KEY

$t_n = t_1 + (n-1)d$ $\qquad$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad$ $S_n = \frac{n}{2}(t_1 + t_n)$

L66 – Arithmetic Sequences and Series

Part 1 – Arithmetic Sequences

Use the following information to answer Q1-Q2:

An Arithmetic Sequence is shown below: $\quad$ $\overset{+3}{\curvearrowright}$ 58, 61, 64, 67… $\quad$ $t_1 = 58$, $d = 3$

Q1: Determine the 20th term in the sequence.

$t_n = t_1 + (n-1)d$

$t_{20} = 58 + (20-1)(3)$

$= 115$

Q2: What term is the number 196?

$t_n = t_1 + (n-1)d$

$196 = 58 + (n-1)(3)$

$196 = 58 + 3n - 3$

$196 = 55 + 3n$

$141 = 3n$

$n = 47$

Use the following information to answer Q3-Q4:

An Arithmetic Sequence is shown below: $\quad$ $\overset{-4}{\curvearrowright}$ 17, 13, 9, 5… $\quad$ $t_1 = 17$, $n = -4$

Q3: Determine the 50th term in the sequence.

$t_n = t_1 + (n-1)d$

$= 17 + (50-1)(-4)$

$= -179$

Q4: What term is the number -67?

$t_n = t_1 + (n-1)d$

$-67 = 17 + (n-1)(-4)$

$-67 = 17 - 4n + 4$

$-67 = 21 - 4n$

$-88 = -4n$

$n = 22$

KEY

$t_n = t_1 + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\quad$ $S_n = \frac{n}{2}(t_1 + t_n)$

Use the following information to answer Q5-7:

The 17th term of an Arithmetic Sequence is 76. The 28th term is 131.

Q5: What is the Common Difference?

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{131 - 76}{28 - 17} = 5$$

Q6: What is the first term in the sequence?

$t_n = t_1 + (n-1)d$ $\quad$ Use 17th term is 76

$76 = t_1 + (17-1)(5)$

$76 = t_1 + 80$

$t_1 = -4$

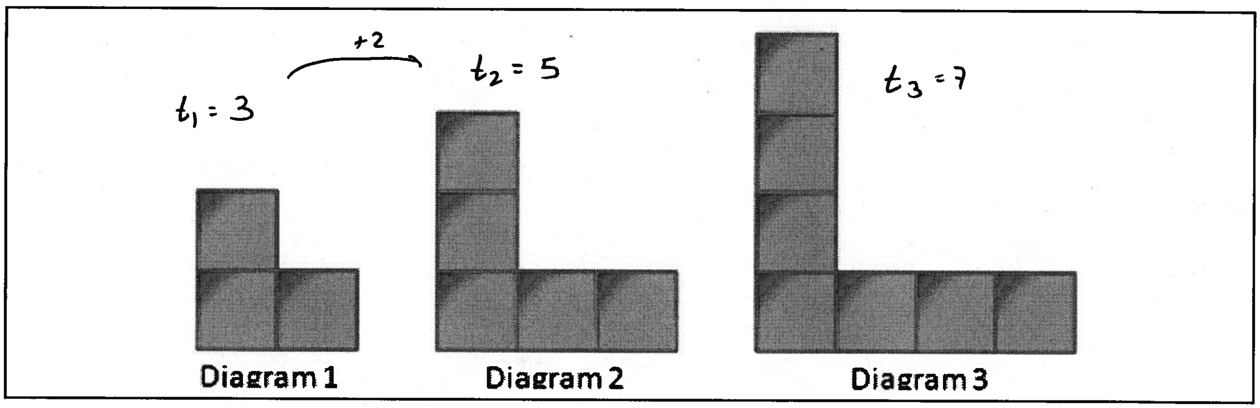
Q7: What is the 100th term in the sequence?

$t_n = t_1 + (n-1)d$

$t_{100} = -4 + (100-1)(5)$

$t_{100} = 491$

Use the following information to answer Q8:



Q8: Write an equation to describe the Arithmetic Sequence.

$t_n = t_1 + (n-1)d$

$t_n = 3 + (n-1)(2)$

KEY

$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $S_n = \frac{n}{2}(t_1 + t_n)$

Part 2 – Arithmetic Series

Use the following information to answer Q9-10:

An Arithmetic Sequence is shown below:

$$6 \overset{+4}{\frown} 10 + 14 + 18 + \ldots$$

Q9: What is the Common Difference?

+4

Q10: What is the sum of the first 20 terms?

$S_{20} = \frac{20}{2}[2(6) + (20-1)(4)]$

$= 10[12 + 76]$

$= 880$

Use the following information to answer Q11-12:

An Arithmetic Sequence is shown below:

$$6 + 2 - 2 - 6 + \ldots$$

-4 -4 -4

Q11: What is the Common Difference?

-4

Q12: What is the sum of the first 20 terms?

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$

$S_{20} = \frac{20}{2}[2(6) + (20-1)(-4)]$

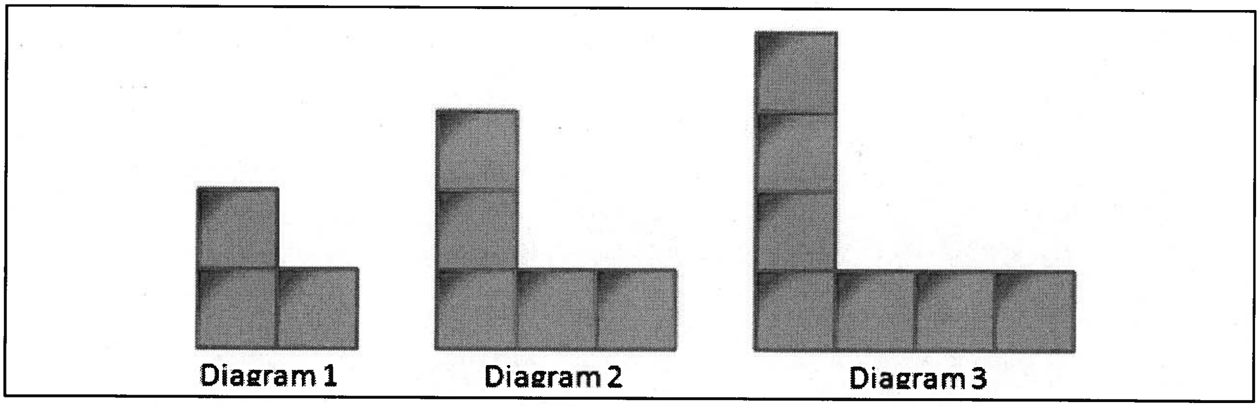
$= 10[12 - 76]$

$= -640$

KEY

$t_n = t_1 + (n-1)d$ $\qquad$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad$ $S_n = \frac{n}{2}(t_1 + t_n)$

Use the following information to answer Q13:



Q13: How many block are required to display Diagrams 1-10 all at the same time?

$t_1 = 3$
$d = 2$

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$
$S_{10} = \frac{10}{2}[2(3) + (10-1)(2)]$
$= 5[6 + 18]$
$= 120$

Q14: In an Arithmetic Series, the first term is 5 and the tenth term is 59. What is the sum of the first 10 terms? Determine the answer using two different methods.

$S_n = \frac{n}{2}(t_1 + t_n)$
$= \frac{10}{2}(5 + 59)$
$= 320$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{59-5}{10-1} = \frac{54}{9} = 6$

$t_1 = 5$
$d = 6$
$n = 10$

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$
$S_{10} = \frac{10}{2}[2(5) + (10-1)(6)]$
$= 5[10 + 54]$
$= 320$

KEY

$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $S_n = \frac{n}{2}(t_1 + t_n)$

Part 3 – Review – Chapter 6 Rational Expressions

Q15: Simplify the expression $\frac{x^2+x-20}{x^2+x-6} \div \frac{x^2-10x+24}{x^2+4x+3}$ and identify all non-permissible values.

$$= \frac{(x+5)(x-4)}{(x+3)(x-2)} \div \frac{(x-4)(x-6)}{(x+1)(x+3)}$$

$x \neq 4$ $x \neq 6$

$x \neq -3$ $x \neq 2$ $x \neq -1$ $x \neq -3$

$$= \frac{(x+5)\cancel{(x-4)}}{\cancel{(x+3)}(x-2)} \cdot \frac{(x+1)\cancel{(x+3)}}{\cancel{(x-4)}(x-6)}$$

$$= \frac{(x+5)(x+1)}{(x-2)(x-6)} \quad \text{where } x \neq -3, -1, 2, 4, 6$$

Q16: Solve the rational equation $1 = \frac{3}{m+3} + \frac{3m}{m+3}$ and identify all non-permissible values.

$$\frac{1}{1}\left(\frac{m+3}{m+3}\right) = \frac{3}{m+3} + \frac{3m}{m+3}$$

$$\frac{m+3}{m+3} = \frac{3}{m+3} + \frac{3m}{m+3}$$

$$m+3 = 3 + 3m$$

$-m$ $-m$

$$3 = 3 + 2m$$

-3 -3

$$0 = 2m$$

$\div 2$ $\div 2$

$$\boxed{0 = m}$$

KEY

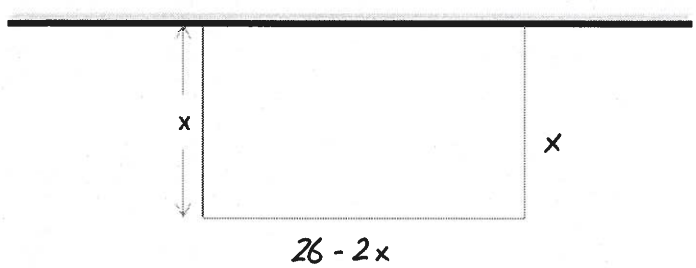
$t_n = t_1 + (n-1)d$ $\qquad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad S_n = \frac{n}{2}(t_1 + t_n)$

Part 4 – Review – Chapter 3 Quadratic Functions

Q17: Convert $f(x) = 3x^2 - 5x + 1000$ into Vertex Form using fractions.

$$\begin{aligned}
y &= (3x^2 - 5x) + 1000 \\
&= 3\left(x^2 - \tfrac{5}{3}x\right) + 1000 \\
&= 3\left(x^2 - \tfrac{5}{6}x - \tfrac{5}{6}x\right) + 1000 \\
&= 3\left(x^2 - \tfrac{5}{6}x - \tfrac{5}{6}x + \tfrac{25}{36}\right) + 1000 - \tfrac{25}{12} \\
&= 3\left(x - \tfrac{5}{6}\right)^2 + \tfrac{11975}{12}
\end{aligned}$$

Q18: A house owner is building a fence against their barn for chickens to run around. They have 26m of fence. What is the maximum area they can build for their chicken coup?



$$\begin{aligned}
A &= (L)(W) \\
&= (x)(26 - 2x) \\
&= -2x^2 + 26x + 0
\end{aligned}$$

$$\begin{aligned}
A(x) &= (-2x^2 + 26x) + 0 \\
&= -2(x^2 - 13x) + 0 \\
&= -2\left(x^2 - \tfrac{13}{2}x - \tfrac{13}{2}x\right) + 0 \\
&= -2\left(x^2 - \tfrac{13}{2}x - \tfrac{13}{2}x + \tfrac{169}{4}\right) + 0 + \tfrac{169}{2} \\
&= -2\left(x - \tfrac{13}{2}\right)^2 + \tfrac{169}{2}
\end{aligned}$$

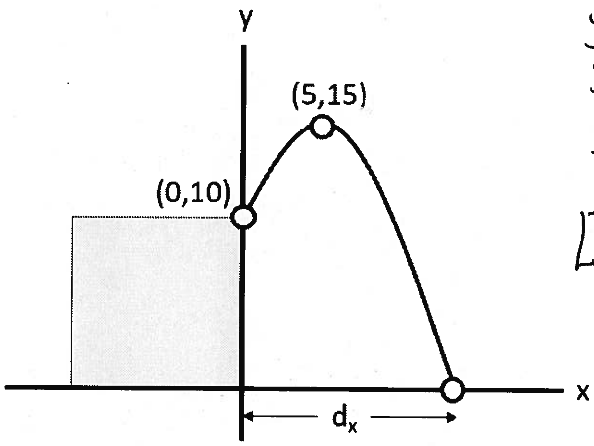
Vertex is at $\left(\frac{13}{2}, \frac{169}{2}\right)$.

So max area is $\frac{169}{2}$ m^2

KEY

$t_n = t_1 + (n-1)d$ $\qquad$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad$ $S_n = \frac{n}{2}(t_1 + t_n)$

Q19: A cat jumps off a 10ft tall ledge, reaching a maximum height of 15m after travelling a horizontal distance of 5m. What horizontal distance, d_x, does the cat cover before landing?



$y = a(x-h)^2 + k$

$y = a(x-5)^2 + 15$ $\qquad$ Use (0,10)

$10 = a(0-5)^2 + 15$

$-5 = a(25)$

$a = -\frac{1}{5}$

$\boxed{y = -\frac{1}{5}(x-5)^2 + 15}$

$0 = -\frac{1}{5}(x-5)^2 + 15$

$-15 = -\frac{1}{5}(x-5)^2$

$75 = (x-5)^2$

$\pm\sqrt{75} = x - 5$

$x = 5 \pm \sqrt{75}$

$x = 5 \pm 5\sqrt{3}$

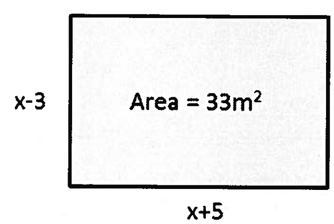
75
3 25
5 5

So $d_x = 5 + 5\sqrt{3}$ or 13.66 m

Part 5 – Review – Chapter 4 Quadratic Equations

Use the following information to answer Q20:

A patio has a total area of $33m^2$, and is proportioned per the diagram below.



Q20: Set up a quadratic equation and solve to determine the value(s) of ***x***.

$A = (l)(w)$

$33 = (x-3)(x+5)$

$33 = x^2 + 2x - 15$

$0 = x^2 + 2x - 48$

$0 = (x+8)(x-6)$

$x = -8$ $\qquad$ $x = 6$ $\qquad$ So $\boxed{x = 6m}$

Key

$t_n = t_1 + (n-1)d$ $\qquad$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad$ $S_n = \frac{n}{2}(t_1 + t_n)$

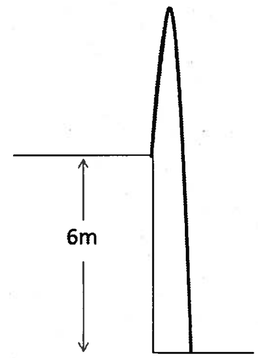
Use the following information to answer Q21:

An object is launched vertically at 13m/s off a 6m tall ledge. The *Physics 20* equation that can be used to determine time is given by:

$$\Delta d = v_i t + \frac{1}{2}at^2$$

Where the acceleration due to gravity is approximately -10 m/s^2. In Math 20-1, we can write this function as follows:

$$h(t) = -5t^2 + 13t + 6$$



Q21: How long does it take the object to land? Support your work algebraically.

Using Factoring

$0 = -5t^2 + 13t + 6$

$0 = -1(5t^2 - 13t - 6)$

$\div(-1)$ $\quad$ $\div(-1)$

$0 = 5t^2 - 13t - 6$ $\qquad$ -15 $+2$ $\quad$ $\square + \square = -13$ $\quad$ $\square \times \square = -30$

$0 = 5t^2 - 15t + 2t - 6$

$0 = (5t^2 - 15t) + (2t - 6)$

$0 = 5t(t-3) + 2(t-3)$

$0 = (t-3)(5t+2)$

$t = 3$ $\qquad$ $t = -2/5$

Takes 3sec.

Using Vertex Form

$0 = -5t^2 + 13t + 6$

$0 = (-5t^2 + 13t) + 6$

$0 = -5(t^2 - \frac{13}{5}t) + 6$

$0 = -5(t^2 - \frac{13}{10}t - \frac{13}{10}t) + 6$

$0 = -5(t^2 - \frac{13}{10}t - \frac{13}{10}t + \frac{169}{100}) + 6 + \frac{169}{20}$

$0 = -5(t - \frac{13}{10})^2 + \frac{289}{20}$

$-\frac{289}{20}$ $\qquad$ $-\frac{289}{20}$

$-\frac{289}{20} = -5(t - \frac{13}{10})^2$

$\frac{289}{100} = (t - \frac{13}{10})^2$

$\pm\frac{17}{10} = t - \frac{13}{10}$

$t_1 = \frac{13}{10} + \frac{17}{10}$ $\qquad$ $t_2 = \frac{13}{10} - \frac{17}{10}$

$t_1 = 3$ $\qquad$ $t_2 = \frac{-4}{10} = -\frac{2}{5}$

Takes 3 sec.

Using Quadratic Formula

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-13 \pm \sqrt{169 - 4(-5)(6)}}{2(-5)}$

$t = \frac{-13 \pm \sqrt{289}}{-10} = \frac{-13 \pm 17}{-10}$

$t_1 = \frac{-13+17}{-10}$ $\qquad$ $t_2 = \frac{-13-17}{-10}$

$t_1 = -\frac{2}{5}$ $\qquad$ $t_2 = 3$

Takes 3 sec.

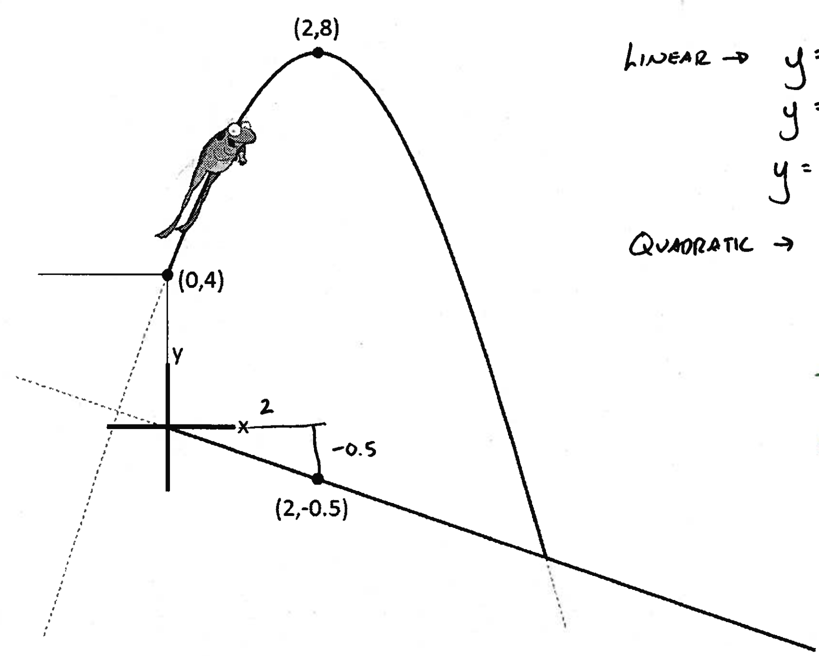
KEY

$t_n = t_1 + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\quad$ $S_n = \frac{n}{2}(t_1 + t_n)$

Part 6 – Review – Chapter 8 Systems of Equations

Use the following information to answer Q22:

A frog jumps off of a 4cm tall ledge onto a downward sloped hill, per the diagram below.



Linear → $y = mx + b$

$y = \frac{-0.5}{2}x + 0$

$y = -\frac{1}{4}x$

Quadratic → $y = a(x-h)^2 + k$

$y = a(x-2)^2 + 8$ Use (0,4)

$4 = a(0-2)^2 + 8$

$-8 \qquad -8$

$-4 = a(4)$

$\div 4 \quad \div 4$

$-1 = a$

$y = -1(x-2)^2 + 8$

All coordinates are in centimeters.

Q22: The frog lands on the hill after travelling a horizontal distance of ***a.bc*** centimeters, where ***a, b,*** and ***c*** are ____, ____, and ____.

(Record your 3-digit answer in the Numerical Response boxes below)

5	0	4	

$y = y$

$-\frac{1}{4}x = -1(x-2)^2 + 8$

$-\frac{1}{4}x = -1(x-2)(x-2) + 8$

$-\frac{1}{4}x = -1(x^2 - 4x + 4) + 8$

$-\frac{1}{4}x = -x^2 + 4x - 4 + 8$

$0 = -x^2 + \frac{17}{4}x + 4$

$\cdot(-4) \quad \cdot(-4) \quad \cdot(-4) \quad \cdot(-4)$

$0 = 4x^2 - 17x - 16$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{17 \pm \sqrt{289 - 4(4)(-16)}}{2(4)}$

$x = \frac{17 \pm \sqrt{545}}{8}$

$x = \frac{17 \pm 23.345}{8}$

$x_1 = 5.04$ a.bc $\qquad x_2 = -0.79$

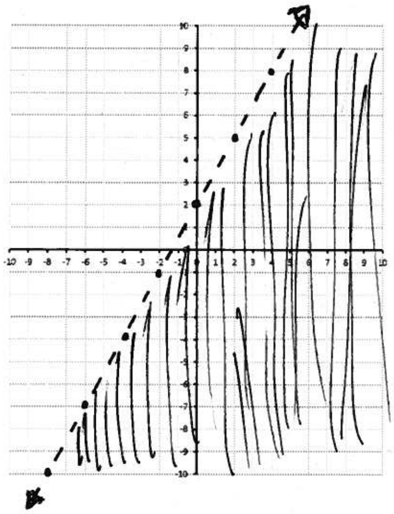
KEY

$t_n = t_1 + (n-1)d$ $\qquad$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad$ $S_n = \frac{n}{2}(t_1 + t_n)$

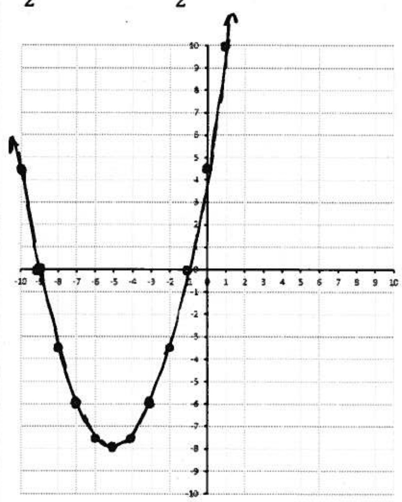
Part 7 – Review – Chapter 9 Inequalities

Q23: Solve each of the following inequalities.

$y < \frac{3}{2}x + 2$

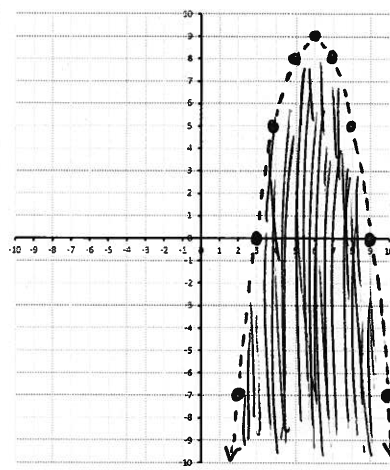


$\frac{1}{2}x^2 + 5x + \frac{9}{2} \geq 0$



$\{x \mid x \leq -9 \text{ or } x \geq -1, x \in \mathbb{R}\}$

$y < -x^2 + 12x - 27$



KEY

$t_n = t_1 + (n-1)d$ $\qquad$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad$ $S_n = \frac{n}{2}(t_1 + t_n)$

Part 8 – Review – Chapter 7 Absolute Values

Q24: Solve $|x^2 + 2x - 8| = x + 4$ algebraically. Verify your answers. Confirm by graphing.

$+(x^2+2x-8) = x+4$

$x^2+2x-8 = x+4$

$\quad -x \; -4 \quad -x \; -4$

$x^2+x-12 = 0$

$(x+4)(x-3) = 0$

$x=-4 \qquad x=3$

$-(x^2+2x-8) = x+4$

$-x^2-2x+8 = x+4$

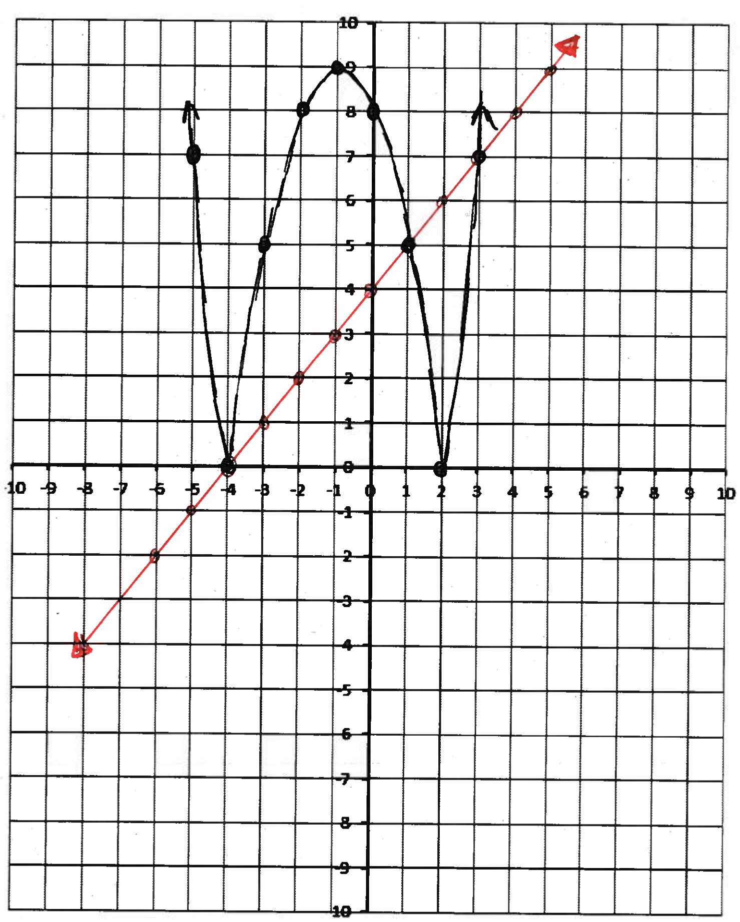
$0 = x^2+3x-4$

$0 = (x+4)(x-1)$

$x=-4 \qquad x=1$

All 3 values verify in the original equation.

$\boxed{x = -4, 1, 3}$

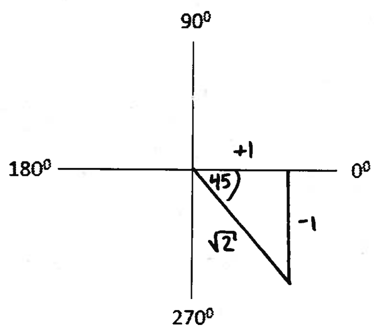


KEY

$t_n = t_1 + (n-1)d$ $\qquad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad S_n = \frac{n}{2}(t_1 + t_n)$

Part 9 – Review – Chapter 2 Trigonometry

Q25: Determine the exact values for the Sin 315⁰, Cos 315⁰, and Tan 315⁰ ratios.



$\sin 315^\circ = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

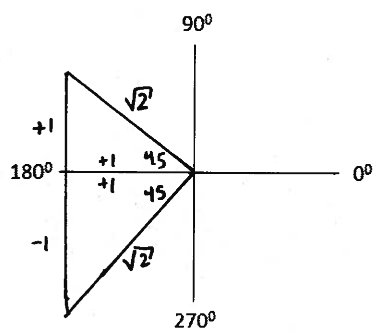
$\cos 315^\circ = \frac{+1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 315^\circ = \frac{-1}{+1} = -1$

Q26: Solve $Cos\theta = -\frac{\sqrt{2}}{2}$ equation, for $0 \leq \theta < 360\ deg$, using a diagram involving a special right triangle.

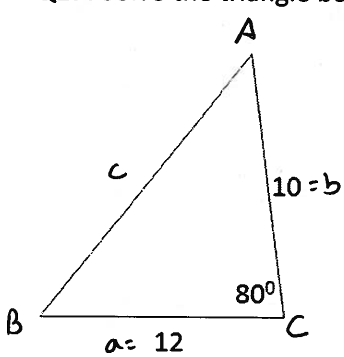
$\cos\theta = \frac{-\sqrt{2}}{2}$ $\qquad$ a is negative

+O or -O



$\theta = 135^\circ$ or 225°

Q27: Solve the triangle below.



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 12^2 + 10^2 - 2(12)(10)\cos 80$

$c^2 = 144 + 100 - 41.67..$

$c^2 = 202.32$

$\boxed{c = 14.2}$

$\frac{\sin A}{12} = \frac{\sin 80}{14.2}$

$\boxed{\angle A = 56.3^\circ}$

$\boxed{\angle B = 43.7^\circ}$

KEY

$t_n = t_1 + (n-1)d$ $\qquad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad S_n = \frac{n}{2}(t_1 + t_n)$

Part 10 – Review – Chapter 5 Radicals

Q28: The expression $\sqrt{60} + \sqrt{36} + \sqrt{135}$ simplifies to $a\sqrt{bc} + d$, where ***a, b, c,*** and ***d*** are ___, ___, ___, and ___.

*(Record your **four digit** answer in the Numerical Response boxes below)*

5	1	5	6

60 → 6, 10 → 2, 3, 2, 5 $\qquad \sqrt{36} = 6$ $\qquad$ 135 → 5, 27 → 3, 9 → 3, 3

$\sqrt{2^2 \cdot 3 \cdot 5} + \sqrt{36} + \sqrt{3^2 \cdot 3 \cdot 5}$

$2\sqrt{15} + 6 + 3\sqrt{15}$

$5\sqrt{15} + 6$

$a\sqrt{bc} + d$

Q29: The expression $\frac{\sqrt{150}}{\sqrt{12}}$ simplifies to $\frac{a\sqrt{b}}{c}$, where ***a, b,*** and ***c*** are ___, ___, and ___.

*(Record your **three digit** answer in the Numerical Response boxes below)*

5	2	2	

$\frac{\sqrt{150}}{\sqrt{12}} = \sqrt{\frac{150}{12}} = \sqrt{\frac{25}{2}} = \frac{\sqrt{25}}{\sqrt{2}} = \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

$\frac{a\sqrt{b}}{c}$

Q30: State the restrictions on x and solve:

$\sqrt{x+3} = 2x + 3$

$x + 3 \geq 0$

$x \geq -3$

$x + 3 = (2x+3)^2$

$x + 3 = (2x+3)(2x+3)$

$x + 3 = 4x^2 + 12x + 9$

$0 = 4x^2 + 11x + 6$

$+3 \quad +8$

$\square + \square = 11$

$\square \times \square = 24$

$0 = 4x^2 + 3x + 8x + 6$

$0 = (4x^2 + 3x) + (8x + 6)$

$0 = x(4x+3) + 2(4x+3)$

$0 = (4x+3)(x+2)$

$4x + 3 = 0$ $\qquad x + 2 = 0$

$x = -3/4$ $\qquad x = -2$

$\sqrt{-\frac{3}{4} + 3} = 2(-3/4) + 3$

$\sqrt{\frac{9}{4}} = -\frac{3}{2} + 3$

$+\frac{3}{2} = \frac{3}{2}$

Yes! Remember, don't take the negative value of the root.

$\sqrt{-2+3} = 2(-2) + 3$

$\sqrt{1} = -4 + 3$

$+1 = -1$

Nope!

$\boxed{\text{So } x = -3/4}$