KEY

$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $t_n = t_1 r^{n-1}$ $S_n = \frac{t_1(r^n-1)}{r-1}, r \neq 0$ $S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

L69 – 1.5 Infinite Geometric Series

Part 1 – Key Ideas

Arithmetic Sequence and Series

Sequence:	2, 6, 10, 14…
Series:	2 + 6 + 10 + 14 +

$t_n = t_1 + (n-1)d$

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$

$S_n = \frac{n}{2}(t_1 + t_n)$

t_1 is the first term

n is the number of terms ($n \in N$)

d is the common difference

t_n is the general term or n^{th} term

S_n is the sum to n terms

Geometric Sequences and Series

Sequence:	3, 6, 12, 24…
Series:	3 + 6 + 12 + 24 +

$r = \frac{t_n}{t_{n-1}}$

$t_n = t_1 r^{n-1}$

$S_n = \frac{t_1(r^n-1)}{r-1}, r \neq 0$

$S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$

$S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

t_1 is the first term

n is the number of terms

r is the common ratio

t_n is the general term or the n^{th} term

S_n is the sum to n terms

Infinite Geometric Series:

- **Convergent Series:** A series with an infinite number of terms, in which the sequence of partial sums approaches a fixed value.
- **Divergent Series:** A series with an infinite number of terms, in which the sequence of partial sums does not approach a fixed value.

KEY

$t_n = t_1 + (n-1)d$ $\qquad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\qquad t_n = t_1 r^{n-1}$ $\qquad S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\qquad S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $\qquad S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

Part 2 – Convergent or Divergent Series?

Use the following information to answer Q1-Q2:

A Geometric Sequence is shown below:

$2 \xrightarrow{\cdot 2} 4 \xrightarrow{\cdot 2} 8 \xrightarrow{\cdot 2} 16 \xrightarrow{\cdot 2} 32 + \ldots$

$2 + 4 + 8 + 16 + 32 + \ldots$

$r = \frac{t_3}{t_2} = \frac{8}{4} = 2$

$t_1 = 2$

Q1: Determine the value of S_{10}, S_{20}, and S_{30}. Is this a convergent or divergent series?

$S_n = \frac{t_1(r^n - 1)}{r-1}$

$S_{10} = \frac{2(2^{10} - 1)}{2-1}$

$S_{10} = 2046$

$S_n = \frac{t_1(r^n - 1)}{r-1}$

$S_{20} = \frac{2(2^{20} - 1)}{2-1}$

$S_{20} = 2{,}097{,}150$

$S_n = \frac{t_1(r^n - 1)}{r-1}$

$S_{30} = \frac{2(2^{30} - 1)}{2-1}$

$S_{30} = 2{,}147{,}483{,}646$

Divergent

$r = 2$

Q2: Determine the value of S_∞.

$S_\infty = \infty$

(It just keeps getting larger)

Use the following information to answer Q3-Q4:

A Geometric Sequence is shown below:

$10 \xrightarrow{\cdot 0.5} 5 \xrightarrow{\cdot 0.5} 2.5 \xrightarrow{\cdot 0.5} 1.25 + 0.625 + \ldots$

$10 + 5 + 2.5 + 1.25 + 0.625 + \ldots$

$r = \frac{t_4}{t_3} = \frac{1.25}{2.5} = 0.5$

Q3: Determine the value of S_{10}, S_{20}, and S_{30}. Is this a convergent or divergent series?

$S_n = \frac{t_1(r^n - 1)}{r-1}$

$S_{10} = \frac{10(0.5^{10} - 1)}{0.5 - 1}$

$S_{10} = 19.98046875\ldots$

$S_n = \frac{t_1(r^n - 1)}{r-1}$

$S_{20} = \frac{10(0.5^{20} - 1)}{0.5 - 1}$

$S_{20} = 19.9999809265$

$S_n = \frac{t_1(r^n - 1)}{r-1}$

$S_{30} = \frac{10(0.5^{30} - 1)}{0.5 - 1}$

$S_{30} = 19.9999999814$

Convergent

$r = 0.5$

$-1 < r < 1$

Q4: Determine the value of S_∞.

$S_\infty = \frac{t_1}{1-r}$ if $-1 < r < 1$

$S_\infty = \frac{10}{1 - 0.5}$

$S_\infty = 20$

KEY

$t_n = t_1 + (n-1)d$ $\quad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\quad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\quad t_n = t_1 r^{n-1}$ $\quad S_n = \frac{t_1(r^n-1)}{r-1}, r \neq 0$ $\quad S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $\quad S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

Use the following information to answer Q5-Q6:

A Geometric Sequence is shown below:

$$1 \xrightarrow{\cdot(-3)} -3 \xrightarrow{\cdot(-3)} +9 \xrightarrow{\cdot(-3)} -27 + \ldots$$

$r = \frac{t_2}{t_1} = \frac{-3}{1} = -3$

$t_1 = 1$

Q5: Determine the value of S_{10}, S_{20}, and S_{30}. Is this a convergent or divergent series?

$S_n = \frac{t_1(r^n-1)}{r-1}$

$S_{10} = \frac{1[(-3)^{10}-1]}{(-3)-1}$

$S_{10} = -14,762$

$S_n = \frac{t_1(r^n-1)}{r-1}$

$S_{20} = \frac{1[(-3)^{20}-1]}{(-3)-1}$

$S_{20} = -871,696,100$

$S_n = \frac{t_1(r^n-1)}{r-1}$

$S_{30} = \frac{1[(-3)^{30}-1]}{(-3)-1}$

$S_{30} = -5.147\ldots \times 10^{13}$

Divergent

$r = -3$

Q6: Determine the value of S_∞.

N/A

Flip flops between positive and negative... but the absolute value (or magnitude) keeps increasing.

Use the following information to answer Q7-Q8:

A Geometric Sequence is shown below:

$$100 \xrightarrow{\cdot(-5)} -20 \xrightarrow{\cdot(-5)} +4 \xrightarrow{\cdot(-5)} -0.8 + \ldots$$

$r = \frac{t_3}{t_2} = \frac{4}{-20} = -0.2$

$t_1 = 100$

Q7: Determine the value of S_{10}, S_{20}, and S_{30}. Is this a convergent or divergent series?

$S_n = \frac{t_1(r^n-1)}{r-1}$

$S_{10} = \frac{100[(-0.2)^{10}-1]}{(-0.2)-1}$

$S_{10} = 83.3333248$

$S_n = \frac{t_1(r^n-1)}{r-1}$

$S_{20} = \frac{100[(-0.2)^{20}-1]}{(-0.2)-1}$

$S_{20} = 83.3333333333$

$S_n = \frac{t_1(r^n-1)}{r-1}$

$S_{30} = \frac{100[(-0.2)^{30}-1]}{(-0.2)-1}$

$S_{30} = 83.3333333333$

Convergent

$r = -0.2$

$-1 < r < 1$

Q8: Determine the value of S_∞.

$S_\infty = \frac{t_1}{1-r}$ if $-1 < r < 1$

$S_\infty = \frac{100}{1-[-0.2]}$

$S_\infty = 83.\overline{3}$

KEY

$t_n = t_1 + (n-1)d$ $\quad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\quad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\quad t_n = t_1 r^{n-1}$ $\quad S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\quad S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $\quad S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

Part 3 – Word Problems

Q9: As a Christmas gift, Sally receives a gift from her Aunt. On Christmas day she receives $200, and every day afterwards she receives half of the previous days amount. How much money does she receive in total?

$200 + 100 + 50 + 25 + \ldots$

$t_1 = 200$
$r = 0.5$

$S_\infty = \frac{t_1}{1-r}$

$S_\infty = \frac{200}{1-0.5}$

$S_\infty = \$400$ in total.

Part 4 – Quick Review of Arithmetic and Geometric Sequences and Series

Use the following information to answer Q10-11:

+4 +4 +4

Arithmetic Sequence:	2, 6, 10, 14...	$d = 4$
Arithmetic Series:	2 + 6 + 10 + 14 + ...	$t_1 = 2$

Q10: Given the *Arithmetic Sequence*, calculate the 8[th] term.

$t_n = t_1 + (n-1)d$

$t_8 = 2 + (8-1)(4)$

$t_8 = 30$

Q11: Given the *Arithmetic Sequence*, calculate the sum of the first 8 terms.

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$

$S_8 = \frac{8}{2}[2(2) + (8-1)(4)]$

$S_8 = 128$

Use the following information to answer Q12-13:

•2 •2 •2

Geometric Sequence:	3, 6, 12, 24...	$r = 2$
Geometric Series:	3 + 6 + 12 + 24 + ...	$t_1 = 3$

Q12: Given the *Geometric Sequence*, calculate the 8[th] term.

$t_n = t_1 r^{n-1}$

$t_8 = (3)(2)^{8-1}$

$t_8 = 384$

Q13: Given the *Geometric Series,* calculate the sum of the first 8 terms.

$S_n = \frac{t_1(r^n - 1)}{r-1}$

$S_8 = \frac{3(2^8 - 1)}{2-1}$

$S_8 = 765$