Math 20-1

KEY

L66.5 – Review of School Year

Unit 1 – Rational Expressions (Ch6)

L04Q5: Multiply, simplify, and determine any Non-Permissible Values (NPV).

$$\frac{2x^2+5x-3}{x^2-x-6} * \frac{x^2+x-2}{2x^2+x-1}$$

$$= \frac{\cancel{(2x-1)}(x+3)}{\cancel{(x+2)}(x-3)} \cdot \frac{(x-1)\cancel{(x+2)}}{\cancel{(2x-1)}(x+1)}$$

$x \neq -2$ $x \neq 3$ $x \neq \frac{1}{2}$ $x \neq -1$

$$= \frac{(x+3)(x-1)}{(x-3)(x+1)}$$

where $x \neq -2, -1, \frac{1}{2}, 3$

$2x^2 + 5x - 3$

-1 $+6$

$\square + \square = 5$

$\square \times \square = -6$

$2x^2 - 1x + 6x - 3$

$(2x^2 - 1x) + (6x - 3)$

$x(2x-1) + 3(2x-1)$

$(2x-1)(x+3)$

$x^2 - x - 6$

$+2$ -3

$\square + \square = -1$

$\square \times \square = -6$

$(x+2)(x-3)$

$x^2 + x - 2$

-1 $+2$

$\square + \square = 1$

$\square \times \square = -2$

$(x-1)(x+2)$

$2x^2 + x - 1$

-1 $+2$

$\square + \square = 1$

$\square \times \square = -2$

$2x^2 - 1x + 2x - 1$

$(2x^2 - 1x) + (2x - 1)$

$x(2x-1) + 1(2x-1)$

$(2x-1)(x+1)$

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L05Q7: Simplify the following expressions. State NPV's.

$$\frac{(x-3)}{(x+1)} - \frac{(x-4)}{(x+2)}$$

$x \neq -1$ $\quad$ $x \neq -2$

$$= \frac{(x-3)(x+2)}{(x+1)(x+2)} - \frac{(x-4)(x+1)}{(x+2)(x+1)}$$

$$= \frac{x^2-x-6}{(x+1)(x+2)} - \frac{x^2-3x-4}{(x+2)(x+1)}$$

$$= \frac{(x^2-x-6)-(x^2-3x-4)}{(x+1)(x+2)}$$

$$= \frac{2x-2}{(x+1)(x+2)} \quad \text{where } x \neq -2, -1$$

$$\frac{(x-6)}{(x+1)} - \frac{(x-2)}{(x+2)}$$

$x \neq -1$ $\quad$ $x \neq -2$

$$= \frac{(x-6)(x+2)}{(x+1)(x+2)} - \frac{(x-2)(x+1)}{(x+2)(x+1)}$$

$$= \frac{x^2-4x-12}{(x+1)(x+2)} - \frac{x^2-x-2}{(x+2)(x+1)}$$

$$= \frac{(x^2-4x-12)-(x^2-x-2)}{(x+1)(x+2)}$$

$$= \frac{-3x-10}{(x+1)(x+2)} \quad \text{where } x \neq -2, -1$$

L07Q4: Solve for the variable.

$$\frac{1}{3x^2} = \frac{x+3}{2x^2} - \frac{1}{6x^2} \quad \rightarrow x \neq 0$$

$$\frac{1}{3x^2}\left(\frac{2}{2}\right) = \frac{(x+3)}{2x^2}\left(\frac{3}{3}\right) - \frac{1}{6x^2}$$

$$\frac{2}{6x^2} = \frac{3x+9}{6x^2} - \frac{1}{6x^2}$$

Now look at numerator

$$2 = (3x+9) - (1)$$

$$2 = 3x + 8$$

$$-6 = 3x$$

$$x = -2$$

$$\frac{2}{x^2-4} + \frac{10}{6x+12} = \frac{1}{x-2}$$

$$\frac{2}{(x+2)(x-2)} + \frac{10}{6(x+2)} = \frac{1}{(x-2)}$$

$$\frac{2}{(x+2)(x-2)}\left(\frac{6}{6}\right) + \frac{10}{6(x+2)}\left(\frac{x-2}{x-2}\right) = \frac{1}{(x-2)}\left(\frac{x+2}{x+2}\right)\left(\frac{6}{6}\right)$$

$$\frac{12}{6(x+2)(x-2)} + \frac{10x-20}{6(x+2)(x-2)} = \frac{6x+12}{6(x+2)(x-2)}$$

$$(12) + (10x-20) = (6x+12)$$

$$10x - 8 = 6x + 12$$

$$4x = 20$$

$$x = 5$$

KEY

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Unit 1 – Quadratic Functions (Ch3)

Use the following information to answer L11Q11-L11Q15:

$$g(x) = 1(x-1)^2 - 9$$

$g(x) = a(x- +1)^2 + -9$

L11Q11: Determine the coordinate of the Vertex.

$(1, -9)$

L11Q12: Determine the equation of the Axis of Symmetry.

$x = 1$

L11Q13: Convert to Standard Form to find the y-Intercept.

$g(x) = 1(x-1)(x-1) - 9$
$= 1(x^2 - 2x + 1) - 9$
$= x^2 - 2x - 8$

y-intercept is -8.

L11Q14: Determine the zeroes.

From Vertex Form	*From Standard Form*
$g(x) = 1(x-1)^2 - 9$ x-int → Set $y = 0$ $0 = 1(x-1)^2 - 9$ $9 = (x-1)^2$ $\sqrt{9} = x - 1$ $+3 = x - 1$ (+1, +1) → $\boxed{4 = x}$ $-3 = x - 1$ (+1, +1) → $\boxed{-2 = x}$	$g(x) = x^2 - 2x - 8$ x-int → Set $y = 0$ $0 = x^2 - 2x - 8$ $0 = (x+2)(x-4)$ $x + 2 = 0$ → $\boxed{x = -2}$ $x - 4 = 0$ → $\boxed{x = 4}$

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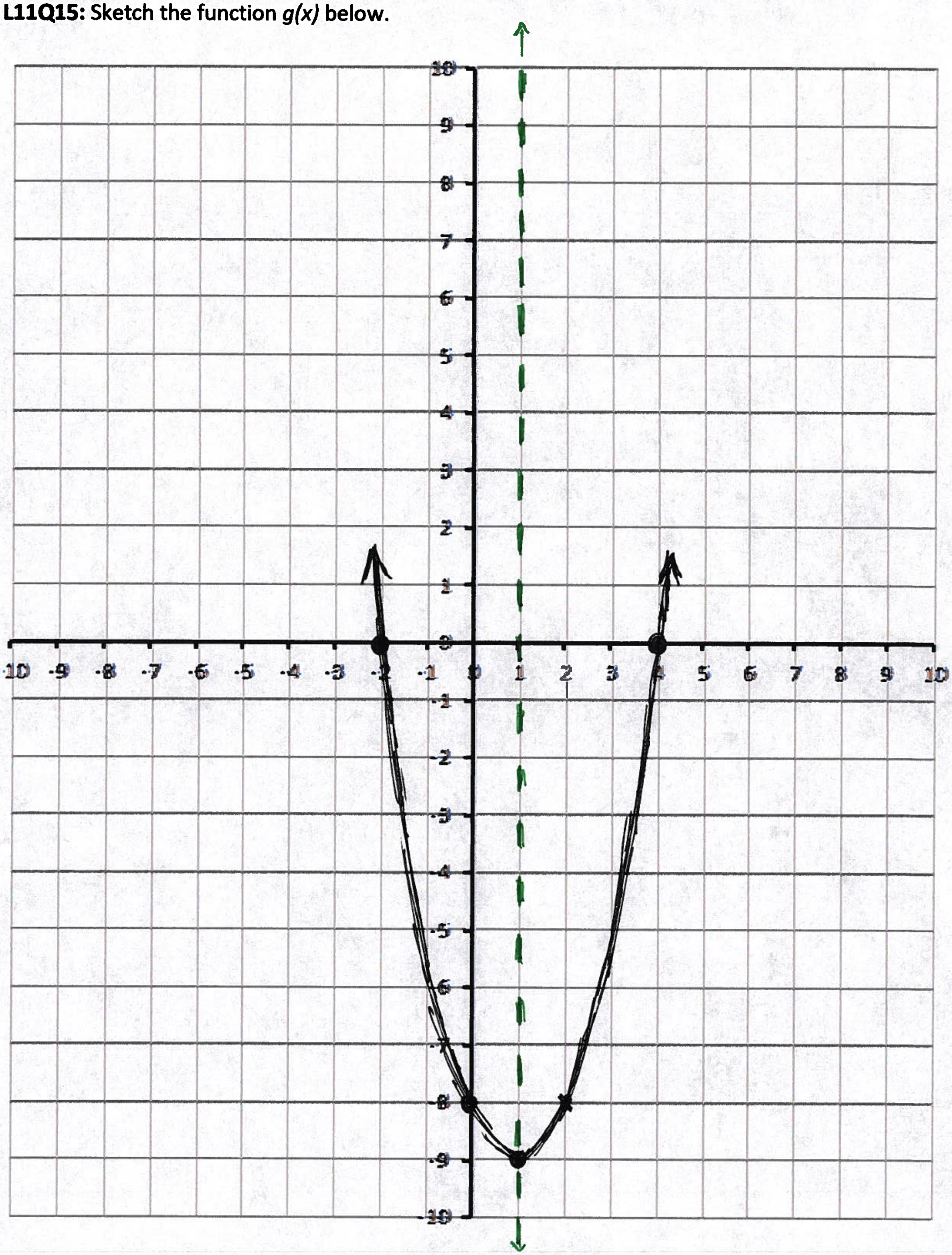
L11Q15: Sketch the function *g(x)* below.



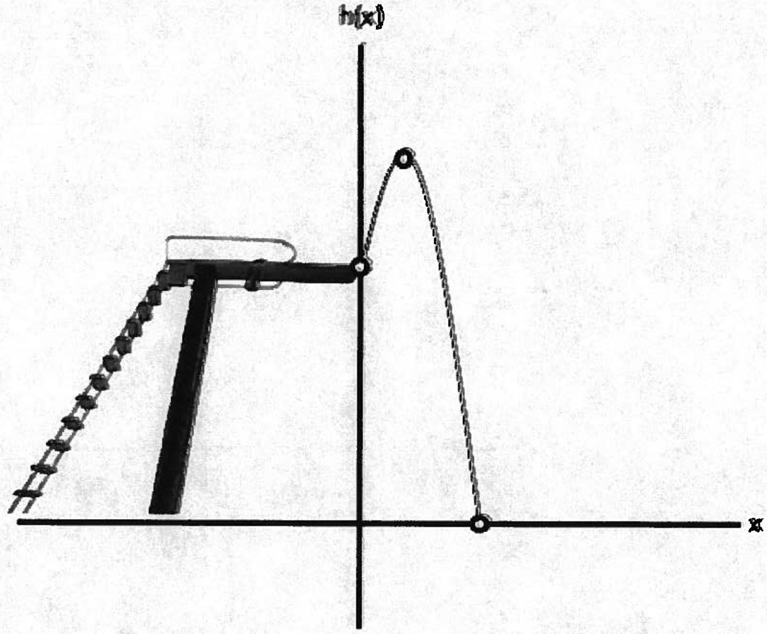
Math 20-1

Unit 1 – Quadratic Equations (Ch4)

Use the following information to answer L12Q8-L12Q9:

A diver jumps off of a 8m tall diving board. The equation that models their height, *h*, as a function of horizontal distance, *x*, is given by:

$$h(x) = -x^2 + 2x + 8$$



L12Q8: Working with Standard Form...

a. Factor to find the zeroes. Which one is significant, and why?
b. Determine the y-intercept. What is the significance?
c. Use the zeroes to find the axis of symmetry.
d. Use the axis of symmetry to find the Vertex. What is the significance?

(A) $0 = -x^2 + 2x + 8$
$0 = x^2 - 2x - 8$
$0 = (x-4)(x+2)$
$\boxed{x=4}$ ✓ Landing Position
$\boxed{x=-2}$ Gibberish

(B) y-intercept is 8. Height of diving board.

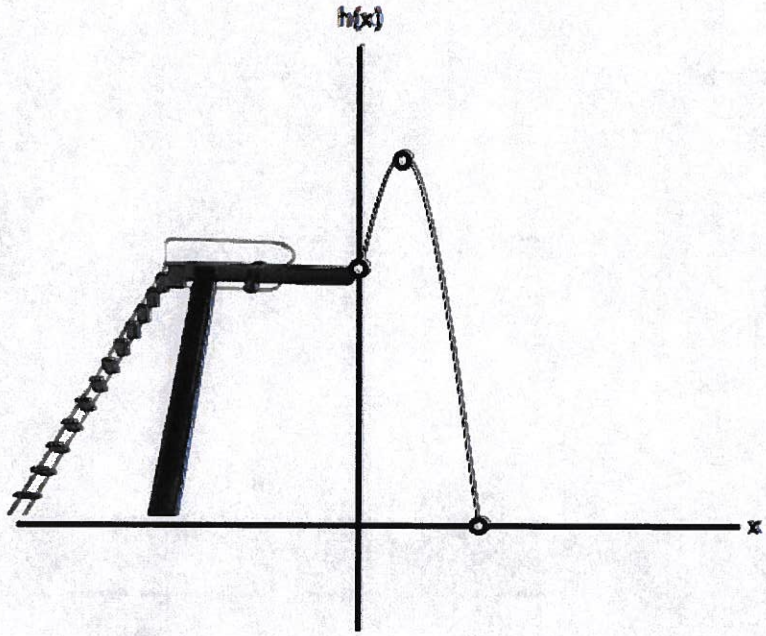
(C) $\frac{(4)+(-2)}{2} = 1$ $\boxed{x=1}$

(D) $h(1) = -(1)^2 + 2(1) + 8$
$= -1 + 2 + 8$
$= 9$
Vertex (1,9). Max height.

Use the following information to answer L12Q8-L12Q9:

A diver jumps off of a 8m tall diving board. The equation that models their height, *h*, as a function of horizontal distance, *x*, is given by:

$$h(x) = -x^2 + 2x + 8$$



L12Q9: Working with Vertex Form…

a. Convert the function to Vertex Form.
b. Find the zeroes. Which one is significant, and why?
c. Find the Vertex. What is the significance?

(A)
$$
\begin{aligned}
h(x) &= -1x^2 + 2x + 8 \\
&= (-1x^2 + 2x) + 8 \\
&= -1(x^2 - 2x) + 8 \\
&= -1(x^2 - 1x - 1x) + 8 \\
&= -1(x^2 - 1x - 1x + 1) + 8 \; +1 \\
&= -1(x-1)^2 + 9
\end{aligned}
$$

(C) Vertex at (1, 9). Max Height.

(B)
$$
\begin{aligned}
0 &= -1(x-1)^2 + 9 \\
-9 & \qquad\qquad -9 \\
-9 &= -1(x-1)^2 \\
9 &= (x-1)^2 \\
\sqrt{9} &= x - 1
\end{aligned}
$$

$+3 = x - 1$ $\quad$ $+1 \quad +1$ $\quad$ $\boxed{4 = x}$

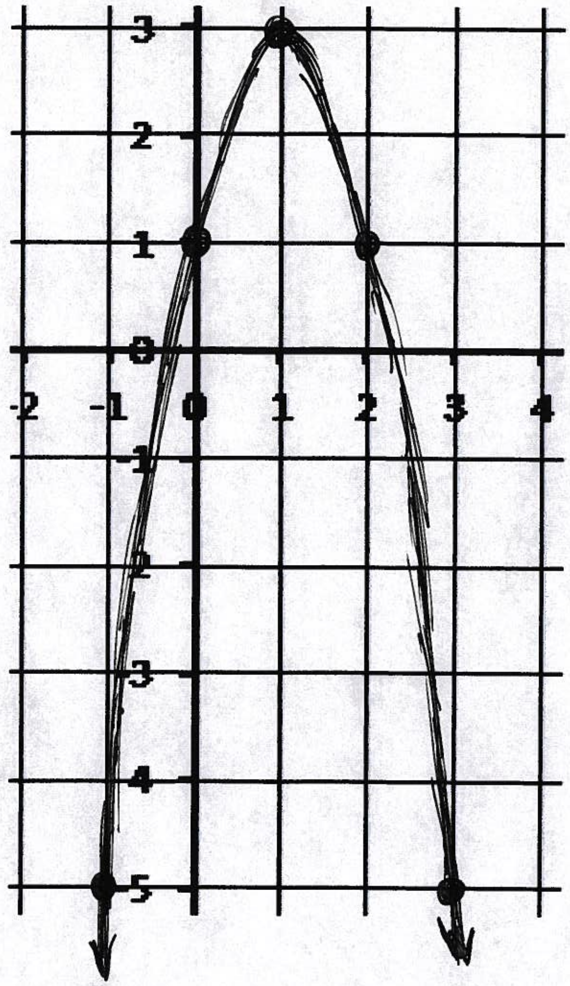
$-3 = x - 1$ $\quad$ $+1 \quad +1$ $\quad$ $\boxed{-2 = x}$

Unit 2 – Systems of Equations (Ch8)

L16Q3: Solve the equation $-2x^2 + 4x + 1 = 0$ graphically.

x	y
-1	-5
0	1
1	3
2	1
3	-5

$\boxed{x \approx -0.2, +2.2}$



L17Q3: Factor the following:

$(x-2)^2 + 5(x-2) - 6$

Let $y = x - 2$

$y^2 + 5y - 6$

$\overset{-1}{\square} + \overset{+6}{\square} = 5$

$\square \times \square = -6$

$(y-1)(y+6)$

$(x-2-1)(x-2+6)$

$(x-3)(x+4)$

$\frac{1}{2}(x+3)^2 + \frac{1}{2}(x+3) - 10$

$\frac{1}{2}\left[(x+3)^2 + (x+3) - 20\right]$

Let $y = x+3$

$\overset{-4}{\square} + \overset{+5}{\square} = 1$

$\square \times \square = -20$

$\frac{1}{2}\left[y^2 + y - 20\right]$

$\frac{1}{2}\left[(y-4)(y+5)\right]$

$\frac{1}{2}\left[(x+3-4)(x+3+5)\right]$

$\frac{1}{2}\left[(x-1)(x+8)\right]$

$\frac{1}{2}(x-1)(x+8)$

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L18Q1: Given the equation $x^2 + 8x + 15 = 0$, determine the roots.

Standard Form	*Vertex Form*
$x^2+8x+15=0$ $+3 \quad +5$ $\square+\square=8$ $\square\times\square=15$ $(x+3)(x+5)=0$ $\boxed{x=-3}$ $\quad$ $\boxed{x=-5}$	$(x^2+8x)+15=0$ $1(x^2+4x+4x)+15=0$ $1(x^2+4x+4x+16)+15-16=0$ $\boxed{1(x+4)^2-1=0}$ Vertex Form $+1 \quad +1$ $(x+4)^2=1$ $x+4=\sqrt{1}$ $x+4=+1$ $\quad$ $x+4=-1$ $-4 \quad -4$ $\quad$ $-4 \quad -4$ $\boxed{x=-3}$ $\quad$ $\boxed{x=-5}$

L20Q1: Solve $x^2 + 13x + 40 = 0$ using each method.

Factoring	*Converting to Vertex Form*	*Quadratic Formula*
$x^2+13x+40=0$ $(x+5)(x+8)=0$ $\boxed{x=-5}$ $\quad$ $\boxed{x=-8}$	$x^2+13x+40=0$ $(x^2+13x)+40=0$ $(x^2+\frac{13}{2}x+\frac{13}{2}x)+40=0$ $(x^2+\frac{13}{2}x+\frac{13}{2}x+\frac{169}{4})+40-\frac{169}{4}=0$ $\boxed{(x+\frac{13}{2})^2-\frac{9}{4}=0}$ Vertex Form $+\frac{9}{4} \quad +\frac{9}{4}$ $(x+\frac{13}{2})^2=\frac{9}{4}$ $x+\frac{13}{2}=\sqrt{\frac{9}{4}}$ $x+\frac{13}{2}=+\frac{3}{2}$ $\quad$ $x+\frac{13}{2}=-\frac{3}{2}$ $-\frac{13}{2} \quad -\frac{13}{2}$ $\quad$ $-\frac{13}{2} \quad -\frac{13}{2}$ $\boxed{x=-5}$ $\quad$ $\boxed{x=-8}$	$1x^2+13x+40=0$ $a=1 \quad b=13 \quad c=40$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-(13)\pm\sqrt{13^2-4(1)(40)}}{2(1)}$ $x=\frac{-13\pm\sqrt{169-160}}{2}$ $x=\frac{-13\pm\sqrt{9}}{2}$ $x=\frac{-13+3}{2}$ $\quad$ $x=\frac{-13-3}{2}$ $\boxed{x=-5}$ $\quad$ $\boxed{x=-8}$

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L28&29Q2: Solve the system of equations graphically.

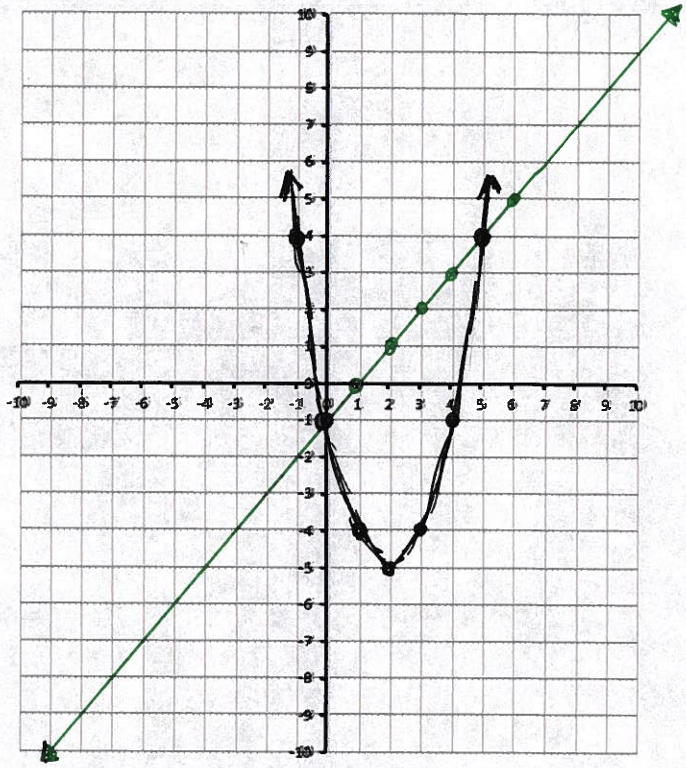
$f(x) = 1(x-2)^2 - 5$

$g(x) = x - 1$

Verify using your calculator.

x	f(x)
0	-1
1	-4
2	-5
3	-4
4	-1
5	4
6	11

x	g(x)
0	-1
1	0
2	1
3	2
4	3
5	4
6	5



$\boxed{\text{Solns are } (0,-1) \text{ and } (5,4)}$

L30&31Q8: Solve using Substitution.

$y = x^2 + 2x - 5$

$y = -2x^2 - 7x - 11$

$y = y$

$x^2 + 2x - 5 = -2x^2 - 7x - 11$

$3x^2 + 9x + 6 = 0$

$3(x^2 + 3x + 2) = 0$

$3(x+2)(x+1) = 0$

$\boxed{x = -2}$ $\boxed{x = -1}$

$y = (-2)^2 + 2(-2) - 5$

$\boxed{y = -5}$

$\boxed{\text{Soln is } (-2,-5)}$

$y = (-1)^2 + 2(-1) - 5$

$\boxed{y = -6}$

$\boxed{\text{Soln is } (-1,-6)}$

L30&31Q9: Solve using Elimination.

$y = 3x^2 + 2x + 5$

$-(y = 2x^2 + 4x + 13)$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$\boxed{x = 4}$ $\boxed{x = -2}$

$y = 2(4)^2 + 4(4) + 13$

$\boxed{y = 61}$

$\boxed{\text{Soln is } (4,61)}$

$y = 2(-2)^2 + 4(-2) + 13$

$\boxed{y = 13}$

$\boxed{\text{Soln is } (-2,13)}$

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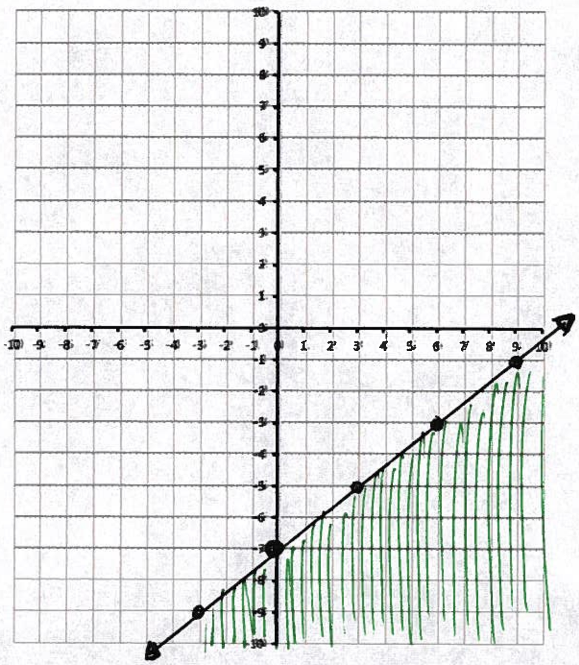
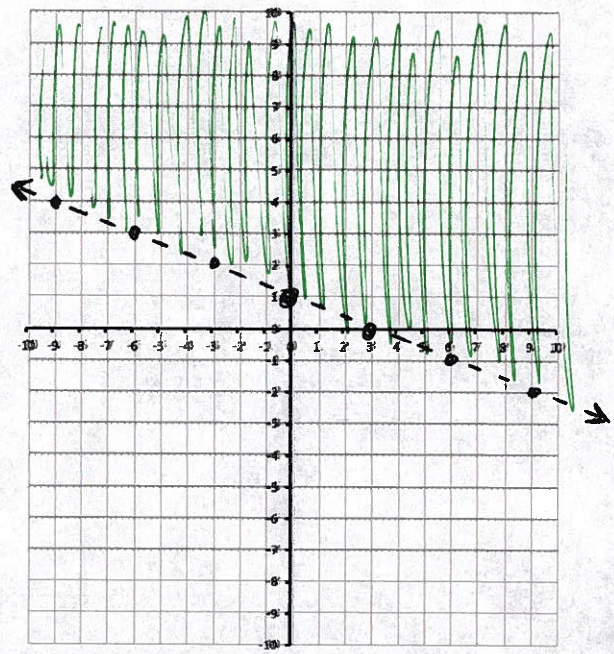
Unit 2 – Inequalities (Ch9)

L34Q4: Graph the following equations:

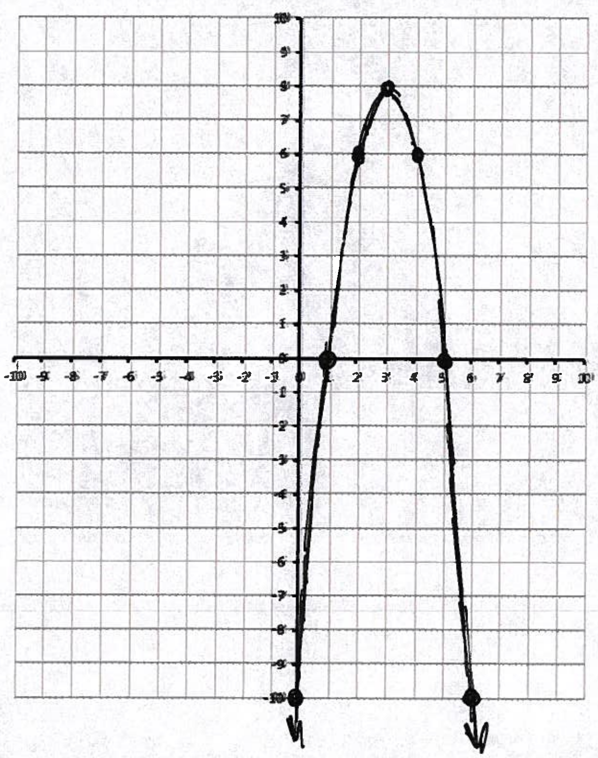
$y \le \frac{2}{3}x - 7$

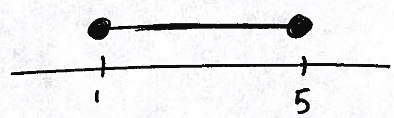


$y > -\frac{1}{3}x + 1$



L35Q4: Use graphing to solve the inequality $-2x^2 + 12x - 10 \ge 0$.





$\{x \mid 1 \le x \le 5, x \in R\}$

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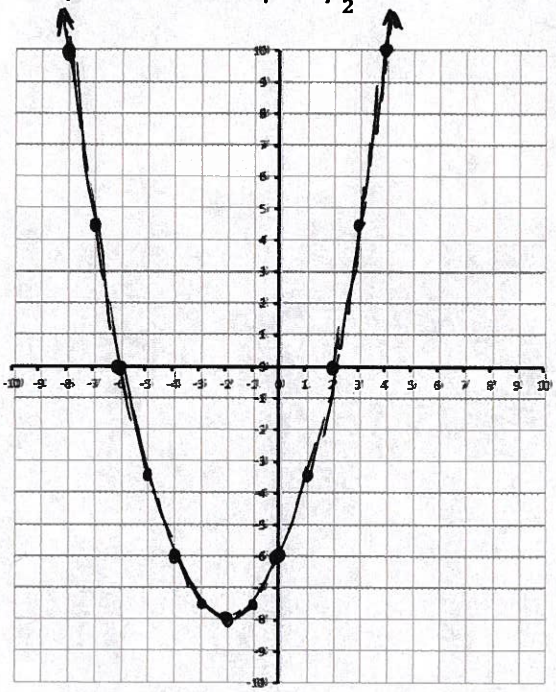
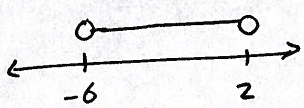
L36Q2: Solve the inequality $\frac{1}{2}x^2 + 2x - 6 < 0$

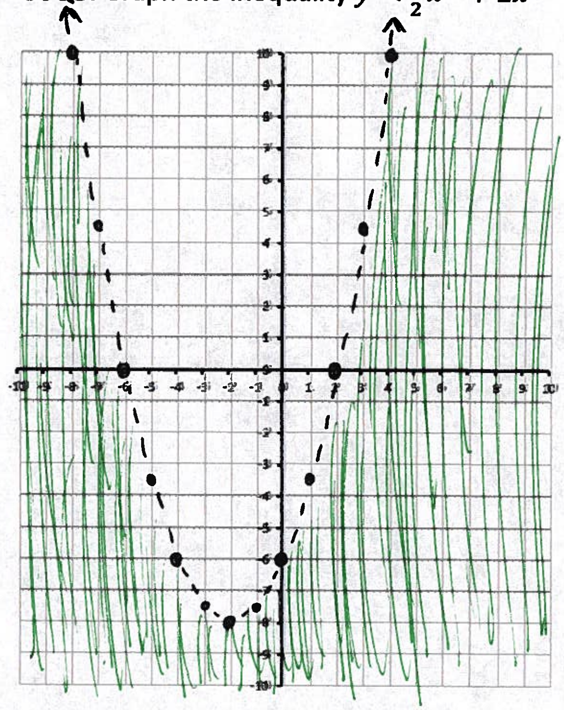


Graph $y = \frac{1}{2}x^2 + 2x - 6$



$\{x \mid -6 < x < 2, x \in \mathbb{R}\}$

L36Q3: Graph the inequality $y < \frac{1}{2}x^2 + 2x - 6$



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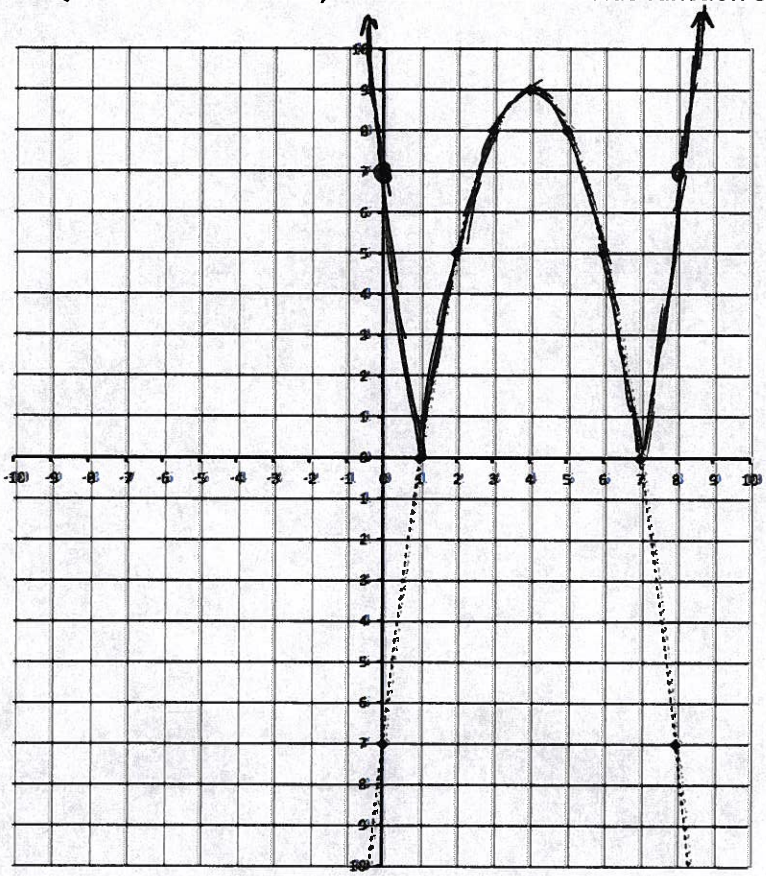
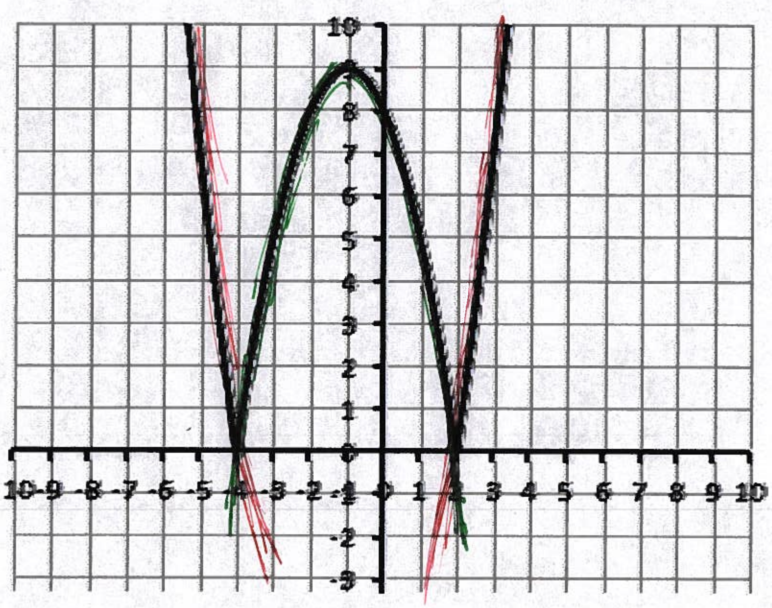
Unit 3 – Absolute Values (Ch7)

L42Q3: Given the function, sketch the absolute value function on the same graph paper.



L42Q4: For the given graph, determine the piecewise function.



$y = 1(x+1)^2 - 9$ for $x \leq -4$

$y = -1(x+1)^2 + 9$ for $-4 < x < 2$

$y = 1(x+1)^2 - 9$ for $x \geq 2$

Key

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L43Q4: Solve $|x^2 + 4x - 6| = 6$

$+(x^2+4x-6) = 6$

$x^2+4x-6 = 6$

$x^2+4x-12 = 0$

$(x+6)(x-2) = 0$

$\boxed{x=-6}$ $\boxed{x=2}$

Verify!

$|(-6)^2+4(-6)-6| = 6$

$|36-24-6| = 6$

$|6| = 6$

Yes!

Verify!

$|2^2+4(2)-6| = 6$

$|4+8-6| = 6$

$|6| = 6$

Yes!

$-(x^2+4x-6) = 6$

$-x^2-4x+6 = 6$

$-x^2-4x = 0$

$-x(x+4) = 0$

$\boxed{x=0}$ $\boxed{x=-4}$

Verify!

$|0^2+4(0)-6| = 6$

$|-6| = 6$

Yes!

Verify!

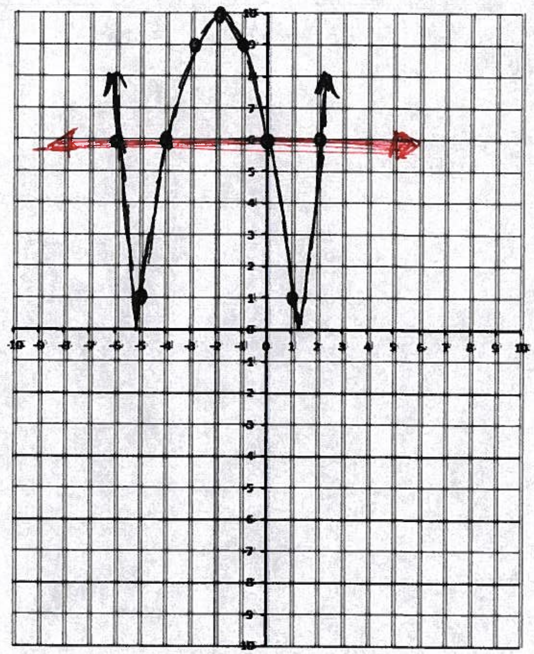
$|(-4)^2+4(-4)-6| = 6$

$|16-16-6| = 6$

$|-6| = 6$

Yes!

$\boxed{\text{Solns are } \quad x = -6, -4, 2, 0}$



KEY

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L45Q3: Given the function $f(x) = 2x + 1$

a. Build a table for $y = f(x)$ and $y = \frac{1}{f(x)}$.

x	$y = f(x)$	$y = \frac{1}{f(x)}$
-3	-5	-0.2
-2	-3	$-0.\overline{3}$
-1	-1	-1
0	1	1
1	3	$0.\overline{3}$
2	5	0.2
3	7	$0.\overline{142857}$

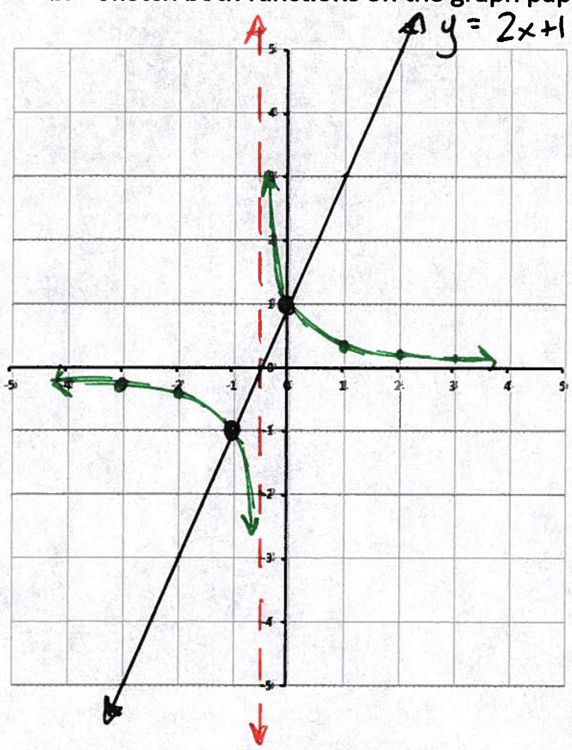
$y = \frac{1}{2x+1}$

$2x+1 \neq 0$

$\boxed{x \neq -\frac{1}{2}}$ Non-Permissible Value

Asymptote at $x = -\frac{1}{2}$

b. Sketch both functions on the graph paper.



A $x \neq -\frac{1}{2}$

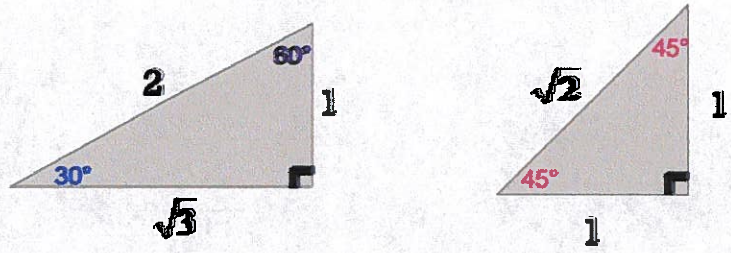
B $x = -\frac{1}{2}$

C $(-1,-1)$ and $(0,1)$

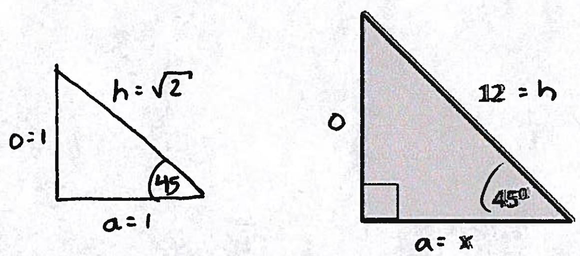
c. Identify (a) the NPV's for the reciprocal function, (b) the equation of the vertical asymptote of the reciprocal function, (c) the Invariant Point(s).

Unit 3 – Trigonometry (Ch2)

Use the following information to answer L48Q13:



L48Q13: Given the triangles below, determine the exact value of x.

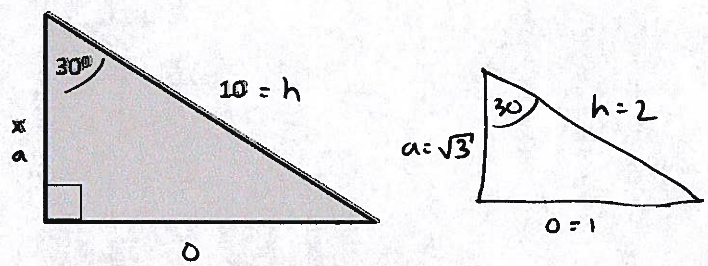


$\cos 45 = \frac{a}{h}$

$\frac{1}{\sqrt{2}} = \frac{x}{12}$

$x = \frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$

$x = 6\sqrt{2}$



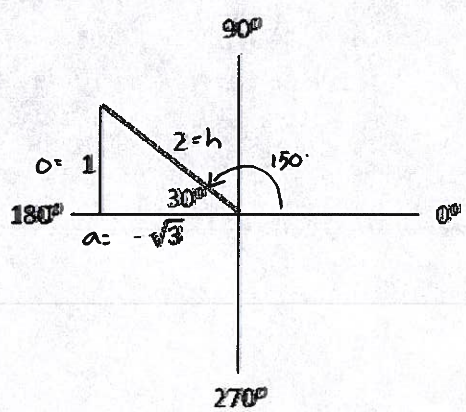
$\cos 30 = \frac{a}{h}$

$\frac{\sqrt{3}}{2} = \frac{x}{10}$

$\frac{10\sqrt{3}}{2} = x$

$\boxed{x = 5\sqrt{3}}$

L49Q2: Determine the trigonometric ratios for each triangle.



$\sin\theta = \frac{o}{h}$

$\sin 150° = \frac{1}{2}$

$\cos\theta = \frac{a}{h}$

$\cos 150 = -\frac{\sqrt{3}}{2}$

$\tan\theta = \frac{o}{a}$

$\tan 150 = \frac{1}{-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\tan 150 = -\frac{\sqrt{3}}{3}$

Key

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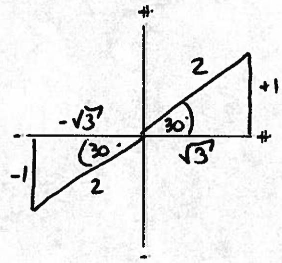
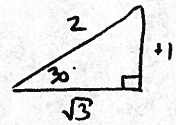
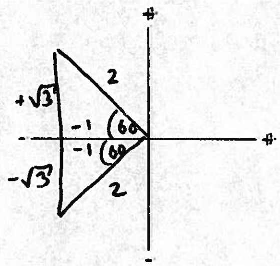
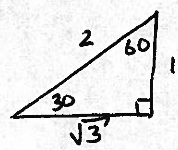
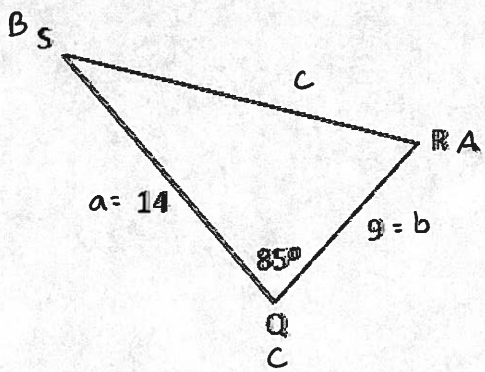
L49Q4: Solve each equation, for $0 \le \theta < 360\ deg$, using a diagram involving a special right triangle.

	Sketch of Angle in Standard Position	Sketch of Special Right Triangle	Solution to Equation
$Tan\theta = \frac{+o}{+a}$ $Tan\theta = \frac{-o}{-a}$ $Tan\theta = \frac{1}{\sqrt{3}}$			$\theta = 30°, 210°$
$Cos\theta = \frac{-a}{h}$ +o or -o $Cos\theta = -\frac{1}{2}$			$\theta = 120°, 240°$

L51Q5: Solve the triangle.



$c^2 = a^2 + b^2 - 2abCosC$

$c^2 = 14^2 + 9^2 - 2(14)(9)Cos85$

$c^2 = 196 + 81 - 21.96$

$c^2 = 255.04$

$\boxed{c = 16.0} \Rightarrow \boxed{RS = 16.0}$

$\frac{SinC}{c} = \frac{SinA}{a}$

$\frac{Sin85}{16.0} = \frac{SinA}{14}$

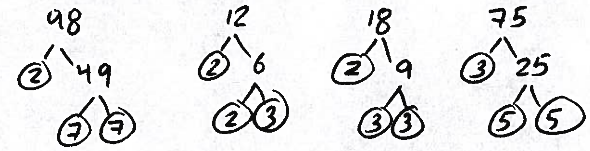
$\boxed{\angle A = 60.7°} \Rightarrow \boxed{\angle R = 60.7°}$

$\angle A + \angle B + \angle C = 180°$

$\boxed{\angle B = 34.3°} \Rightarrow \boxed{\angle S = 34.3°}$

Unit 4 – Radicals (Ch5)

L58Q9: Simplify the expression $\sqrt{98}+\sqrt{12}+\sqrt{18}-\sqrt{75}$

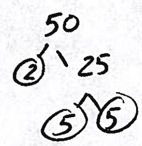


$\sqrt{2\cdot 7^2}+\sqrt{2^2\cdot 3}+\sqrt{2\cdot 3^2}-\sqrt{3\cdot 5^2}$

$7\sqrt{2}+2\sqrt{3}+3\sqrt{2}-5\sqrt{3}$

$10\sqrt{2}-3\sqrt{3}$

L59Q3: Simplify each of the following:



$\sqrt{50x^2}=\sqrt{2\cdot 5^2\cdot x^2}$

$=5x\sqrt{2}$

$(\sqrt{50x^2})(5\sqrt{3x}+\sqrt{2})$

$(5x\sqrt{2})(5\sqrt{3x}+\sqrt{2})$

$25x\sqrt{6x}+5x\sqrt{4}$

$25x\sqrt{6x}+5x(2)$

$25x\sqrt{6x}+10x$

L60Q1: Simplify the following expressions:

$\dfrac{\sqrt{40}}{\sqrt{10}}=\sqrt{\dfrac{40}{10}}=\sqrt{4}=2$

$\dfrac{2\sqrt{18}}{4\sqrt{2}}=\dfrac{1\sqrt{18}}{2\sqrt{2}}=\dfrac{1}{2}\sqrt{9}$

$=\dfrac{3}{2}$

$\dfrac{3\sqrt{26}}{\sqrt{13}}=\dfrac{3}{1}\sqrt{\dfrac{26}{13}}=\dfrac{3}{1}\sqrt{2}$

$=3\sqrt{2}$

Math 20-1

L60Q4: Rationalize the Denominator:

$$\frac{(2+\sqrt{5})}{(\sqrt{5}+3)} \cdot \frac{(\sqrt{5}-3)}{(\sqrt{5}-3)} = \frac{2\sqrt{5}-6+5-3\sqrt{5}}{5-3\sqrt{5}+3\sqrt{5}-9}$$

$$= \frac{-\sqrt{5}-1}{-4} = \frac{\sqrt{5}+1}{4}$$

$$\frac{(\sqrt{3}-2)}{(5-\sqrt{3})} \cdot \frac{(5+\sqrt{3})}{(5+\sqrt{3})} = \frac{5\sqrt{3}+3-10-2\sqrt{3}}{25+5\sqrt{3}-5\sqrt{3}-3}$$

$$= \frac{3\sqrt{3}-7}{22}$$

L62Q4: Solve the radical equation $\sqrt{-2x+13} = x+1$ and verify your solution(s).

$$-2x+13 = (x+1)^2$$
$$-2x+13 = (x+1)(x+1)$$
$$-2x+13 = x^2+2x+1$$
$$0 = x^2+4x-12$$
$$0 = (x+6)(x-2)$$

$x = -6$ $\qquad$ $x = 2$

$$\sqrt{-2(-6)+13} = (-6)+1$$
$$\sqrt{12+13} = -5$$
$$\sqrt{25} = -5$$

Nope. Positive roots only.

$$\sqrt{-2(2)+13} = 2+1$$
$$\sqrt{-4+13} = 3$$
$$\sqrt{9} = 3$$

Yep!

$\boxed{x=2}$

Math 20-1

Unit 4 – Sequences and Series (Ch1)

Arithmetic Sequences $t_n = t_1 + (n-1)d$

t_1 is the first term

n is the number of terms ($n \in N$)

d is the common difference

t_n is the general term or n^{th} term

Use the following information to answer L65Q3:

An Arithmetic Sequence is shown below: $\overset{-4}{\curvearrowright}$ 100, 96, 92, 88… $t_1 = 100$, $d = -4$

L65Q3: Determine the 18th term in the sequence.

$$t_n = t_1 + (n-1)d$$
$$= 100 + (18-1)(-4)$$
$$= 100 + (17)(-4)$$
$$= 32$$

Use the following information to answer L65Q18:

An Arithmetic Sequence is shown below: $\overset{+5}{\curvearrowright}$ -2 + 3 + 8 + 13 + …

L65Q18: What is the sum of the first 10 terms?

$$S_n = \frac{n}{2}\left[2t_1 + (n-1)d\right]$$
$$S_{10} = \frac{10}{2}\left[2(-2) + (10-1)(5)\right]$$
$$= \frac{10}{2}\left[-4 + 45\right]$$
$$= 205$$