Math 20-1 KEY

63 – Worksheet – Radicals Unit Review

Part 1: Entire to Mixed Radicals

Q1: For each entire radical, write it as a mixed radical.

$\sqrt{75x^3y^2}$

75 → 3, 25 → 5, 5

$\sqrt{3 \cdot 5^2 \cdot x^2 \cdot x \cdot y^2}$

$5xy\sqrt{3x}$

$\sqrt{8x^4y^3z^2}$

8 → 2, 4 → 2, 2

$\sqrt{2 \cdot 2^2 \cdot x^2 \cdot x^2 \cdot y^2 \cdot y \cdot z^2}$

$2 \cdot x \cdot x \cdot y \cdot z\sqrt{2y}$

$2x^2yz\sqrt{2y}$

Part 2: Non-Permissible Values and Restrictions

Q2: For each of the following radicals, state the ***restrictions*** on the variable x.

$\sqrt{2x+1}$

$2x+1 \geq 0$

$2x \geq -1$

$x \geq -\frac{1}{2}$

$\sqrt{3x-1}$

$3x-1 \geq 0$

$3x \geq 1$

$x \geq \frac{1}{3}$

Q3: For each of the following radicals, state the ***restrictions*** on the variable x

$\sqrt{2x+7}$

$2x+7 \geq 0$

$2x \geq -7$

$x \geq -\frac{7}{2}$

$\sqrt{5-3x}$

$5-3x \geq 0$

$-3x \geq -5$

$x \leq \frac{5}{3}$

KEY

Q4: Given the expression $\sqrt{x^2-4}$, which of the following correctly describes the ***restrictions*** on x?

a. $x > 2$
b. $x \geq 2$
c. $x > 2$ or $x < -2$
d. $x \geq 2$ or $x \leq -2$

$x^2 - 4 \geq 0$
$x^2 \geq 4$
so $x \geq 2$ or $x \leq -2$

Part 3: Adding and Subtracting Radicals

Q5: Simplify radicals and combine like terms.

$2\sqrt{5} + 7\sqrt{5}$

$9\sqrt{5}$

$\sqrt{28} + \sqrt{63}$

28
2 14
2 7

63
3 21
3 7

$\sqrt{2^2 \cdot 7} + \sqrt{3^2 \cdot 7}$

$2\sqrt{7} + 3\sqrt{7}$

$5\sqrt{7}$

Q6: Simplify radicals and combine like terms.

$\sqrt{75x} - \sqrt{12x}$

75
3 25
5 5

12
3 4
2 2

$\sqrt{3 \cdot 5^2 \cdot x} - \sqrt{2^2 \cdot 3 \cdot x}$

$5\sqrt{3x} - 2\sqrt{3x}$

$3\sqrt{3x}$

$\sqrt{50} - \sqrt{27} + \sqrt{18} - \sqrt{3}$

50
2 25
5 5

27
3 9
3 3

18
2 9
3 3

$\sqrt{2 \cdot 5^2} - \sqrt{3 \cdot 3^2} + \sqrt{2 \cdot 3^2} - \sqrt{3}$

$5\sqrt{2} - 3\sqrt{3} + 3\sqrt{2} - 1\sqrt{3}$

$8\sqrt{2} - 4\sqrt{3}$

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Q7: The expression $\sqrt{60}+\sqrt{36}+\sqrt{135}$ simplifies to $a\sqrt{bc}+d$, where ***a, b, c,*** and ***d*** are ___, ___, ___, and ___.

(Record your ***four digit*** *answer in the Numerical Response boxes below)*

5	1	5	6

60 → 6, 10 → 2, 3, 2, 5

36 → 6, 6 → 2, 3, 2, 3

135 → 5, 27 → 3, 9 → 3, 3

$\sqrt{2^2\cdot3\cdot5}+\sqrt{2^2\cdot3^2}+\sqrt{3^2\cdot3\cdot5}$

$2\sqrt{15}+6+3\sqrt{15}$

$5\sqrt{15}+6$

$a\sqrt{bc}+d$

$a=5$
$b=1$
$c=5$
$d=6$

Part 4: Rationalizing the Denominator

Q8: For each expression, rationalize the denominator.

$\frac{\sqrt{5}}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}=\frac{\sqrt{35}}{7}$

$\frac{\sqrt{5}-1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{25}-1\sqrt{5}}{5}=\frac{5-\sqrt{5}}{5}$

Q9: For each expression, rationalize the denominator.

$\left(\frac{\sqrt{7}-3}{\sqrt{7}+2}\right)\cdot\left(\frac{\sqrt{7}-2}{\sqrt{7}-2}\right)=\frac{\sqrt{49}-2\sqrt{7}-3\sqrt{7}+6}{\sqrt{49}-2\sqrt{7}+2\sqrt{7}-4}$

$=\frac{7-2\sqrt{7}-3\sqrt{7}+6}{7-2\sqrt{7}+2\sqrt{7}-4}$

$=\frac{13-5\sqrt{7}}{3}$

$\left(\frac{\sqrt{3}-1}{1+\sqrt{6}}\right)\left(\frac{1-\sqrt{6}}{1-\sqrt{6}}\right)=\frac{1\sqrt{3}-\sqrt{18}-1+1\sqrt{6}}{1-1\sqrt{6}+1\sqrt{6}-\sqrt{36}}$

$=\frac{1\sqrt{3}-\sqrt{2\cdot3^2}-1+1\sqrt{6}}{1-1\sqrt{6}+1\sqrt{6}-6}$

$=\frac{\sqrt{3}-3\sqrt{2}-1+\sqrt{6}}{-5}$

$=\frac{-\sqrt{3}+3\sqrt{2}+1-\sqrt{6}}{5}$

KEY

Q10: When rationalizing the denominator of the expression $\frac{\sqrt{5}+3}{\sqrt{5}-1}$, the expression can be simplified to $a+\sqrt{b}$, where a and b are ___ and ___.

*(Record your **two digit** answer in the Numerical Response boxes below)*

2	5		

$$\left(\frac{\sqrt{5}+3}{\sqrt{5}-1}\right)\left(\frac{\sqrt{5}+1}{\sqrt{5}+1}\right) = \frac{\sqrt{25}+1\sqrt{5}+3\sqrt{5}+3}{\sqrt{25}+1\sqrt{5}-1\sqrt{5}-1} = \frac{5+1\sqrt{5}+3\sqrt{5}+3}{5-1}$$

$$= \frac{8+4\sqrt{5}}{4} = 2+\sqrt{5}$$

$a+\sqrt{b}$ $\quad$ $a=2$, $b=5$

Part 5: Multiplying Radicals

Q11: Multiply. Simplify the products where possible.

$(3\sqrt{2})(4\sqrt{6}) = 12\sqrt{12}$

12 → 2, 6; 6 → 2, 3

$12\sqrt{2^2\cdot 3}$

$12(2)\sqrt{3}$

$24\sqrt{3}$

$(\sqrt{5}+1)(3-2\sqrt{2})$

$3\sqrt{5}-2\sqrt{10}+3-2\sqrt{2}$

Q12: Multiply. Simplify the products where possible.

$(2\sqrt{x}+5)(3\sqrt{x}-4)$

$6\sqrt{x^2}-8\sqrt{x}+15\sqrt{x}-20$

$6x-8\sqrt{x}+15\sqrt{x}-20$

$6x+7\sqrt{x}-20$

$(x\sqrt{x}-5)(4+2\sqrt{x})$

$4x\sqrt{x}+2x\sqrt{x^2}-20-10\sqrt{x}$

$4x\sqrt{x}+2x(x)-20-10\sqrt{x}$

$4x\sqrt{x}+2x^2-20-10\sqrt{x}$

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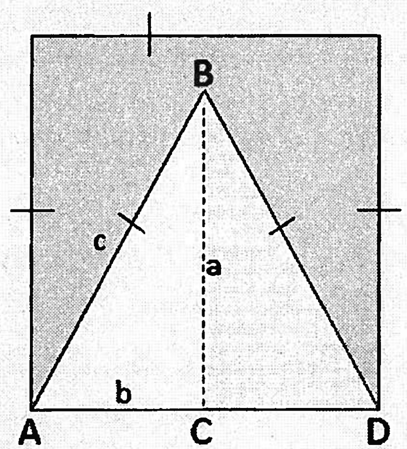
Use the following information to answer Q13-Q15:

An equilateral triangle, ΔABD, is placed inside a square.



$A_{square} = 40\ cm^2$

The area of the square is 40 cm^2.

Q13: The side length of the square, as a mixed radical, is $a\sqrt{bc}$, where ***a, b,*** and ***c*** are ___, ___, and ___.

(Record your four digit answer in the Numerical Response boxes below)

2	1	0	

Area = (L)(w)

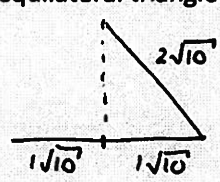
$40 = (x)(x)$

$40 = x^2$

$x = \sqrt{40} = \sqrt{2^2 \cdot 2 \cdot 5} = 2\sqrt{2 \cdot 5} = 2\sqrt{10}$

Q14: The height of the equilateral triangle is

a. $5\sqrt{2}$
b. $3\sqrt{10}$
c. $5\sqrt{10}$
d. $\sqrt{30}$



$a^2 + b^2 = c^2$

$(1\sqrt{10})^2 + b^2 = (2\sqrt{10})$

$10 + b^2 = 4(10)$

$b^2 = 30$

$b = \sqrt{30}$

Q15: The area of the equilateral triangle, ΔABD, is $ab\sqrt{c}$ cm^2, where ***a, b,*** and ***c*** are ___, ___, and ___.

(Record your three digit answer in the Numerical Response boxes below)

1	0	3	

$A = \frac{1}{2}bh$

$= \frac{1}{2}(2\sqrt{10})(\sqrt{30})$

$= 1\sqrt{300}$

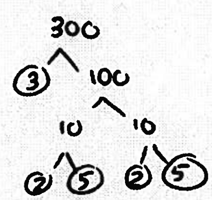
$= \sqrt{2^2 \cdot 3 \cdot 5^2}$

$= 2 \cdot 5 \cdot \sqrt{3}$

$= 10\sqrt{3}$

$= ab\sqrt{c}$

a = 1
b = 0
c = 3



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Part 6: Dividing Radicals

Q16: Simplify each expression.

$$\frac{4\sqrt{10}}{6\sqrt{5}} = \frac{4}{6}\sqrt{\frac{10}{5}} = \frac{2}{3}\sqrt{\frac{2}{1}}$$

$$= \frac{2\sqrt{2}}{3}$$

$$\frac{\sqrt{60}}{\sqrt{20}} = \sqrt{\frac{60}{20}} = \sqrt{3}$$

Q17: Simplify each expression using ***two different*** methods.

$$\frac{\sqrt{90}}{\sqrt{20}} = \sqrt{\frac{90}{20}} = \sqrt{\frac{9}{2}} = \frac{\sqrt{9}}{\sqrt{2}}$$

$$= \frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

$$\frac{\sqrt{90}}{\sqrt{20}} = \frac{\sqrt{2\cdot 3^2\cdot 5}}{\sqrt{2^2\cdot 5}} = \frac{3\sqrt{10}}{2\sqrt{5}}$$

$$= \frac{3}{2}\sqrt{\frac{10}{5}} = \frac{3\sqrt{2}}{2}$$

90
9 10
3 3 2 5

Q18: The expression $\frac{\sqrt{150}}{\sqrt{12}}$ simplifies to $\frac{a\sqrt{b}}{c}$, where ***a, b,*** and ***c*** are ___, ___, and ___.

*(Record your **three digit** answer in the Numerical Response boxes below)*

5	2	2	

$$\frac{\sqrt{150}}{\sqrt{12}} = \frac{\sqrt{2\cdot 3\cdot 5^2}}{\sqrt{2^2\cdot 3}} = \frac{5\sqrt{6}}{2\sqrt{3}}$$

$$= \frac{5}{2}\frac{\sqrt{6}}{\sqrt{3}} = \frac{5}{2}\sqrt{\frac{2}{1}} = \frac{5\sqrt{2}}{2}$$

$$= \frac{a\sqrt{b}}{c}$$

a = 5
b = 2
c = 2

150
10 15
2 5 3 5

12
4 3
2 2

KEY

Part 7: Radical Equations (Factorable)

Q19: State the restrictions on *x* and solve:

$$\sqrt{11x+67} = x+7$$

Restrictions

$11x+67 \geq 0$

$11x \geq -67$

$x \geq \frac{-67}{11}$

$x \geq \approx -6.\overline{09}$

$11x+67 = (x+7)^2$

$11x+67 = (x+7)(x+7)$

$11x+67 = x^2+14x+49$

$0 = x^2+3x-18$

$0 = (x+6)(x-3)$

$x+6=0$, $x=-6$

$x-3=0$, $x=3$

$x = -6$ or 3

Both soln's verify.

$$\sqrt{5x+11} = x+3$$

Restrictions

$5x+11 \geq 0$

$5x \geq -11$

$x \geq -11/5$

$x \geq -2.2$

$5x+11 = (x+3)^2$

$5x+11 = (x+3)(x+3)$

$5x+11 = x^2+6x+9$

$0 = x^2+x-2$

$0 = (x+2)(x-1)$

$x+2=0$, $x=-2$

$x-1=0$, $x=+1$

$x = -2$, or $+1$

Both soln's verify.

Q20: State the restrictions on *x* and solve:

$$\sqrt{-2x+10} = x-1$$

Restrictions

$-2x+10 \geq 0$

$-2x \geq -10$

$x \leq 5$

$-2x+10 = (x-1)^2$

$-2x+10 = x^2-2x+1$

$0 = x^2+0x-9$

$0 = (x+3)(x-3)$

$x+3=0$, $x=-3$

$x-3=0$, $x=+3$

$x = -3$ or $+3$

Doesn't verify because positive roots only.

$\boxed{x=3}$

$$\sqrt{-3x+6} = x-2$$

Restrictions

$-3x+6 \geq 0$

$-3x \geq -6$

$x \leq 2$

$-3x+6 = (x-2)^2$

$-3x+6 = (x-2)(x-2)$

$-3x+6 = x^2-4x+4$

$0 = x^2-x-2$

$0 = (x-2)(x+1)$

$x-2=0$, $x=2$

$x+1=0$, $x=-1$

$x = -1$ or 2

Doesn't verify because only positive roots.

$\boxed{x=2}$

KEY

Q21: State the restrictions on *x* and solve:

$\sqrt{-8x-8} = 2x+1$

Restrictions
$-8x-8 \geq 0$
$-8x \geq 8$
$x \leq -1$

$-8x-8 = (2x+1)^2$
$-8x-8 = 4x^2+4x+1$
$0 = 4x^2+12x+9$
$0 = (2x+3)(2x+3)$
$2x+3=0$
$2x=-3$
$x=-3/2$

$+6 \quad +6$
$\square + \square = 12$
$\square \times \square = 36$
1,36
2,18
3,12
4,9
6,6
$4x^2+6x+6x+9$
$(4x^2+6x)+(6x+9)$
$2x(2x+3)+3(2x+3)$
$(2x+3)(2x+3)$

$x = -3/2$

Doesn't verify because positive roots only.

$\boxed{\text{No solutions}}$

$\sqrt{18x+3} = 3x+2$

$18x+3 = (3x+2)^2$
$18x+3 = 9x^2+12x+4$
$0 = 9x^2-6x+1$
$0 = (3x-1)(3x-1)$
$3x-1=0$
$3x=1$
$x=1/3$

Restrictions
$18x+3 \geq 0$
$18x \geq -3$
$x \geq -3/18$
$x \geq -1/6$

$-3 \quad -3$
$\square + \square = -6$
$\square \times \square = 9$
1,
3
$9x^2-3x-3x$
$3x(3x-1)-1($
$(3x-1)(3x-1$

$x = 1/3$

Soln verifies.

Q22: State the restrictions on *x* and solve:

$\sqrt{x+3} = 2x+3$

Restrictions
$x+3 \geq 0$
$x \geq -3$

$x+3 = (2x+3)^2$
$x+3 = (2x+3)(2x+3)$
$x+3 = 4x^2+12x+9$
$0 = 4x^2+11x+6$
$0 = (4x+3)(x+2)$
$4x+3=0$
$4x = -3$
$x = -3/4$

$x+2=0$
$x=-2$

$+3 \quad +8$
$\square + \square = 11$
$\square \times \square = 24$
1,24
2,12
3,8
4,6
$4x^2+3x+8x+6$
$(4x^2+3x)+(8x+6)$
$x(4x+3)+2(4x+3)$
$(4x+3)(x+2)$

$x = -2$ or $-3/4$

Doesn't verify because positive roots only

$\boxed{x=-3/4}$

$\sqrt{-15x-1} = 3x-1$

Restrictions
$-15x-1 \geq 0$
$-15x \geq 1$
$x \leq -1/15$

$-15x-1 = (3x-1)^2$
$-15x-1 = (3x-1)(3x-1)$
$-15x-1 = 9x^2-6x+1$
$0 = 9x^2+9x+2$
$0 = (3x+1)(3x+2)$
$3x+1=0$
$3x=-1$
$x=-1/3$

$3x+2=0$
$3x=-2$
$x=-2/3$

$+3 \quad +6$
$\square + \square = 9$
$\square \times \square = 18$
1,
2,
3,
$9x^2+3x+6x$
$(9x^2+3x)+(6$
$3x(3x+1)+2($
$(3x+1)(3x+2$

$x = -2/3$ or $-1/3$

Neither solution verifies because positive roots only.

$\boxed{\text{No solution.}}$

[redacted] KEY [redacted]

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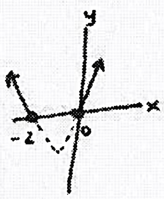
Q23: State the restrictions on *x* and solve:

Restrictions

$x^2+2x \geq 0$

$x(x+2) \geq 0$



$x \leq -2$

$x \geq 0$

You'll learn this in the quadratics unit.

$\sqrt{x^2+2x} = 2x+4$

$x^2+2x = (2x+4)^2$

$x^2+2x = (2x+4)(2x+4)$

$x^2+2x = 4x^2+16x+16$

$0 = 3x^2+14x+16$

$0 = (x+2)(3x+8)$

$x+2=0$ $3x+8=0$

$x=-2$ $3x=-8$

$x=-8/3$

$x = -2$ or $-8/3$

Doesn't verify because positive roots only.

$\boxed{x=-2}$

+6 +8

$\square+\square = 14$

$\square\times\square = 48$

1, 48
2, 24
3, 16
4, 12
6, 8

$3x^2+6x+8x+16$

$(3x^2+6x)+(8x+16)$

$3x(x+2)+8(x+2)$

$(x+2)(3x+8)$

$\sqrt{x^2} = 2x+2$

Restrictions

$x^2 \geq 0$

No restrictions

$x \in \mathbb{R}$

$x^2 = (2x+2)^2$

$x^2 = (2x+2)(2x+2)$

$x^2 = 4x^2+8x+4$

$0 = 3x^2+8x+4$

$0 = (3x+2)(x+2)$

$3x+2=0$ $x+2=0$

$3x=-2$ $x=-2$

$x=-2/3$

$x = -2$ or $-2/3$

Doesn't verify because positive roots only.

$\boxed{x=-2/3}$

+2 +6

$\square+\square = 8$

$\square\times\square = 12$

1, 1
2,
3,

$3x^2+2x+6x+$

$(3x^2+2x)+(6x+4$

$x(3x+2)+2(3x+$

$(3x+2)(x+2)$

Q24: The expression $\sqrt{-12x-2} = 2x-1$ simplifies to $(ax+b)(cx+d)=0$, where the values of ***a*, *b*, *c***, and ***d*** are ___, ___, ___, and ___.

(Record your four digit answer in the Numerical Response boxes below)

2	1	2	3

Restrictions

$-12x-2 \geq 0$

$-12x \geq 2$

$x \leq -2/12$

$x \leq -1/6$

Can also be

2	3	2	1

$\sqrt{-12x-2} = 2x-1$

$-12x-2 = (2x-1)^2$

$-12x-2 = (2x-1)(2x-1)$

$-12x-2 = 4x^2-4x+1$

$0 = 4x^2+8x+3$

$0 = (2x+1)(2x+3)$

$(ax+b)(cx+d)$

+2 +6

$\square+\square = 8$

$\square\times\square = 12$

1, 12
2, 6
3, 4

$4x^2+2x+6x+3$

$(4x^2+2x)+(6x+3)$

$2x(2x+1)+3(2x+1)$

$(2x+1)(2x+3)$

KEY

Part 8: Word Problems

Use the following information to answer Q25:

When analyzing car crashes, police use a formula that relates the stopping distance and the initial velocity of the car prior to braking:

$$v_i = \sqrt{2\mu_k g d}$$

Where v_i is the initial velocity μ_k is the coefficient of kinetic friction, g is the gravitational field strength, and d is the stopping distance.

Additionally, the kinematics equation $d = \left(\frac{v_i+v_f}{2}\right)t$, when modelling a braking object coming to a stop, can simplify to

$$v_i = \frac{2d}{t}$$

These two equations can be set equal to each other to solve for stopping distance:

$$\sqrt{2\mu_k g d} = \frac{2d}{t}$$

Q25: A car is travelling at a moderate velocity on wet concrete ($\mu_k = 0.60$) on Earth ($g = 9.81$) before it slams on its brakes, coming to a stop after 3.20 seconds ($t = 3.20$). Determine the distance it takes the car to stop.

$\mu_k = 0.60$
$g = 9.81$
$t = 3.20$

$$\sqrt{2\mu_k g d} = \frac{2d}{t}$$

$$\sqrt{2(0.60)(9.81)d} = \frac{2d}{3.20}$$

Square both sides.

$$2(0.60)(9.81)d = \frac{4d^2}{(3.20)^2}$$

$$2(0.60)(9.81)(d)(3.20)^2 = 4d^2$$

$$120.54528\,d = 4d^2$$

$$0 = 4d^2 - 120.54528d$$

$$0 = d(4d - 120.54528)$$

$d = 0$

This answer doesn't make sense.

$4d - 120.54528 = 0$
$4d = 120.54528$
$d = 30.13632\text{ m}$

It takes the car ≈ 30.1 m to stop.

KEY

Part 9: Radical Equations (Quadratic Equation)

Q26: State the restrictions on x and solve:

$$\sqrt{3x+\frac{23}{2}} = x+\frac{1}{2}$$

Restrictions

$3x+\frac{23}{2} \geq 0$

$3x \geq -\frac{23}{2}$

$x \geq -\frac{23}{6}$

$x \geq -3.8\bar{3}$

$3x+\frac{23}{2} = (x+\frac{1}{2})^2$

$3x+\frac{23}{2} = (x+\frac{1}{2})(x+\frac{1}{2})$

$3x+\frac{23}{2} = x^2+x+\frac{1}{4}$

$0 = x^2-2x-\frac{45}{4}$

$0 = x^2-2x-11.25$

$0 = ax^2+bx+c$

$a=1$
$b=-2$
$c=-11.25$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(-11.25)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4+45}}{2} = \frac{2 \pm \sqrt{49}}{2}$

$x_1 = \frac{2+\sqrt{49}}{2} = \frac{2+7}{2} = \frac{9}{2}$

$x_2 = \frac{2-\sqrt{49}}{2} = \frac{2-7}{2} = \frac{-5}{2}$

Doesn't verify because positive roots only.

$x = \frac{9}{2}$

$\frac{1}{4} - \frac{23}{2}$

$\frac{1}{4} - \frac{46}{4}$

$-\frac{45}{4}$

Q26: State the restrictions on x and solve:

$$\sqrt{x+\frac{7}{9}} = x+1$$

Restrictions

$x+\frac{7}{9} \geq 0$

$x \geq -\frac{7}{9}$

$x+\frac{7}{9} = (x+1)^2$

$x+\frac{7}{9} = (x+1)(x+1)$

$x+\frac{7}{9} = x^2+2x+1$

$0 = x^2+x+\frac{2}{9}$

$0 = ax^2+bx+c$

$a=1$
$b=1$
$c=\frac{2}{9} = 0.\bar{2}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{1^2-4(1)(0.\bar{2})}}{2(1)}$

$x = \frac{-1 \pm \sqrt{1-0.\bar{8}}}{2} = \frac{-1 \pm \sqrt{0.\bar{1}}}{2}$

$x_1 = \frac{-1+\sqrt{0.\bar{1}}}{2} = \frac{-1+0.\bar{3}}{2} = -0.\bar{3}$ or $-\frac{1}{3}$

$x_2 = \frac{-1-\sqrt{0.\bar{1}}}{2} = \frac{-1-0.\bar{3}}{2} = -0.\bar{6}$ or $-\frac{2}{3}$

$1 - \frac{7}{9}$

$\frac{9}{9} - \frac{7}{9}$

$\frac{2}{9}$

$x = -\frac{2}{3}$ or $\frac{-1}{3}$

Both solutions verify