KEY

$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $t_n = t_1 r^{n-1}$ $S_n = \frac{t_1(r^n-1)}{r-1}, r \neq 0$ $S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$

L68 – Geometric Sequences and Series

Part 1 – Arithmetic Sequences

Use the following information to answer Q1-Q2:

An Arithmetic Sequence is shown below:

$$23 \xrightarrow{+4} 27 \xrightarrow{+4} 31 \xrightarrow{+4} 35 \ldots$$

$t_1 = 23$
$d = 4$

Q1: Determine the 20th term in the sequence.

$t_n = t_1 + (n-1)d$
$t_{20} = 23 + (20-1)(4)$
$= 99$

Q2: What term is the number 131?

$131 = t_1 + (n-1)d$
$131 = 23 + (n-1)(4)$
$131 = 23 + 4n - 4$
$112 = 4n$
$n = 28$

Part 2 – Arithmetic Series

Use the following information to answer Q3-4:

An Arithmetic Series is shown below:

$$-10 \xrightarrow{+6} -4 \xrightarrow{+6} +2 \xrightarrow{+6} +8 + \ldots$$

$t_1 = -10$
$d = 6$

Q3: What is the Common Difference?

$d = 6$

Q4: What is the sum of the first 20 terms?

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$
$S_{20} = \frac{20}{2}[2(-10) + (20-1)(6)]$
$= 10[-20 + 114]$
$= 940$

KEY

$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $t_n = t_1 r^{n-1}$ $S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $S_n = \frac{r t_n - t_1}{r-1}, r \neq 0$

Part 3 – Geometric Sequences

Use the following information to answer Q5-Q6:

A Geometric Sequence is shown below: $\cdot -2 \quad \cdot -2 \quad \cdot -2$ 5, -10, 20, -40... $t_1 = 5$, $r = -2$

Q5: Determine the 20th term in the sequence.

$$t_n = t_1 r^{n-1}$$
$$t_{20} = (5)(-2)^{20-1}$$
$$= (5)(-524,288)$$
$$= -2,621,440$$

Q6: What term is the number 81,920?

$$81,920 = (5)r^{n-1}$$
$$16,384 = r^{n-1}$$
$$16,384 = (-2)^{n-1}$$

Through guess+check, $(-2)^{14} = 16,384$

So $n-1 = 14$

$n = 15$ $t_{15} = 81,920$

Use the following information to answer Q7-Q8:

A Geometric Sequence is shown below: $\cdot 3 \quad \cdot 3 \quad \cdot 3$ -5, -15, -45, -135...

Q7: What is the Common Ratio?

$$r = \frac{t_3}{t_2} = \frac{-45}{-15} = 3$$

Q8: What is the value of the 18th term in the sequence?

$$t_n = t_1 r^{n-1}$$
$$t_{18} = (-5)(3)^{18-1}$$
$$= -645,700,815$$

KEY

$t_n = t_1 + (n-1)d$ $\qquad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\qquad t_n = t_1 r^{n-1}$ $\qquad S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\qquad S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$

Part 4 – Geometric Series

Use the following information to answer Q9-10:

A Geometric Series is shown below:
$\cdot\frac{1}{2} \quad \cdot\frac{1}{2} \quad \cdot\frac{1}{2}$ 20 + 10 + 5 + 2.5 + ...

Q9: What is the Common Ratio?

$$r = \frac{t_2}{t_1} = \frac{10}{20} = \frac{1}{2} \text{ or } 0.5$$

Q10: What is the sum of the first 20 terms?

$$S_n = \frac{t_1(r^n - 1)}{r-1}$$

$$S_{20} = \frac{20(0.5^{20} - 1)}{0.5 - 1}$$

$$= \frac{20(-0.999999046326....)}{-0.5}$$

$S_{20} = 39.999961853...$

$S_{20} \approx 40$

Use the following information to answer Q11-12:

A Geometric Series is shown below:
8 + 12.8 + 20.48 + 32.768 + ...

Q11: What is the Common Ratio?

$$r = \frac{t_3}{t_2} = \frac{20.48}{12.8} = 1.6$$

Q12: What is the sum of the first 15 terms?

$$S_n = \frac{t_1(r^n - 1)}{r-1}$$

$$S_{15} = \frac{8(1.6^{15} - 1)}{1.6 - 1}$$

$$= \frac{8(1151.92150461...)}{0.6}$$

$$= 15,358.953348$$

$$\approx 15,359$$

KEY

$t_n = t_1 + (n-1)d$ $\quad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\quad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\quad t_n = t_1 r^{n-1}$ $\quad S_n = \frac{t_1(r^n-1)}{r-1}, r \neq 0$ $\quad S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$

Part 5 – Mix of Questions

Use the following information to answer Q13-14:

In a sequence, the 5th term is 200 and the 7th term is 450.

Q13: If this was an Arithmetic Sequence, what is the common difference?

(5,200) and (7, 450)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{450-200}{7-5}$

$m = 125$

$d = 125$

$t_5 = t_1 + (5-1)d \Rightarrow 200 = t_1 + 4d$

$t_7 = t_1 + (7-1)d \Rightarrow -(450 = t_1 + 6d)$

$-250 = -2d$

$d = 125$

Q14: If this was a Geometric Sequence, what is the common ratio?

$t_5 \times r = t_6$

$t_6 \times r = t_7$

So $t_5 \times r^2 = t_7$

$(200)(r^2) = 450$

$r^2 = 2.25$

$r = 1.5$

$t_5 = t_1 r^{5-1}$ $\quad t_7 = t_1 r^{7-1}$

$\frac{450}{200} = \frac{t_1 r^6}{t_1 r^4}$

$2.25 = r^2$

$r = 1.5$

Use the following information to answer Q15:

A Series is shown below:

-1/4 -1/8 Not Arithmetic

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

×1/2 ×1/2 Same.. So Geometric

Q15: Determine the sum of the first 12 terms.

Arithmetic or Geometric?

Geometric because same common ratio (not same common difference)

$S_n = \frac{t_1(r^n - 1)}{r-1}$

$S_{12} = \frac{(\frac{1}{2})([1/2]^{12} - 1)}{1/2 - 1}$

$= \frac{-0.499877929688}{-0.5}$

$= 0.999755859376$

$S_{12} \approx 1$

Key

$t_n = t_1 + (n-1)d$ $\qquad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\qquad t_n = t_1 r^{n-1}$ $\qquad S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\qquad S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$

Part 6 – Cumulative Review

L08, Pg 348 #8: The sum of two numbers is 25. The sum of their reciprocals is $\frac{1}{4}$. Determine the two numbers.

Let m and n be our numbers.

$m + n = 25$

$n = 25 - m$

$\frac{1}{m} + \frac{1}{n} = \frac{1}{4} \implies \frac{1}{m} + \frac{1}{25-m} = \frac{1}{4}$

$\frac{1}{m}\left(\frac{25-m}{25-m}\right)\left(\frac{4}{4}\right) + \frac{1}{25-m}\left(\frac{4m}{4m}\right) = \frac{1}{4}\left(\frac{m}{m}\right)\left(\frac{25-m}{25-m}\right)$

$\frac{100-4m}{4m(25-m)} + \frac{4m}{4m(25+m)} = \frac{25m-m^2}{4m(25-m)}$

$(100-4m) + (4m) = (25m - m^2)$

$m^2 - 25m + 100 = 0$

$(m-5)(m-20) = 0$

$m = 5 \qquad m = 20$

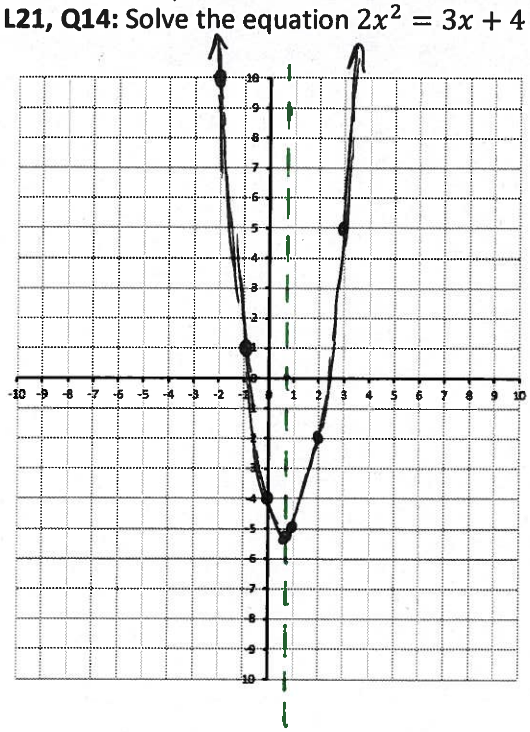
$m + n = 25$

If $m = 5$, then $n = 20$.

If $m = 20$, then $n = 5$

Our two numbers are 5 and 25.

L21, Q14: Solve the equation $2x^2 = 3x + 4$ graphically. *(3 marks)*



$2x^2 - 3x - 4 = 0$

Graph $y = 2x^2 - 3x - 4$ and look for zeroes.

Vertex at $(0.75, -5.13)$

y-intercept $y = -4$

Axis of symmetry $x = 0.75$

x	y
-2	10
-1	1
0	-4
1	-5
2	-2
3	5
4	16

$\boxed{x \approx -0.9, 2.4}$

KEY

$t_n = t_1 + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\quad$ $S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\quad$ $t_n = t_1 r^{n-1}$ $\quad$ $S_n = \frac{t_1(r^n-1)}{r-1}, r \neq 0$ $\quad$ $S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$

L24&25, Q6: The equation $f(x) = x^2 + 2x - 8$ can be written in several different forms:

Standard Form	*Vertex Form*	*Factored Form*
$f(x) = x^2+2x-8$	$f(x) = 1(x^2+2x) - 8$ $= 1(x^2 + kx + lx) - 8$ $= 1(x^2+1x+1x+1) - 8 - 1$ $f(x) = 1(x+1)^2 - 9$	$f(x) = (x+4)(x-2)$

L24&25, Q7: Using each of form of the equation above, complete the table.

Equation Form	Information	How did you find this information?
Standard Form $f(x) = x^2+2x-8$	*y-Intercept* $(0, -8)$	This is our c-value in $y = ax^2 + bx + c$
Vertex Form $f(x) = 1(x+1)^2 - 9$	*Axis of Symmetry* $x = -1$	Axis of symmetry goes through our vertex.
	Vertex Coordinate $(-1, -9)$	IF $f(x) = a(x-h)^2 + k$ Vertex is at (h, k)
Factored Form $f(x) = (x+4)(x-2)$	*Zeroes* $f(x) = (x+4)(x-2)$ $0 = (x+4)(x-2)$ $x = -4$ $\quad$ $x = 2$	When $y = 0$, determine the x-values.
	Axis of Symmetry $\frac{(-4)+(2)}{2} = -1$, so $x = -1$	Average the zeroes to find the halfway point.
	Vertex Coordinate $f(-1) = (-1+4)(-1-2)$ $= (3)(-3)$ $= -9$ so vertex at $(-1, -9)$	This occurs on the axis of symmetry.

KEY

$t_n = t_1 + (n-1)d$ $\quad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\quad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\quad t_n = t_1 r^{n-1}$ $\quad S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\quad S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$

Use the following information to answer L24&25, Q10-14:

$$f(x) = 3(x-1)(x+5)$$

L24&25, Q10: Determine the x-intercept(s) of this function.

x-intercepts (set y = 0)

$0 = 3(x-1)(x+5)$

$x - 1 = 0$ $\quad x + 5$

$x = 1$ $\quad x = -5$

L24&25, Q11: Using the x-intercepts, determine the coordinates of the vertex.

Vertex is halfway between zeroes (x-ints)

$\frac{(1) + (-5)}{2} = -2$ $\quad f(-2) = 3(-2-1)(-2+5)$

$= 3(-3)(3)$

$= -27$

Axis of symmetry at $x = -2$.

L24&25, Q12: Determine the y-intercept of this function.

y-intercept (set x = 0)

$f(0) = 3(0-1)(0+5)$

$= 3(-1)(5)$

$= -15$

y-intercept at $y = -15$

L24&25, Q13: Convert to Standard Form to confirm the y-intercept.

$f(x) = 3(x-1)(x+5)$

$= 3(x^2 + 4x - 5)$

$f(x) = 3x^2 + 12x - 15$

y-intercept

L24&25, Q14: Convert to Vertex Form to confirm the coordinates of the vertex.

$f(x) = 3x^2 + 12x - 15$

$= (3x^2 + 12x) - 15$

$= 3(x^2 + 4x) - 15$

$= 3(x^2 + 2x + 2x) - 15$

$= 3(x^2 + 2x + 2x + 4) - 15 - 12$

$f(x) = 3(x+2)^2 - 27$

Vertex at $(-2, -27)$

KEY

$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $t_n = t_1 r^{n-1}$ $S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$

Use the following information to answer L32, Q11:

Use the following equations for Q11:

$$f(x) = 2x^2 + 5x - 10$$

$$g(x) = -x^2 + 3x + 7$$

L32, Q11: Use *Elimination* to determine the ***exact*** solution to the system of equations.

$$\begin{array}{r} y = 2x^2 + 5x - 10 \\ -\left(y = -x^2 + 3x + 7\right) \\ \hline 0 = 3x^2 + 2x - 17 \end{array}$$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 4(3)(-17)}}{2(3)}$

$x = \frac{-2 \pm \sqrt{208}}{6} = \frac{-2 \pm 4\sqrt{13}}{6} = \frac{-1 \pm 2\sqrt{13}}{3}$

$x_1 = \frac{-1 + 2\sqrt{13}}{3}$ $x_2 = \frac{-1 - 2\sqrt{13}}{3}$

$g(x_1) = -(x_1)^2 + 3(x_1) + 7$

$= -\left(\frac{-1+2\sqrt{13}}{3}\right)\left(\frac{-1+2\sqrt{13}}{3}\right) + 3\left(\frac{-1+2\sqrt{13}}{3}\right) + 7$

$= -1\left(\frac{1 - 4\sqrt{13} + 4(13)}{9}\right) + (-1 + 2\sqrt{13}) + 7$

$= -\frac{1}{9}(53 - 4\sqrt{13}) + 2\sqrt{13} + 6$

$= \frac{-53}{9} + \frac{4}{9}\sqrt{13} + 2\sqrt{13} + 6$

$= \frac{1}{9} + \frac{22}{9}\sqrt{13}$

Soln is $\left(-\frac{1}{3} + \frac{2}{3}\sqrt{13}, \frac{1}{9} + \frac{22}{9}\sqrt{13}\right)$

$g(x_2) = -(x_2)^2 + 3(x_2) + 7$

$= -\left(\frac{-1-2\sqrt{13}}{3}\right)\left(\frac{-1-2\sqrt{13}}{3}\right) + 3\left(\frac{-1-2\sqrt{13}}{3}\right) + 7$

$= -1\left(\frac{1 + 4\sqrt{13} + 4(13)}{9}\right) + (-1 - 2\sqrt{13}) + 7$

$= -\frac{1}{9}(53 + 4\sqrt{13}) - 2\sqrt{13} + 6$

$= \frac{-53}{9} - \frac{4}{9}\sqrt{13} - 2\sqrt{13} + 6$

$= \frac{1}{9} - \frac{22}{9}\sqrt{13}$

Soln is $\left(-\frac{1}{3} - \frac{2}{3}\sqrt{13}, \frac{1}{9} - \frac{22}{9}\sqrt{13}\right)$

KEY

$t_n = t_1 + (n-1)d$ $\quad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\quad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\quad t_n = t_1 r^{n-1}$ $\quad S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\quad S_n = \frac{r t_n - t_1}{r-1}, r \neq 0$

Use the following information to answer L32, Q12

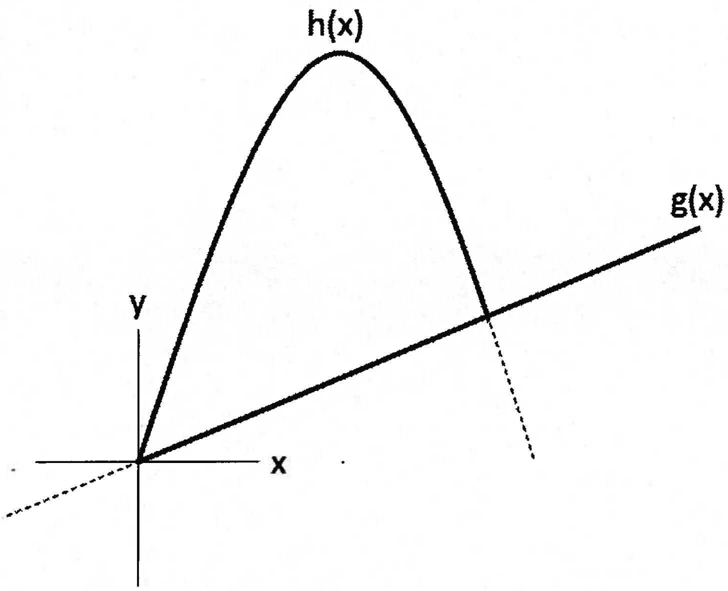
A kangaroo is jumping horizontally. The height as a function of horizontal distance, in meters, is:

$$h(x) = -x^2 + 5x$$

The kangaroo is jumping uphill. The height of the hill as a function of horizontal distance, in meters, is:

$$g(x) = \frac{1}{2}x$$

The quick sketch of the jump is as follows:



L32, Q12: Algebraically determine the horizontal distance travelled by the kangaroo.

$$\begin{array}{r} y = -x^2 + 5x + 0 \\ -\left(y = \frac{1}{2}x + 0\right) \\ \hline 0 = -x^2 + 4.5x + 0 \end{array}$$

$x^2 - 4.5x = 0$

$(x)(x - 4.5) = 0$

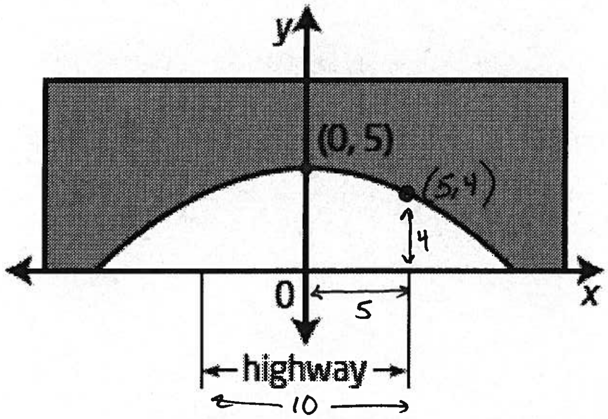
$x = 0$ $\quad x = 4.5 \Rightarrow$ Kangaroo travels a horizontal distance of 4.5m

KEY

$t_n = t_1 + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\quad$ $S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\quad$ $t_n = t_1 r^{n-1}$ $\quad$ $S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\quad$ $S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$

L37, Pg 496 #13: A highway goes under a bridge formed by a parabolic arch, as shown. The highest point of the arch is 5 m high. The road is 10 m wide, and the minimum height of the bridge over the road is 4 m.



a. Determine the quadratic function that models the parabolic arch of the bridge.
b. What is the inequality that represents the space under the bridge in quadrants I and II?

A

$y = a(x-h)^2 + k$

$y = a(x-0)^2 + 5$ $\quad$ Use (5,4)

$4 = a(5-0)^2 + 5$

$-1 = a(25)$

$-\frac{1}{25} = a$

$\boxed{y = -0.04x^2 + 5}$

B

$y < -0.04x^2 + 5$

or

$0 < y < -0.04x^2 + 5$