★ KEY ★

L67 – 1.3 & 1.24 Geometric Sequences and Series

Part 1 – Key Ideas

Arithmetic Sequence and Series

Sequence:	2, 6, 10, 14…
Series:	2 + 6 + 10 + 14 +

$$t_n = t_1 + (n-1)d$$

$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$

$$S_n = \frac{n}{2}(t_1 + t_n)$$

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t_1 is the first term

n is the number of terms ($n \in N$)

d is the common difference

t_n is the general term or n^{th} term

S_n is the sum to n terms

Geometric Sequences and Series

Sequence:	3, 6, 12, 24…
Series:	3 + 6 + 12 + 24 +

$$r = \frac{t_n}{t_{n-1}}$$

$$t_n = t_1 r^{n-1}$$

$$S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$$

$$S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$$

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t_1 is the first term

n is the number of terms

r is the common ratio

t_n is the general term or the nth term

S_n is the sum to n terms

Math 20-1

Part 2 – Geometric Sequences

Geometric Sequences $\qquad r = \dfrac{t_n}{t_{n-1}}$ $\qquad\qquad\qquad t_1$ is the first term

$$t_n = t_1 r^{n-1}$$

n is the number of terms

r is the common ratio

t_n is the general term or the nth term

Use the following information to answer Q1-Q3:

A Geometric Sequence is shown below:

$\overset{\times 5}{\frown}\ \overset{\times 5}{\frown}$

2, 10, 50, 250...

Q1: Determine the common ratio.

$t_2 = 10$ $\qquad r = \dfrac{t_2}{t_1} = \dfrac{10}{2} = 5$

$t_1 = 2$

Q2: Determine the 8th term in the sequence.

$t_n = t_1 r^{n-1}$

$t_8 = 2(5)^{8-1}$

$\quad = 156,250$

Q3: Determine the 12th term in the sequence.

$t_{12} = 2(5)^{12-1}$

$\quad = 97,656,250$

Math 20-1

Geometric Sequences $\qquad r = \dfrac{t_n}{t_{n-1}}$ $\qquad\qquad$ t_1 is the first term

$$t_n = t_1 r^{n-1}$$ $\qquad\qquad$ n is the number of terms

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ r is the common ratio

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ t_n is the general term or the nth term

Use the following information to answer Q4-Q6:

A Geometric Sequence is shown below:

$\qquad\qquad\qquad\qquad\qquad$ ×2 $\quad$ ×2

7, 14, 28, 56...

Q4: Determine the common ratio.

$$r = \frac{t_n}{t_{n-1}} = \frac{t_3}{t_2} = \frac{28}{14} = 2$$

Q5: Determine the 9th term in the sequence.

$$t_n = t_1 r^{n-1}$$
$$t_9 = 7(2)^{9-1}$$
$$= 1792$$

Q6: Determine the 15th term in the sequence.

$$t_n = t_1 r^{n-1}$$
$$t_{15} = 7(2)^{15-1}$$
$$= 114,688$$

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Geometric Sequences $\qquad r = \dfrac{t_n}{t_{n-1}}$ $\qquad\qquad$ t_1 is the first term

$$t_n = t_1 r^{n-1}$$ $\qquad\qquad\qquad$ n is the number of terms

r is the common ratio

t_n is the general term or the nth term

Use the following information to answer Q7-Q8:

In a Geometric Sequence, the 5th term is 324 and the 6th term is 972.

Q7: Determine the common ratio.

$$r = \dfrac{t_n}{t_{n-1}} = \dfrac{t_6}{t_5} = \dfrac{972}{324} = 3$$

Q8: Determine the value of the first term.

$$t_n = t_1 r^{n-1} \qquad \text{Use } 5^{th} \text{ term as } 324$$
$$324 = t_1 (3)^{5-1}$$
$$324 = t_1 (81)$$
$$t_1 = 4$$

Use the following information to answer Q9-Q10:

In a Geometric Sequence, the 4th term is -27 and the 5th term is -243.

Q9: Determine the common ratio.

$$r = \dfrac{t_n}{t_{n-1}} = \dfrac{t_5}{t_4} = \dfrac{-243}{-27} = 9$$

Q10: Determine the value of the first term.

$$t_n = t_1 r^{n-1} \qquad \text{Use } 4^{th} \text{ term as } -27$$
$$-27 = t_1 (9)^{4-1}$$
$$-27 = t_1 (729)$$
$$-\tfrac{1}{27} = t_1$$

Math 20-1

Geometric Sequences

$$r = \frac{t_n}{t_{n-1}}$$

$$t_n = t_1 r^{n-1}$$

t_1 is the first term

n is the number of terms

r is the common ratio

t_n is the general term or the nth term

Use the following information to answer Q11:

In a Geometric Sequence, the 3rd term is 24 and the 5th term is 96.

Q11: Determine the value of the 12th term.

Option #1: Easiest

$t_3 = t_1 r^{3-1}$ and $t_5 = t_1 r^{5-1}$
$24 = t_1 r^2$ $\qquad$ $96 = t_1 r^4$

$$\frac{96}{24} = \frac{t_1 r^4}{t_1 r^2}$$

$4 = r^2$

$r = 2$

Option #2: Harder

$t_3 \xrightarrow{\times r} t_4 \xrightarrow{\times r} t_5$

$t_3 \times r^2 = t_5$
$24 \times r^2 = 96$
$\div 24 \qquad \div 24$
$r^2 = 4$
$r = 2$

$t_n = t_1 r^{n-1}$
$24 = t_1 (2)^2$
$t_1 = 6$

Use 3rd term is 24

$t_n = t_1 r^{n-1}$
$t_{12} = 6(2)^{12-1}$
$\qquad = 12,288$

Option #3: Easiest (if you are awesome at math)

$r = \frac{t_n}{t_{n-1}}$ or $t_n = r(t_{n-1})$

so $t_n = r^2(t_{n-2})$
$\qquad = r^3(t_{n-3}) \dots$ etc

$$\boxed{r^x = \frac{t_n}{t_{n-x}}}$$

Math 20-1

Geometric Sequences $\qquad r = \dfrac{t_n}{t_{n-1}}$ $\qquad\qquad t_1$ is the first term

$$t_n = t_1 r^{n-1}$$ $\qquad\qquad$ n is the number of terms

r is the common ratio

t_n is the general term or the nth term

Use the following information to answer Q12:

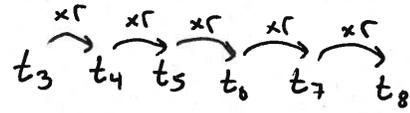In a Geometric Sequence, the 3rd term is 448 and the 8th term is 458752.

Q12: Determine the value of the 12th term.

$t_3 = t_1 r^2$ $\qquad\qquad$ $t_8 = t_1 r^7$

$448 = t_1 r^2$ $\qquad\qquad$ $458{,}752 = t_1 r^7$ $\quad$ or $\quad$

$t_3 \xrightarrow{\times r} t_4 \xrightarrow{\times r} t_5 \xrightarrow{\times r} t_6 \xrightarrow{\times r} t_7 \xrightarrow{\times r} t_8$

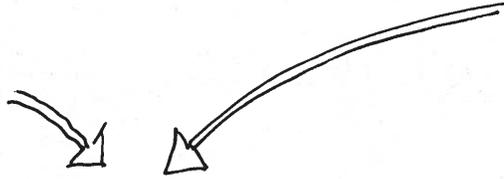$\dfrac{458{,}752}{448} = \dfrac{t_1 r^7}{t_1 r^2}$

$448 \times r^5 = 458{,}752$

$r^5 = 1024$

$r = 4$

$1024 = r^5$

$4 = r$

$t_n = t_1 r^{n-1}$ $\qquad$ Use 3rd term is 448

$448 = t_1 (4)^{3-1}$

$448 = t_1 (16)$

$t_1 = 28$

$t_n = t_1 r^{n-1}$

$t_{12} = t_1 r^{12-1}$

$t_{12} = 28 (4)^{11}$

$= 117{,}440{,}512$

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Part 3 – Geometric Series

Geometric Series

$$S_n = \frac{t_1(r^n - 1)}{r - 1}, r \neq 0$$

$$S_n = \frac{rt_n - t_1}{r - 1}, r \neq 0$$

$$S_\infty = \frac{t_1}{1 - r}, -1 < r < 1$$

t_1 is the first term

n is the number of terms

r is the common ratio

t_n is the general term or the nth term

S_n is the sum to n terms

Use the following information to answer Q13-Q14:

A Geometric Series is shown below:

×2   ×2

7 + 14 + 28 + 56...

Q13: Determine the common ratio.

$$r = \frac{t_n}{t_{n-1}} = \frac{t_4}{t_3} = \frac{56}{28} = 2$$

Q14: Determine the sum of the first 10 terms.

$$S_n = \frac{t_1(r^n - 1)}{r - 1} = \frac{7(2^{10} - 1)}{2 - 1}$$

$$S_{10} = 7161$$

Math 20-1

Geometric Series

$$S_n = \frac{t_1(r^n - 1)}{r - 1}, r \neq 0$$ t_1 is the first term

$$S_n = \frac{rt_n - t_1}{r - 1}, r \neq 0$$ n is the number of terms

$$S_\infty = \frac{t_1}{1 - r}, -1 < r < 1$$ r is the common ratio

t_n is the general term or the nth term

S_n is the sum to n terms

Use the following information to answer Q15-Q16:

A Geometric Series is shown below:

$$\overset{\times 5 \qquad \times 5}{\curvearrowright \quad \curvearrowright}$$
2, 10, 50, 250...

Q15: Determine the common ratio.

$$r = \frac{t_n}{t_{n-1}} = \frac{t_3}{t_2} = \frac{50}{10} = 5$$

Q16: Determine the sum of the first 10 terms.

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{2(5^{10} - 1)}{5 - 1} = 4,882,812$$

Key

Part 4 – Distinguishing between Arithmetic and Geometric Sequences and Series

Use the following information to answer Q17:

A sequence is shown below:

$$\overset{-2\quad -2\quad -2}{5, 3, 1, -1...}$$

Q17: Determine the value of the 10th term.

$$t_n = t_1 + (n-1)d$$
$$t_{10} = 5 + (10-1)(-2)$$
$$= -13$$

Use the following information to answer Q18:

A sequence is shown below:

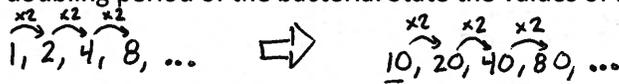$$\overset{\times(-2)\quad \cdot(-2)\quad \cdot(-2)}{-6, 12, -24, 48...}$$

Q18: Determine the value of the 10th term.

$$r = \frac{t_n}{t_{n-1}} = \frac{t_4}{t_3} = \frac{48}{-24} = -2$$

$$t_n = t_1 r^{n-1}$$
$$= (-6)(-2)^{10-1}$$
$$= 3072$$

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Part 5 – Textbook Examples

Pg 34, Example #1: In nature, many single-celled organisms, such as bacteria, reproduce by splitting in two so that one cell gives rise to 2, then 4, then 8 cells, and so on, producing a geometric sequence. Suppose there were 10 bacteria originally present in the bacteria sample. Determine the general term that relates the number of bacteria to the doubling period of the bacteria. State the values of t_1 and r in the geometric sequence produced.

$$\overset{\times 2}{\overset{\frown}{}}\ \overset{\times 2}{\overset{\frown}{}}\ \overset{\times 2}{\overset{\frown}{}}$$
$$1, 2, 4, 8, \dots \quad \Rightarrow \quad \overset{\times 2}{\overset{\frown}{}}\ \overset{\times 2}{\overset{\frown}{}}\ \overset{\times 2}{\overset{\frown}{}}$$
$$10, 20, 40, 80, \dots$$

Start with 10.

$$t_n = t_1 r^{n-1}$$
$$t_n = 10(2)^{n-1}$$

Pg 35, Example #2: Sometimes you use a photocopier to create enlargements or reductions. Suppose the actual length of a photograph is 25 cm and the smallest size that a copier can make is 67% of the original. What is the shortest possible length of the photograph after 5 reductions? Express your answer to the nearest tenth of a centimeter.

$$\overset{\times 0.67}{\overset{\frown}{}}\ \overset{\times 0.67}{\overset{\frown}{}}$$
$$25, 16.75, 11.2225, \text{ etc.}$$

Item #6
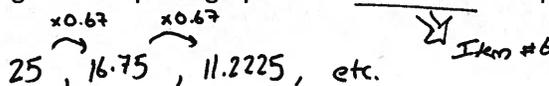
$$t_n = t_1 r^{n-1}$$
$$t_6 = 25(0.67)^{6-1}$$
$$= 3.4$$

So 3.4 cm.

Pg 52. Example #3: The Western Scrabble Network is an organization whose goal is to promote the game of Scrabble. It offers Internet tournaments throughout the year that WSN members participate in. The format of these tournaments is such that the losers of each round are eliminated from the next round. The winners continue to play until a final match determinist the champion. If there are 256 entries in an Internet Scrabble tournament, what is the total number of matches that will be played in the tournament?

256 players, so 128 initial matches.

$$128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 = 255 \text{ matches}$$

or $$S_n = \frac{r t_n - t_1}{r - 1} = \frac{(0.5)(1) - (128)}{(0.5) - 1} = \frac{-127.5}{-0.5} = 255$$

So $S_n = 255$ (even if we don't know what "n" is).