Math 20-1 KEY

L65 – 1.1 & 1.2 Arithmetic Sequences and Series

Part 1 – Key Ideas

Sequence: An ordered list of elements.

Arithmetic Sequence: A sequence in which the difference between the consecutive terms is constant.

Common Difference: The difference between successive terms in an arithmetic sequence. The difference may be positive or negative.

General term: An expression for directly determining any term of a sequence.

Arithmetic Series: A sum of terms that form an arithmetic sequence.

Part 2 – Arithmetic Sequences

Arithmetic Sequences $t_n = t_1 + (n-1)d$

t_1 is the first term

n is the number of terms ($n \in N$)

d is the common difference

t_n is the general term or n^{th} term

Use the following information to answer Q1-Q2:

An Arithmetic Sequence is shown below:

$t_1 = 15$
$d = 3$

$\overset{+3}{\frown}$
15, 18, 21, 24...

Q1: Determine the 12th term in the sequence.

$$t_n = t_1 + (n-1)d$$
$$t_{12} = 15 + (12-1)(3)$$
$$= 15 + (11)(3)$$
$$= 48$$

Q2: Determine the 30th term in the sequence.

$$t_n = t_1 + (n-1)d$$
$$t_{30} = 15 + (30-1)(3)$$
$$= 15 + (29)(3)$$
$$= 102$$

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Use the following information to answer Q3-Q6:

An Arithmetic Sequence is shown below:

-4

100, 96, 92, 88...

$t_1 = 100$
$d = -4$

Q3: Determine the 18th term in the sequence.

$t_n = t_1 + (n-1)d$
$= 100 + (18-1)(-4)$
$= 100 + (17)(-4)$
$= 32$

Q4: Determine the 50th term in the sequence.

$t_n = t_1 + (n-1)d$
$= 100 + (50-1)(-4)$
$= 100 + (49)(-4)$
$= -96$

Q5: What term is the number 0?

$t_n = t_1 + (n-1)d$
$0 = 100 + (n-1)(-4)$
$0 = 100 - 4n + 4$
$+4n \quad +4n$
$4n = 104$
$n = 26$

Q6: What term is the number -248?

$t_n = t_1 + (n-1)d$
$-248 = 100 + (n-1)(-4)$
$-248 = 100 - 4n + 4$
$+4n \quad +4n$
$4n - 248 = 104$
$+248 \quad +248$
$4n = 352$
$\div 4 \quad \div 4$
$n = 88$

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Use the following information to answer Q7-Q10:

An Arithmetic Sequence is shown below: $\overset{+2}{\curvearrowright}$ 25, 27, 29, 31... $t_1 = 25$, $d = 2$

Q7: Determine the 9th term in the sequence.

$t_n = t_1 + (n-1)d$

$t_9 = 25 + (9-1)(2)$

$t_9 = 25 + (8)(2)$

$t_9 = 41$

Q8: Determine the 40th term in the sequence.

$t_n = t_1 + (n-1)d$

$t_{40} = 25 + (40-1)(2)$

$= 25 + (39)(2)$

$= 103$

Q9: What term is the number 211?

$t_n = t_1 + (n-1)d$

$211 = 25 + (n-1)(2)$

$211 = 25 + 2n - 2$

$211 = 23 + 2n$

$-23 \quad -23$

$188 = 2n$

$n = 94$

Q10: What term is the number 57?

$t_n = t_1 + (n-1)d$

$57 = 25 + (n-1)(2)$

$57 = 25 + 2n - 2$

$-25 \quad -25$

$32 = 2n - 2$

$+2 \quad +2$

$34 = 2n$

$n = 17$

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Use the following information to answer Q11-13:

The 19th term of an Arithmetic Sequence is 58. The 22nd term is 76.

Q11: What is the Common Difference?

(19, 58) and (22, 76) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{76-58}{22-19} = 6$

Common Difference = 6

Q12: What is the first term in the sequence?

$t_n = t_1 + (n-1)d$ Use 19th term is 58

$58 = t_1 + (19-1)(6)$

$58 = t_1 + (18)(6)$

$58 = t_1 + 108$

$-108 \quad -108$

$-50 = t_1$

Q13: What is the 10th term in the sequence?

$t_n = t_1 + (n-1)d$

$t_{10} = -50 + (10-1)(6)$

$= -50 + (9)(6)$

$= 4$

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Use the following information to answer Q14-16:

The 10th term of an Arithmetic Sequence is 18. The 14th term is 2.

Q14: What is the Common Difference?

(10, 18) and (14, 2) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-18}{14-10} = -4$

Common Difference is −4.

Q15: What is the first term in the sequence?

$t_n = t_1 + (n-1)d$ Use 10th term is 18.

$18 = t_1 + (10-1)(-4)$

$18 = t_1 + (9)(-4)$

$18 = t_1 - 36$

$+36 \qquad +36$

$t_1 = 54$

Q16: What is the 20th term in the sequence?

$t_n = t_1 + (n-1)d$

$t_{20} = 54 + (20-1)(-4)$

$= 54 + (19)(-4)$

$= -22$

KEY

Part 3 – Arithmetic Series

Arithmetic Series

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$

$S_n = \frac{n}{2}(t_1 + t_n)$

t_1 is the first term

n is number of terms

d is the common difference

t_n is the n^{th} term

S_n is the sum to n terms

Use the following information to answer Q17-19:

An Arithmetic Sequence is shown below:

$-2 \xrightarrow{+5} 3 + 8 + 13 + \ldots$

$t_1 = -2$
$d = 5$

Q17: What is the Common Difference?

$d = 5$

Q18: What is the sum of the first 10 terms?

$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$
$$S_{10} = \frac{10}{2}[2(-2) + (10-1)(5)]$$
$$= \frac{10}{2}[-4 + 45]$$
$$= 205$$

Q19: What is the sum of the first 20 terms?

$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$
$$S_{20} = \frac{20}{2}[2(-2) + (20-1)(5)]$$
$$= 10[-4 + 95]$$
$$= 910$$

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Arithmetic Sequences $t_n = t_1 + (n-1)d$

t_1 is the first term

n is the number of terms ($n \in N$)

d is the common difference

t_n is the general term or n^{th} term

Arithmetic Series $S_n = \frac{n}{2}[2t_1 + (n-1)d]$

$S_n = \frac{n}{2}(t_1 + t_n)$

t_1 is the first term

n is number of terms

d is the common difference

t_n is the n^{th} term

S_n is the sum to n terms

Use the following information to answer Q20-21:

An Arithmetic Sequence is shown below:

$t_1 = 51$

$d = -3$

$\overset{-3}{51 + 48} + 45 + 42 + \ldots$

Q20: What is the 10th term?

$t_n = t_1 + (n-1)d$

$t_{10} = 51 + (10-1)(-3)$

$= 51 + (9)(-3)$

$= 24$

Q21: What is the sum of the first 10 terms?

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$

$S_{10} = \frac{10}{2}[2(51) + (10-1)(-3)]$

$= 5[102 - 27]$

$= 375$

KEY

Part 4 – Textbook Examples

Pg10: A visual and performing arts group wants to hire a community events leader. The person will be paid $12.00 for the first hour for work, $19 for two hours of work, $26 for 3 hours of work, and so on.

a. Write the general term that you could use to determine the pay for any number of hours worked.

b. What will the person get paid for 6 hours of work?

$12 \xrightarrow{+7} 19, 26 \ldots$ $\quad t_1 = 12$, $d = 7$

A $t_n = t_1 + (n-1)d$

$t_n = 12 + (n-1)(7)$

B $t_6 = 12 + (6-1)(7)$

$= 12 + 35$

$= 47$

Gets paid $47.00

Pg12: The musk-ox and the caribou of northern Canada are hoofed mammals that survived the Pleistocene Era, which ended 10,000 years ago. In 1955 the Banks Island musk-ox population was approximately 9250 animals. Suppose that in subsequent years, the growth of the musk-ox population generated an arithmetic sequence, in which the number of musk-ox increased by approximately 1650 each year. How many years would it take for the musk-ox population to reach 100,000?

Year	1	2	
Pop.	9250	10900	...

+1650

$t_1 = 9250$

$d = 1650$

$t_n = t_1 + (n-1)d$

$100{,}000 = 9250 + (n-1)(1650)$

$100{,}000 = 9250 + 1650n - 1650$

$100{,}000 = 1650n + 7600$

$-7600 \qquad -7600$

$92{,}400 = 1650n$

$n = 56$

So 56th term... or 56 years from 1955

KEY

Pg25: Male fireflies flash in various patterns to signal location or to ward off predators. Different species of fireflies have different flash characteristics, such as the intensity of the flash, the rate of the flash, and the shape of the flash. Suppose that under certain circumstances, a particular firefly flashes twice in the first minute, four times in the second minute, and six times in the 3rd minute.

a. If this pattern continues, what is the number of flashes in the 30th minute?
b. What is the total number of flashes in 30 minutes?

MIN	1	2	3
FLASH	2	4	6

$+2$

$t_1 = 2$
$n = 2$

(A) $t_n = t_1 + (n-1)d$
$t_{30} = 2 + (30-1)(2)$
$= 2 + (29)(2)$
$= 60$ flashes.

(B) $S_n = \frac{n}{2}[2t_1 + (n-1)d]$
$= \frac{30}{2}[2(2) + (30-1)(2)]$
$= 15[4 + 58]$
$= 930$ flashes

Pg26: The sum of the first two terms of an arithmetic series is 13 and the sum of the first four terms is 46. Determine the first six terms of the series and the sum to six terms.

METHOD #1

$(t_1) + (t_1 + d) = 13$
$2t_1 + d = 13$

$(t_1) + (t_1 + d) + (t_1 + d + d) + (t_1 + d + d + d) = 46$
$4t_1 + 6d = 46$

$2(2t_1 + d = 13)$
$4t_1 + 6d = 46$

$\rightarrow$

$4t_1 + 2d = 26$
$-(4t_1 + 6d = 46)$
$-4d = -20$
$\boxed{d = 5}$

$\rightarrow$

$2t_1 + (5) = 13$
$2t_1 = 8$
$\boxed{t_1 = 4}$

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$
$S_6 = \frac{6}{2}[2(4) + (6-1)(5)]$
$= 3[8 + 25]$
$\boxed{S_6 = 99}$

METHOD #2

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$
$S_2 = \frac{2}{2}[2t_1 + (2-1)d]$
$13 = 2t_1 + d$

$S_4 = \frac{4}{2}[2t_1 + (4-1)d]$
$S_4 = 2[2t_1 + 3d]$
$46 = 4t_1 + 6d$

Same eqns as previous.
Rest of solution is the same.