + 2bx - x^2$$

$$a^2 + b^2 = c^2 + 2bx$$

$$a^2 + b^2 - 2bx = c^2$$

or

$$c^2 = a^2 + b^2 - 2bx$$

$$c^2 = a^2 + b^2 - 2bx$$

$$c^2 = a^2 + b^2 - 2b(a \cos C)$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$