Math 20-1 KEY

L72 - Worksheet - Unit Review

Part 1: Entire to Mixed Radicals

Q1: For each entire radical, write it as a mixed radical.

$\sqrt{75x^3y^2}$

75 → 3, 25 → 5, 5

$\sqrt{3 \cdot 5^2 \cdot x^2 \cdot x \cdot y^2}$

$5xy\sqrt{3x}$

$\sqrt{8x^4y^3z^2}$

8 → 2, 4 → 2, 2

$\sqrt{2 \cdot 2^2 \cdot x^2 \cdot x^2 \cdot y^2 \cdot y \cdot z^2}$

$2 \cdot x \cdot x \cdot y \cdot z\sqrt{2y}$

$2x^2yz\sqrt{2y}$

Part 2: Non-Permissible Values and Restrictions

Q2: For each of the following radicals, state the ***restrictions*** on the variable x.

$\sqrt{2x+1}$

$2x+1 \geq 0$

$2x \geq -1$

$x \geq -\frac{1}{2}$

$\sqrt{3x-1}$

$3x-1 \geq 0$

$3x \geq 1$

$x \geq \frac{1}{3}$

Q3: For each of the following radicals, state the ***restrictions*** on the variable x

$\sqrt{2x+7}$

$2x+7 \geq 0$

$2x \geq -7$

$x \geq -\frac{7}{2}$

$\sqrt{5-3x}$

$5-3x \geq 0$

$-3x \geq -5$

$x \leq \frac{5}{3}$

KEY

Math 20-1

Q4: Given the expression $\sqrt{x^2-4}$, which of the following correctly describes the ***restrictions*** on x?

a. $x > 2$
b. $x \geq 2$
c. $x > 2$ or $x < -2$
d. $x \geq 2$ or $x \leq -2$

$x^2 - 4 \geq 0$
$x^2 \geq 4$

so $x \geq 2$ or $x \leq -2$

Part 2: Adding and Subtracting Radicals

Q5: Simplify radicals and combine like terms.

$2\sqrt{5} + 7\sqrt{5}$

$9\sqrt{5}$

$\sqrt{28} + \sqrt{63}$

28: 2, 14; 14: 2, 7

63: 3, 21; 21: 3, 7

$\sqrt{2^2 \cdot 7} + \sqrt{3^2 \cdot 7}$

$2\sqrt{7} + 3\sqrt{7}$

$5\sqrt{7}$

Q6: Simplify radicals and combine like terms.

$\sqrt{75x} - \sqrt{12x}$

75: 3, 25; 25: 5, 5

12: 3, 4; 4: 2, 2

$\sqrt{3 \cdot 5^2 \cdot x} - \sqrt{2^2 \cdot 3 \cdot x}$

$5\sqrt{3x} - 2\sqrt{3x}$

$3\sqrt{3x}$

$\sqrt{50} - \sqrt{27} + \sqrt{18} - \sqrt{3}$

50: 2, 25; 25: 5, 5

27: 3, 9; 9: 3, 3

18: 2, 9; 9: 3, 3

$\sqrt{2 \cdot 5^2} - \sqrt{3 \cdot 3^2} + \sqrt{2 \cdot 3^2} - \sqrt{3}$

$5\sqrt{2} - 3\sqrt{3} + 3\sqrt{2} - 1\sqrt{3}$

$8\sqrt{2} - 4\sqrt{3}$

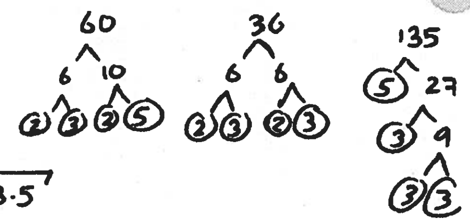
KEY

Math 20-1

Q7: The expression $\sqrt{60}+\sqrt{36}+\sqrt{135}$ simplifies to $a\sqrt{bc}+d$, where ***a, b, c,*** and ***d*** are ___, ___, ___, and ___.

(Record your four digit answer in the Numerical Response boxes below)

5	1	5	6



$\sqrt{2^2\cdot3\cdot5}+\sqrt{2^2\cdot3^2}+\sqrt{3^2\cdot3\cdot5}$

$2\sqrt{15}+6+3\sqrt{15}$

$5\sqrt{15}+6$

$a\sqrt{bc}+d$

$a=5$
$b=1$
$c=5$
$d=6$

Part 4: Rationalizing the Denominator

Q8: For each expression, rationalize the denominator.

$$\frac{\sqrt{5}}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}=\frac{\sqrt{35}}{7}$$

$$\frac{\sqrt{5}-1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{25}-1\sqrt{5}}{5}=\frac{5-\sqrt{5}}{5}$$

Q9: For each expression, rationalize the denominator.

$$\left(\frac{\sqrt{7}-3}{\sqrt{7}+2}\right)\cdot\left(\frac{\sqrt{7}-2}{\sqrt{7}-2}\right)=\frac{\sqrt{49}-2\sqrt{7}-3\sqrt{7}+6}{\sqrt{49}-2\sqrt{7}+2\sqrt{7}-4}$$

$$=\frac{7-2\sqrt{7}-3\sqrt{7}+6}{7-2\sqrt{7}+2\sqrt{7}-4}$$

$$=\frac{13-5\sqrt{7}}{3}$$

$$\left(\frac{\sqrt{3}-1}{1+\sqrt{6}}\right)\left(\frac{1-\sqrt{6}}{1-\sqrt{6}}\right)=\frac{1\sqrt{3}-\sqrt{18}-1+1\sqrt{6}}{1-1\sqrt{6}+1\sqrt{6}-\sqrt{36}}$$

$$=\frac{1\sqrt{3}-\sqrt{2\cdot3^2}-1+1\sqrt{6}}{1-1\sqrt{6}+1\sqrt{6}-6}$$

$$=\frac{\sqrt{3}-3\sqrt{2}-1+\sqrt{6}}{-5}$$

$$=\frac{-\sqrt{3}+3\sqrt{2}+1-\sqrt{6}}{5}$$

Q10: When rationalizing the denominator of the expression $\frac{\sqrt{5}+3}{\sqrt{5}-1}$, the expression can be simplified to $a+\sqrt{b}$, where a and b are ___ and ___.

(Record your two digit answer in the Numerical Response boxes below)

2	5		

$$\left(\frac{\sqrt{5}+3}{\sqrt{5}-1}\right)\left(\frac{\sqrt{5}+1}{\sqrt{5}+1}\right) = \frac{\sqrt{25}+1\sqrt{5}+3\sqrt{5}+3}{\sqrt{25}+1\sqrt{5}-1\sqrt{5}-1} = \frac{5+1\sqrt{5}+3\sqrt{5}+3}{5-1}$$

$$= \frac{8+4\sqrt{5}}{4} = 2+\sqrt{5}$$

$a+\sqrt{b}$

$a=2$
$b=5$

Part 5: Multiplying Radicals

Q11: Multiply. Simplify the products where possible.

$(3\sqrt{2})(4\sqrt{6}) = 12\sqrt{12}$

12
2 6
2 3

$12\sqrt{2^2 \cdot 3}$

$12(2)\sqrt{3}$

$24\sqrt{3}$

$(\sqrt{5}+1)(3-2\sqrt{2})$

$3\sqrt{5} - 2\sqrt{10} + 3 - 2\sqrt{2}$

Q12: Multiply. Simplify the products where possible.

$(2\sqrt{x}+5)(3\sqrt{x}-4)$

$6\sqrt{x^2} - 8\sqrt{x} + 15\sqrt{x} - 20$

$6x - 8\sqrt{x} + 15\sqrt{x} - 20$

$6x + 7\sqrt{x} - 20$

$(x\sqrt{x}-5)(4+2\sqrt{x})$

$4x\sqrt{x} + 2x\sqrt{x^2} - 20 - 10\sqrt{x}$

$4x\sqrt{x} + 2x(x) - 20 - 10\sqrt{x}$

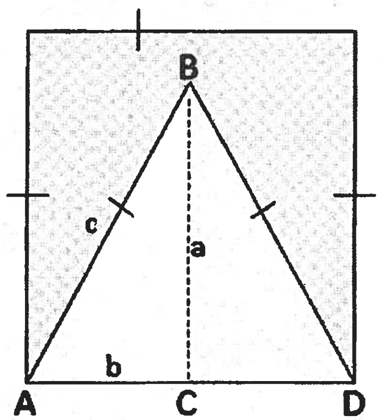
$4x\sqrt{x} + 2x^2 - 20 - 10\sqrt{x}$

KEY

Math 20-1

Use the following information to answer Q13-Q15:

An equilateral triangle, ΔABD, is placed inside a square.



$A_{square} = 40 \text{ cm}^2$

The area of the square is 40 cm².

Q13: The side length of the square, as a mixed radical, is $a\sqrt{bc}$, where ***a***, ***b***, and ***c*** are ___, ___, and ___.

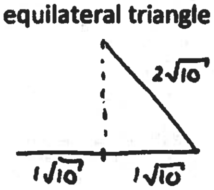
(Record your four digit answer in the Numerical Response boxes below)

2	1	0	

Area = (L)(w)
$40 = (x)(x)$
$40 = x^2$
$x = \sqrt{40} = \sqrt{2^2 \cdot 2 \cdot 5} = 2\sqrt{2 \cdot 5} = 2\sqrt{10}$

Q14: The height of the equilateral triangle is

a. $5\sqrt{2}$
b. $3\sqrt{10}$
c. $5\sqrt{10}$
d. $\sqrt{30}$



$a^2 + b^2 = c^2$
$(1\sqrt{10})^2 + b^2 = (2\sqrt{10})$
$10 + b^2 = 4(10)$
$b^2 = 30$
$b = \sqrt{30}$

Q15: The area of the equilateral triangle, ΔABD, is $ab\sqrt{c}$ cm², where ***a***, ***b***, and ***c*** are ___, ___, and ___.

(Record your three digit answer in the Numerical Response boxes below)

1	0	3	

$A = \frac{1}{2}bh$
$= \frac{1}{2}(2\sqrt{10})(\sqrt{30})$
$= 1\sqrt{300}$
$= \sqrt{2^2 \cdot 3 \cdot 5^2}$
$= 2 \cdot 5 \cdot \sqrt{3}$
$= 10\sqrt{3}$
$= ab\sqrt{c}$

$a = 1$
$b = 0$
$c = 3$

300
3 100
10 10
2 5 2 5

Math 20-1

Key

Part 6: Dividing Radicals

Q16: Simplify each expression.

$$\frac{4\sqrt{10}}{6\sqrt{5}} = \frac{4}{6}\sqrt{\frac{10}{5}} = \frac{2}{3}\sqrt{\frac{2}{1}}$$

$$= \frac{2\sqrt{2}}{3}$$

$$\frac{\sqrt{60}}{\sqrt{20}} = \sqrt{\frac{60}{20}} = \sqrt{3}$$

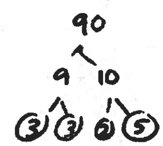
Q17: Simplify each expression using ***two different*** methods.

$$\frac{\sqrt{90}}{\sqrt{20}} = \sqrt{\frac{90}{20}} = \sqrt{\frac{9}{2}} = \frac{\sqrt{9}}{\sqrt{2}}$$

$$= \frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

$$\frac{\sqrt{90}}{\sqrt{20}} = \frac{\sqrt{2 \cdot 3^2 \cdot 5}}{\sqrt{2^2 \cdot 5}} = \frac{3\sqrt{10}}{2\sqrt{5}}$$

$$= \frac{3}{2}\sqrt{\frac{10}{5}} = \frac{3\sqrt{2}}{2}$$



Q18: The expression $\frac{\sqrt{150}}{\sqrt{12}}$ simplifies to $\frac{a\sqrt{b}}{c}$, where ***a, b,*** and ***c*** are ___, ___, and ___.

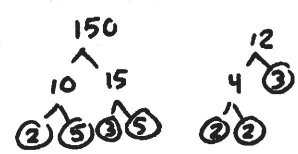
(Record your three digit answer in the Numerical Response boxes below)

5	2	2	

$$\frac{\sqrt{150}}{\sqrt{12}} = \frac{\sqrt{2 \cdot 3 \cdot 5^2}}{\sqrt{2^2 \cdot 3}} = \frac{5\sqrt{6}}{2\sqrt{3}}$$

$$= \frac{5}{2}\sqrt{\frac{6}{3}} = \frac{5}{2}\sqrt{\frac{2}{1}} = \frac{5\sqrt{2}}{2}$$

$$= \frac{a\sqrt{b}}{c}$$



a = 5
b = 2
c = 2

KEY

Part 7: Radical Equations (Factorable)

Q19: State the restrictions on *x* and solve:

$$\sqrt{11x+67} = x+7$$

Restrictions

$11x+67 \geq 0$

$11x \geq -67$

$x \geq \frac{-67}{11}$

$x \geq \approx -6.\overline{09}$

$11x+67 = (x+7)^2$

$11x+67 = (x+7)(x+7)$

$11x+67 = x^2+14x+49$

$0 = x^2+3x-18$

$0 = (x+6)(x-3)$

$x+6=0$, $x=-6$

$x-3=0$, $x=3$

$x = -6$ or 3

Both soln's verify.

$$\sqrt{5x+11} = x+3$$

$5x+11 = (x+3)^2$

$5x+11 = (x+3)(x+3)$

$5x+11 = x^2+6x+9$

$0 = x^2+x-2$

$0 = (x+2)(x-1)$

$x+2=0$, $x=-2$

$x-1=0$, $x=+1$

$x = -2$, or $+1$

Both soln's verify.

Restrictions

$5x+11 \geq 0$

$5x \geq -11$

$x \geq -11/5$

$x \geq -2.2$

Q20: State the restrictions on *x* and solve:

$$\sqrt{-2x+10} = x-1$$

Restrictions

$-2x+10 \geq 0$

$-2x \geq -10$

$x \leq 5$

$-2x+10 = (x-1)^2$

$-2x+10 = x^2-2x+1$

$0 = x^2+0x-9$

$0 = (x+3)(x-3)$

$x+3=0$, $x=-3$

$x-3=0$, $x=+3$

$x = -3$ or $+3$

Doesn't verify because positive roots only.

$\boxed{x=3}$

$$\sqrt{-3x+6} = x-2$$

$-3x+6 = (x-2)^2$

$-3x+6 = (x-2)(x-2)$

$-3x+6 = x^2-4x+4$

$0 = x^2-x-2$

$0 = (x-2)(x+1)$

$x-2=0$, $x=2$

$x+1=0$, $x=-1$

$x = -1$ or 2

Doesn't verify because only positive roots.

$\boxed{x=2}$

Restrictions

$-3x+6 \geq 0$

$-3x \geq -6$

$x \leq 2$

KEY

Math 20-1

Q21: State the restrictions on *x* and solve:

Restrictions
$-8x-8 \geq 0$
$-8x \geq 8$
$x \leq -1$

$\sqrt{-8x-8} = 2x+1$
$-8x-8 = (2x+1)^2$
$-8x-8 = 4x^2+4x+1$
$0 = 4x^2+12x+9$
$0 = (2x+3)(2x+3)$
$2x+3=0$
$2x=-3$
$x=-3/2$

$x=-3/2$
Doesn't verify because positive roots only.

$\boxed{\text{No solutions}}$

+6 +6
$\square+\square=12$
$\square\times\square=36$
1,36
2,18
3,12
4,9
6,6
$4x^2+6x+6x+9$
$(4x^2+6x)+(6x+9)$
$2x(2x+3)+3(2x+3)$
$(2x+3)(2x+3)$

$\sqrt{18x+3} = 3x+2$
$18x+3 = (3x+2)^2$
$18x+3 = 9x^2+12x+4$
$0 = 9x^2-6x+1$
$0 = (3x-1)(3x-1)$
$3x-1=0$
$3x=1$
$x=1/3$

$x=1/3$
soln verifies.

Restrictions
$18x+3 \geq 0$
$18x \geq -3$
$x \geq -3/18$
$x \geq -1/6$

-3 -3
$\square+\square=-6$
$\square\times\square=9$
1, 3
$9x^2-3x-3x$
$3x(3x-1)-1($
$(3x-1)(3x-1$

Q22: State the restrictions on *x* and solve:

Restrictions
$x+3 \geq 0$
$x \geq -3$

$\sqrt{x+3} = 2x+3$
$x+3 = (2x+3)^2$
$x+3 = (2x+3)(2x+3)$
$x+3 = 4x^2+12x+9$
$0 = 4x^2+11x+6$
$0 = (4x+3)(x+2)$
$4x+3=0$, $4x=-3$, $x=-3/4$
$x+2=0$, $x=-2$

$x=-2$ or $-3/4$
Doesn't verify because positive roots only

$\boxed{x=-3/4}$

+3 +8
$\square+\square=11$
$\square\times\square=24$
1,24
2,12
3,8
4,6
$4x^2+3x+8x+6$
$(4x^2+3x)+(8x+6)$
$x(4x+3)+2(4x+3)$
$(4x+3)(x+2)$

$\sqrt{-15x-1} = 3x-1$
$-15x-1 = (3x-1)^2$
$-15x-1 = (3x-1)(3x-1)$
$-15x-1 = 9x^2-6x+1$
$0 = 9x^2+9x+2$
$0 = (3x+1)(3x+2)$
$3x+1=0$, $3x=-1$, $x=-1/3$
$3x+2=0$, $3x=-2$, $x=-2/3$

$x=-2/3$ or $-1/3$
Neither solution verifies because positive roots only.

$\boxed{\text{No solution.}}$

Restrictions
$-15x-1 \geq 0$
$-15x \geq 1$
$x \leq -1/15$

+3 +6
$\square+\square=9$
$\square\times\square=18$
1,
2,
3,
$9x^2+3x+6x$
$(9x^2+3x)+(6$
$3x(3x+1)+2($
$(3x+1)(3x+2$

KEY

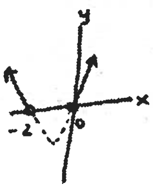
Math 20-1

Q23: State the restrictions on x and solve:

Restrictions

$x^2+2x \geq 0$

$x(x+2) \geq 0$



$x \leq -2$

$x \geq 0$

You'll learn this in the quadratics unit.

$\sqrt{x^2+2x} = 2x+4$

$x^2+2x = (2x+4)^2$

$x^2+2x = (2x+4)(2x+4)$

$x^2+2x = 4x^2+16x+16$

$0 = 3x^2+14x+16$

$0 = (x+2)(3x+8)$

$x+2=0$, $x=-2$

$3x+8=0$, $3x=-8$, $x=-8/3$

$x = -2$ or $-8/3$

Doesn't verify because positive roots only.

$\boxed{x=-2}$

$+6 \quad +8$

$\square + \square = 14$

$\square \times \square = 48$

1, 48
2, 24
3, 16
4, 12
6, 8

$3x^2+6x+8x+16$

$(3x^2+6x)+(8x+16)$

$3x(x+2)+8(x+2)$

$(x+2)(3x+8)$

$\sqrt{x^2} = 2x+2$

Restrictions

$x^2 \geq 0$

No restrictions

$x \in \mathbb{R}$

$x^2 = (2x+2)^2$

$x^2 = (2x+2)(2x+2)$

$x^2 = 4x^2+8x+4$

$0 = 3x^2+8x+4$

$0 = (3x+2)(x+2)$

$3x+2=0$, $3x=-2$, $x=-2/3$

$x+2=0$, $x=-2$

$x = -2$ or $-2/3$

Doesn't verify because positive roots only.

$\boxed{x=-2/3}$

$+2 \quad +6$

$\square + \square = 8$

$\square \times \square = 12$

1, 1
2,
3,

$3x^2+2x+6x+4$

$(3x^2+2x)+(6x+4)$

$x(3x+2)+2(3x+2)$

$(3x+2)(x+2)$

Q24: The expression $\sqrt{-12x-2} = 2x-1$ simplifies to $(ax+b)(cx+d)=0$, where the values of ***a***, ***b***, ***c***, and ***d*** are ___, ___, ___, and ___.

(Record your four digit answer in the Numerical Response boxes below)

2	1	2	3

Restrictions

$-12x-2 \geq 0$

$-12x \geq 2$

$x \leq -2/12$

$x \leq -1/6$

Can also be

2	3	2	1

$\sqrt{-12x-2} = 2x-1$

$-12x-2 = (2x-1)^2$

$-12x-2 = (2x-1)(2x-1)$

$-12x-2 = 4x^2-4x+1$

$0 = 4x^2+8x+3$

$0 = (2x+1)(2x+3)$

$(ax+b)(cx+d)$

$+2 \quad +6$

$\square + \square = 8$

$\square \times \square = 12$

1, 12
2, 6
3, 4

$4x^2+2x+6x+3$

$(4x^2+2x)+(6x+3)$

$2x(2x+1)+3(2x+1)$

$(2x+1)(2x+3)$

KEY

Part 8: Word Problems

Use the following information to answer Q25:

When analyzing car crashes, police use a formula that relates the stopping distance and the initial velocity of the car prior to braking:

$$v_i = \sqrt{2\mu_k g d}$$

Where v_i is the initial velocity μ_k is the coefficient of kinetic friction, g is the gravitational field strength, and d is the stopping distance.

Additionally, the kinematics equation $d = \left(\frac{v_i+v_f}{2}\right)t$, when modelling a braking object coming to a stop, can simplify to

$$v_i = \frac{2d}{t}$$

These two equations can be set equal to each other to solve for stopping distance:

$$\sqrt{2\mu_k g d} = \frac{2d}{t}$$

Q25: A car is travelling at a moderate velocity on wet concrete ($\mu_k = 0.60$) on Earth ($g = 9.81$) before it slams on its brakes, coming to a stop after 3.20 seconds ($t = 3.20$). Determine the distance it takes the car to stop.

$\mu_k = 0.60$
$g = 9.81$
$t = 3.20$

$$\sqrt{2\mu_k g d} = \frac{2d}{t}$$

$$\sqrt{2(0.60)(9.81)d} = \frac{2d}{3.20}$$

Square both sides.

$$2(0.60)(9.81)d = \frac{4d^2}{(3.20)^2}$$

$$2(0.60)(9.81)(d)(3.20)^2 = 4d^2$$

$$120.54528\,d = 4d^2$$

$$0 = 4d^2 - 120.54528d$$

$$0 = d(4d - 120.54528)$$

$d = 0$

This answer doesn't make sense.

$4d - 120.54528 = 0$
$4d = 120.54528$
$d = 30.13632\text{m}$

It takes the car ≈ 30.1m to stop.

Math 20-1

KEY

Part 9: Radical Equations (Quadratic Equation)

Q26: State the restrictions on *x* and solve:

$$\sqrt{3x+\frac{23}{2}} = x+\frac{1}{2}$$

Restrictions

$3x+\frac{23}{2} \geq 0$

$3x \geq -\frac{23}{2}$

$x \geq \frac{-23}{6}$

$x \geq -3.8\bar{3}$

$3x+\frac{23}{2} = (x+\frac{1}{2})^2$

$3x+\frac{23}{2} = (x+\frac{1}{2})(x+\frac{1}{2})$

$3x+\frac{23}{2} = x^2+x+\frac{1}{4}$

$0 = x^2-2x-\frac{45}{4}$

$0 = x^2-2x-11.25$

$0 = ax^2+bx+c$

$a=1$
$b=-2$
$c=-11.25$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(-11.25)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4+45}}{2} = \frac{2 \pm \sqrt{49}}{2}$

$x_1 = \frac{2+\sqrt{49}}{2} = \frac{2+7}{2} = \frac{9}{2}$

$x_2 = \frac{2-\sqrt{49}}{2} = \frac{2-7}{2} = \frac{-5}{2}$

Doesn't verify because positive roots only.

$x = \frac{9}{2}$

$\frac{1}{4} - \frac{23}{2}$

$\frac{1}{4} - \frac{46}{4}$

$\frac{-45}{4}$

Q27: State the restrictions on *x* and solve:

$$\sqrt{x+\frac{7}{9}} = x+1$$

Restrictions

$x+\frac{7}{9} \geq 0$

$x \geq -\frac{7}{9}$

$x+\frac{7}{9} = (x+1)^2$

$x+\frac{7}{9} = (x+1)(x+1)$

$x+\frac{7}{9} = x^2+2x+1$

$0 = x^2+x+\frac{2}{9}$

$0 = ax^2+bx+c$

$a=1$
$b=1$
$c=\frac{2}{9} \approx 0.\bar{2}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{1^2-4(1)(0.\bar{2})}}{2(1)}$

$x = \frac{-1 \pm \sqrt{1 - 0.\bar{8}}}{2} = \frac{-1 \pm \sqrt{0.\bar{1}}}{2}$

$x_1 = \frac{-1+\sqrt{0.\bar{1}}}{2} = \frac{-1+0.\bar{3}}{2} = -0.\bar{3}$ or $-\frac{1}{3}$

$x_2 = \frac{-1-\sqrt{0.\bar{1}}}{2} = \frac{-1-0.\bar{3}}{2} = -0.\bar{6}$ or $-\frac{2}{3}$

$x = -\frac{2}{3}$ or $\frac{-1}{3}$

Both solutions verify

$1 - \frac{7}{9}$

$\frac{9}{9} - \frac{7}{9}$

$\frac{2}{9}$

KEY

$t_n = t_1 + (n-1)d$ $\quad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\quad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\quad t_n = t_1 r^{n-1}$ $\quad S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\quad S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $\quad S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

Part 10 – Arithmetic Sequences and Series (Easy)

Use the following information to answer Q28 - Q30:

A sequence and series are shown below:		$d = 1.6$ $t_1 = 5.2$
Arithmetic Sequence:	5.2, 6.8, 8.4, 10, … ($+1.6$, $+1.6$)	
Arithmetic Series:	5.2 + 6.8 + 8.4 + 10 + …	

Q28: What is the value of the fifth term, to the nearest tenth?

(Record your answer in the Numerical Response boxes below) $\quad 10 + 1.6 = 11.6$

1	1	.	6

Q29: The value of the 12th term is $t_{12} = ab.c$, where ***a, b,*** and ***c*** are ____, ____, and ____.

*(Record your **three-digit** answer in the Numerical Response boxes below)*

2	2	8	

$t_n = t_1 + (n-1)d$

$t_{12} = 5.2 + (12-1)(1.6)$

$t_{12} = 22.8$

Q30: The sum of the first 15 terms is

a. 27.6

b. 74.4

c. 246

d. 570

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$

$S_{15} = \frac{15}{2}[2(5.2) + (15-1)(1.6)]$

$S_{15} = 246$

KEY

$t_n = t_1 + (n-1)d$ $\qquad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\qquad t_n = t_1 r^{n-1}$ $\qquad S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\qquad S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $\qquad S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

Part 11 – Geometric Sequences and Series (Easy)

Use the following information to answer Q31 - Q33 :

A sequence and series are shown below:

Geometric Sequence: 40, 60, 90, 135, ...

Geometric Series: 40 + 60 + 90 + 135 + ...

$r = \frac{t_2}{t_1} = \frac{60}{40} = 1.5$

Q31: What is the value of the fifth term?

a. 155
b. 165
c. 180
d. 202.5

$135 \times 1.5 = 202.5$

Q32: Determine the value of the 12th term, rounded to the nearest whole number.

(Record your three-digit answer in the Numerical Response boxes below)

3	4	6	0

$t_n = t_1 r^{n-1}$

$t_{12} = (40)(1.5)^{12-1}$

$t_{12} = 3459.9023...$

$t_{12} \simeq 3460$

Q33: Determine the sum of the first 8 terms, rounded to the nearest whole number.

(Record your three-digit answer in the Numerical Response boxes below)

1	9	7	0

$S_n = \frac{t_1(r^n - 1)}{r-1}$

$S_8 = \frac{40(1.5^8 - 1)}{1.5 - 1}$

$S_8 = \frac{985.15625}{0.5}$

$S_8 = 1970.3125$

$S_8 \approx 1970$

KEY

$t_n = t_1 + (n-1)d$ $\qquad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\qquad t_n = t_1 r^{n-1}$ $\qquad S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\qquad S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $\qquad S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

Part 12 – Infinite Geometric Series (Easy)

Use the following information to answer Q34-Q36:

Several Series are listed below:

Series A:	5 + 10 + 20 + 40 + ...	$r = 2$
Series B:	20 + 10 + 5 + 2.5 + ...	$r = 0.5$
Series C:	81 – 54 + 36 – 24 + ...	$r = -2/3$
Series D:	10 + 10.5 + 11 + 11.5 + ...	$d = 0.5$ ⇨ Actually an Arithmetic Series.

Q34: How many of the series listed above are converging series?

a. 1
b. 2
c. 3
d. 4

Converging if $-1 < r < 1$
so just Series B and Series C

Q35: Determine the sum of ***Series B***.

*(Record your **three-digit** answer in the Numerical Response boxes below)*

4	0	.	0

$t_1 = 20$
$r = 0.5$

$S_\infty = \frac{t_1}{1-r}$

$S_\infty = \frac{20}{1-0.5}$

$S_\infty = 40$

Q36: Determine the sum of ***Series C.***

*(Record your **three-digit** answer in the Numerical Response boxes below)*

4	8	.	6

$t_1 = 81$
$r = -2/3$

$S_\infty = \frac{t_1}{1-r}$

$S_\infty = \frac{81}{1-(-2/3)}$

$S_\infty = 48.6$

KEY

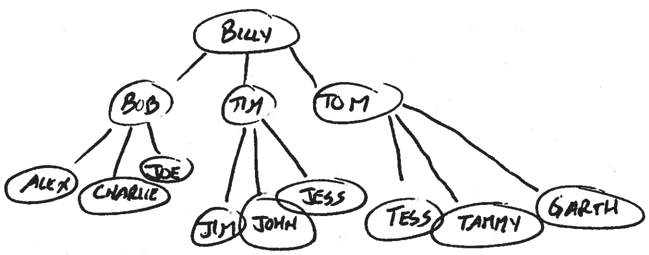
$t_n = t_1 + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\quad$ $S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\quad$ $t_n = t_1 r^{n-1}$ $\quad$ $S_n = \frac{t_1(r^n-1)}{r-1}, r \neq 0$ $\quad$ $S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $\quad$ $S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

Part 13 – Word Problems (Medium)

Q37: Billy decides to do 3 good deeds. Every recipient of one of his good deeds decides to "pay it forward" and do 3 good deeds of their own.

a. Build a Series showing the first 4 terms, in the form $S_n = t_1 + t_2 + t_3 + t_4 + \cdots$
b. Is this an Arithmetic or Geometric Series? Explain.
c. Sally is the 10th person in the chain to commit 3 good deeds. Once she completes her 3 good deeds, how many good deeds have been done, in total?



To help visualize
(but not necessary)

(A) Good deeds:

$S_n = 3 + 9 + 27 + 81 + \ldots$

(B) Geometric Series. The number of good deeds done triples at every level.

(C) $t_1 = 3$
$r = 3$

$S_n = \frac{t_1(r^n - 1)}{r-1}$

$S_{10} = \frac{3(3^{10}-1)}{3-1}$

$S_{10} = 88,572$

$\therefore$ 88,572 good deeds done in total.

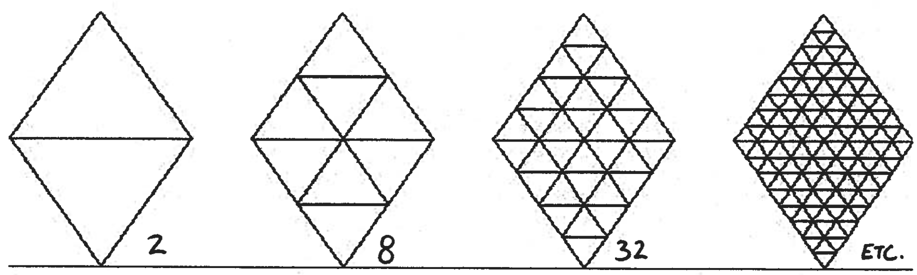
$t_n = t_1 + (n-1)d$ $\qquad$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad$ $S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\qquad$ $t_n = t_1 r^{n-1}$ $\qquad$ $S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\qquad$ $S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $\qquad$ $S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

Use the following information to answer Q 38 :

A ***fractal*** is a curve or geometric figure, each part of which has the same statistical character as the whole. Fractals are useful in modeling structures (such as eroded coastlines or snowflakes) in which similar patterns recur at progressively smaller scales, and in describing partly random or chaotic phenomena such as crystal growth, fluid turbulence, and galaxy formation.

A fractal pattern is depicted below:



Q38: How many ***tiny*** triangles are required to make the 8th shape in the sequence?

$t_1 = 2$
$r = 4$

$t_n = t_1 r^{n-1}$
$t_8 = (2)(4)^{8-1}$
$t_8 = 32,768$

So 32,768 tiny triangles required.

Use the following information to answer Q39:

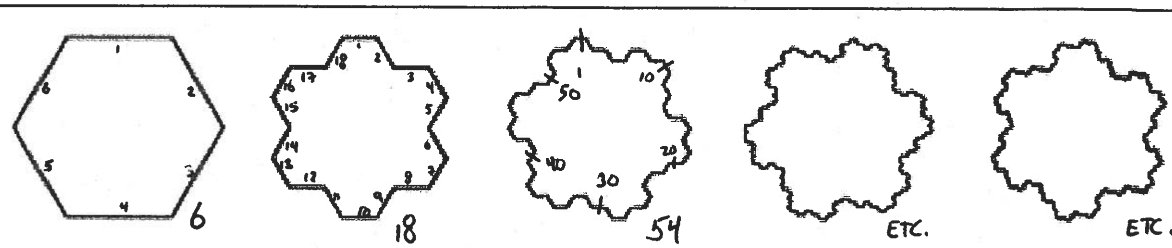


Figure 1 $\quad$ *Figure 2* $\quad$ *Figure 3* $\quad$ *Figure 4* $\quad$ *Figure 5*

Q39: In the fractal pattern shown above, how many sides does *Figure 10* have?

$t_1 = 6$
$r = 3$

$t_n = t_1 r^{n-1}$
$t_{10} = (6)(3)^{10-1}$
$t_{10} = 118,098$

Figure 10 would have 118,098 sides.

KEY

$t_n = t_1 + (n-1)d$ $\quad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\quad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\quad t_n = t_1 r^{n-1}$ $\quad S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\quad S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $\quad S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

Part 7 – Arithmetic and Geometric Sequences and Series (Hard)

Use the following information to answer Q40 - Q42:

$\frac{5}{8} - \frac{5}{12} + \frac{5}{18} - \frac{5}{27} + \cdots$ $\quad r = -2/3$

Q40: Which best describes the information in the textbox above?

a. Arithmetic Sequence
b. Arithmetic Series
c. Geometric Sequence
d. Geometric Series (Diverging)
e. Geometric Series (Converging) $\quad -1 < r < 1$
f. None of the above

Q41: The value of the 5th term is $t_5 = \frac{ab}{cd}$, where ***a, b, c,*** and ***d*** are ___, ___, ___, and ___.

*(Record your **four-digit** answer in the Numerical Response boxes below)*

1	0	8	1

$\frac{-5}{27} \cdot \frac{-2}{3} = \frac{10}{81}$

Q42: The sum of the first 8 terms is approximately ***a.bc***, where ***a, b,*** and ***c*** are ___, ___, and ___.

*(Record your **three-digit** answer in the Numerical Response boxes below)*

0	3	6	

$r = -2/3$

$t_1 = 5/8$

$S_n = \frac{t_1(r^n - 1)}{r-1}$

$S_8 = \frac{(5/8)[(-2/3)^8 - 1]}{(-2/3) - 1}$

$S_8 = \frac{-0.600613473556}{-1.\bar{6}}$

$S_8 = 0.360368084133$

$S_8 \approx 0.36$

KEY

$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $t_n = t_1 r^{n-1}$ $S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

Use the following information to answer Q43-Q44:

The fourth term in a sequence is 192. The seventh term is 1029.

Q43: If the sequence is an ***Arithmetic Sequence***, determine the sum of the first 10 terms.

OPTION #1

(4,192) and (7,1029)

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1029-192}{7-4} = 279$

OPTION #2

$t_n = t_1 + (n-1)d \rightarrow 192 = t_1 + (4-1)d$

$\rightarrow 1029 = t_1 + (7-1)d$

$192 = t_1 + 3d$

$-(1029 = t_1 + 6d)$

$-837 = -3d$

$d = 279$

$t_n = t_1 + (n-1)d$ Use $t_4 = 192$

$192 = t_1 + (4-1)(279)$

$192 = t_1 + 837$

$t_1 = -645$

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$

$S_{10} = \frac{10}{2}[2(-645) + (10-1)(279)]$

$\boxed{S_{10} = 6105}$

Q44: If the sequence is a ***Geometric Sequence***, determine the sum of the first 10 terms.

$t_n = t_1 r^{n-1}$

$192 = t_1 r^{4-1}$ and $1029 = t_1 r^{7-1}$

$\frac{1029}{192} = \frac{t_1 r^6}{t_1 r^3}$

$5.359375 = r^3$

$r = 1.75$

$t_n = t_1 r^{n-1}$

$192 = t_1 (1.75)^{4-1}$

$192 = t_1 (5.359375)$

$t_1 = 35.8250728863$

$S_n = \frac{t_1(r^n - 1)}{r-1}$

$S_{10} = \frac{(35.8250728863)(1.75^{10} - 1)}{1.75 - 1}$

$S_{10} = \frac{9615.06945837}{0.75}$

$S_{10} \approx 12820$

KEY

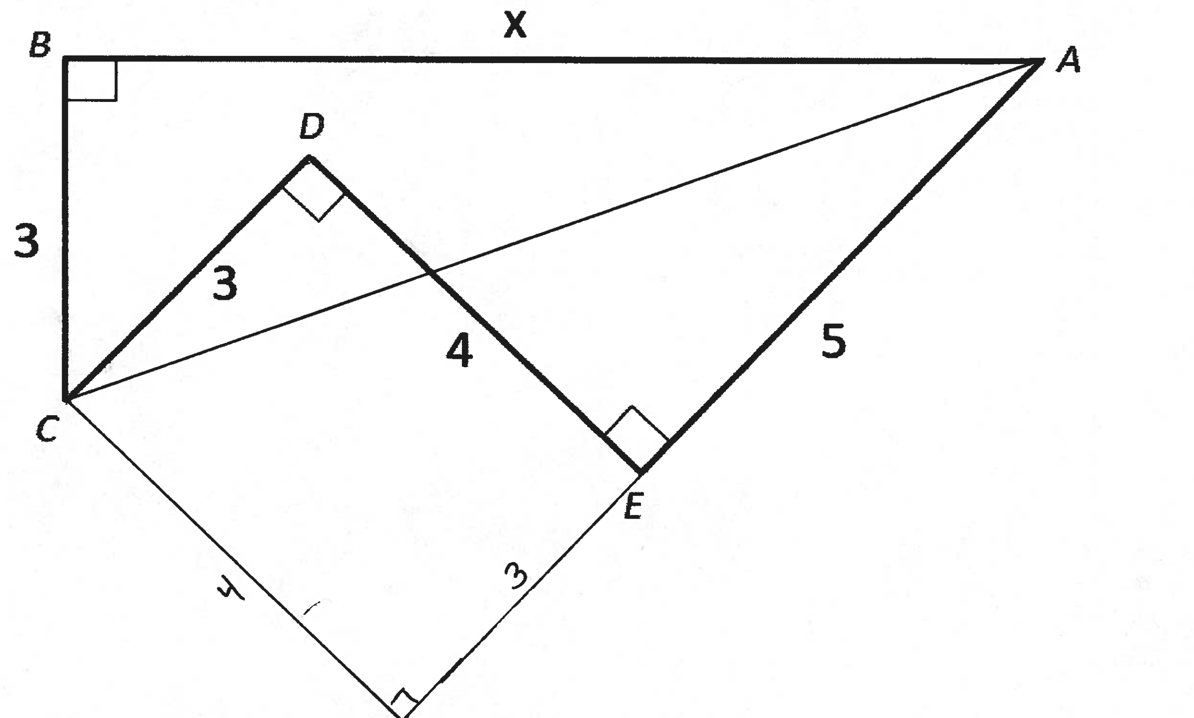
$t_n = t_1 + (n-1)d$ $\qquad$ $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad$ $S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\qquad$ $t_n = t_1 r^{n-1}$ $\qquad$ $S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\qquad$ $S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $\qquad$ $S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

Part 15: Challenge Questions (Very hard... especially the last one)

Use the following information to answer Q45:

Students will determine the length of side *AB*, conveniently labelled *x*.

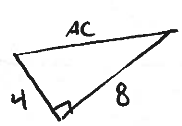


Original problem posted by "MindYourDecisions" on YouTube.
(https://www.youtube.com/watch?v=kfijuP5HDjU)

Q45: Students will determine the length of side *AB*.

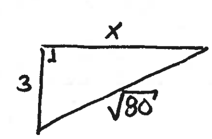
a. Draw line *AC*.
b. Draw a triangle, with *AC* as its hypotenuse and one side passing through point *E*, and determine the length and height of the triangle.
c. Use Pythagoras Theorem to determine the length of *AC*.
d. Use triangle *ABC* to determine the length of side *AB*.

(c)



$a^2 + b^2 = c^2$
$4^2 + 8^2 = c^2$
$c = \sqrt{80}$

(D)



$a^2 + b^2 = c^2$
$x^2 + 3^2 = (\sqrt{80})^2$
$x^2 + 9 = 80$
$x^2 = 71$
$x = \sqrt{71}$

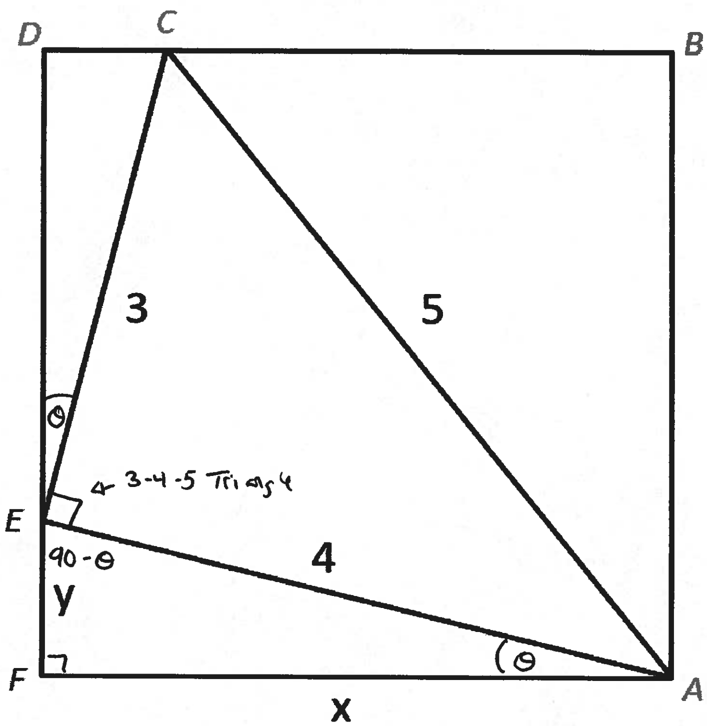
KEY

$t_n = t_1 + (n-1)d$ $\quad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\quad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\quad t_n = t_1 r^{n-1}$ $\quad S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\quad S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $\quad S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

Use the following information to answer Q46

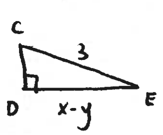
Determine the side length of the square, *AF*, conveniently labelled *x*.

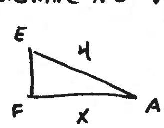


Original problem posted by "MindYourDecisions" on YouTube.
(https://www.youtube.com/watch?v=aQsb5jfqSYo)

Q46: Students will determine the length of the square, *AF*.

a. Prove that ∠EAF has the same value as ∠CED.
b. Determine the length of side DE in terms of *x* and *y*. $\quad DE = x - y$
c. Using similar triangles ΔEAF and ΔCED, write y in terms of x.
d. Use ΔEAF to determine the value of x.

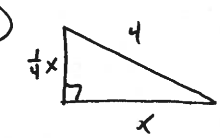




(c) $\frac{3}{x-y} = \frac{4}{x}$

$3x = 4(x-y)$
$3x = 4x - 4y$
$4y = x$
$y = \frac{1}{4}x$

(d)



$a^2 + b^2 = c^2$
$x^2 + (\frac{1}{4}x)^2 = 4^2$
$x^2 + \frac{1}{16}x^2 = 16$
$\frac{17}{16}x^2 = 16$
$x^2 = \frac{(16)(16)}{17}$
$x = \frac{16}{\sqrt{17}}$

$x = \frac{16}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}}$

$\boxed{x = \frac{16\sqrt{17}}{17}}$

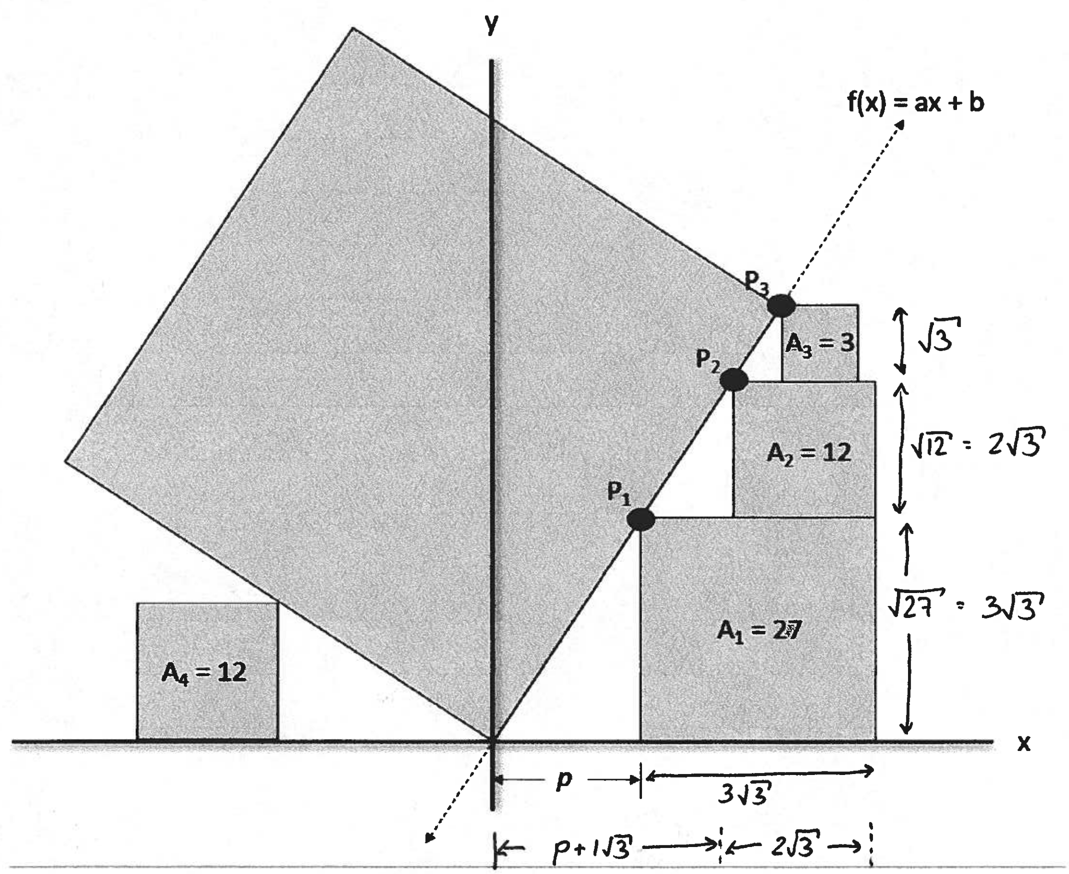
KEY

$t_n = t_1 + (n-1)d$ $\quad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\quad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\quad t_n = t_1 r^{n-1}$ $\quad S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\quad S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $\quad S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

Use the following information to answer Q47:

Five squares are depicted below. Determine the area of the largest square.



Original problem posed by "Flammable Maths" on YouTube.
(https://www.youtube.com/watch?v=cD9FsAAaKnE)

Q47: Students will determine the area of the largest square.

a. Determine the side length, as a mixed radical, for the three small squares on the right side.
b. Determine the coordinates of P_1 and P_2 in terms of the value p.
c. Use points P_1 and P_2, along with your Slope Equation, to determine the function $f(x)$.
d. Use the function $f(x)$ to determine the coordinates of point P_3.
e. Use the coordinates of point P_3 to determine the side length of the largest square.
f. Use the side length of the largest square to determine its area.

(A) On Diagram

(B) Also see diagram.

P_1 is $(p, 3\sqrt{3})$

P_2 is $(p+\sqrt{3}, 5\sqrt{3})$

(C) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5\sqrt{3} - 3\sqrt{3}}{(p+\sqrt{3}) - (p)} = \frac{2\sqrt{3}}{\sqrt{3}} = 2$

$f(x) = 2x + 0$

Key

$t_n = t_1 + (n-1)d$ $\qquad S_n = \frac{n}{2}[2t_1 + (n-1)d]$ $\qquad S_n = \frac{n}{2}(t_1 + t_n)$

$r = \frac{t_n}{t_{n-1}}$ $\qquad t_n = t_1 r^{n-1}$ $\qquad S_n = \frac{t_1(r^n - 1)}{r-1}, r \neq 0$ $\qquad S_n = \frac{rt_n - t_1}{r-1}, r \neq 0$ $\qquad S_\infty = \frac{t_1}{1-r}, -1 < r < 1$

Space to work on Q47.

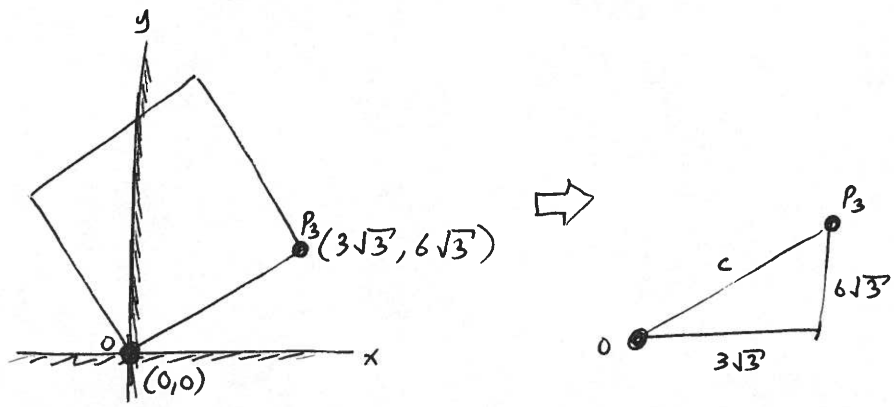
D $\quad y = 2x + 0 \quad$ Use Point P_3 as $(x, 6\sqrt{3})$

$6\sqrt{3} = 2x + 0$

$x = 3\sqrt{3}$

So Point P_3 is $(3\sqrt{3}, 6\sqrt{3})$

E



$a^2 + b^2 = c^2$

$(3\sqrt{3})^2 + (6\sqrt{3})^2 = c^2$

$9(3) + 36(3) = c^2$

$27 + 108 = c^2$

$135 = c^2$

$c = \sqrt{135}$

Side length is $\sqrt{135}$

F $\quad$ Area of square is given by $\quad A = LW$

$= (\sqrt{135})(\sqrt{135})$

$\boxed{\text{Area} = 135}$