Chapter 4 – Exponents and Radicals

4.1   Square Roots and Cube Roots

22. What is the value of each expression?

a) $\sqrt{4}$ = 2

b) $\sqrt{16}$ = 4

c) $\sqrt{25}$ = 5

d) $\sqrt{144}$ = 12

23. Find each square root to two decimal places.

a) $\sqrt{18}$ = 4.24

b) $\sqrt{33}$ = 5.74

c) $\sqrt{95}$ = 9.75

d) $\sqrt{61}$ = 7.81

24. A square lot has an area of 79 m².
    a) Calculate the length of each side to one decimal place.
    b) How much fencing, to one decimal place, would be required to enclose the lot?

Ⓐ $\sqrt{79}$ = 8.9m

Ⓑ $4(8.9)$ = 35.6m (Perimeter of fence)

25. The diameter of a circle is given by the formula $d = 113\sqrt{A}$ where d is the diameter of the circle and A is the area of the circle. Find the diameter of a circle with an area of 155 cm², to the nearest tenth.

$d = 113\sqrt{155}$
$= 113(12.4499)$
$= 1406.83\,cm$

Is this right?
$A = \pi r^2$
$r^2 = \frac{A}{\pi}$

$r = \sqrt{\frac{A}{\pi}}$

$d = 2\sqrt{\frac{A}{\pi}}$

$d = \frac{2}{\sqrt{\pi}}\sqrt{A}$

$d \approx 1.13\sqrt{A}$

No, it is not correct.

26. Evaluate each of the following.

   a) $\sqrt[3]{8}$ $= 2$

   b) $\sqrt[3]{27}$ $= 3$

   c) $\sqrt[3]{1000}$ $= 10$

   d) $\sqrt[3]{0.001}$ $= 0.1$

27. In one day a gold mine produces approximately $1600\ cm^3$ of gold. If five day's gold production is cast into a cube, how long is one of its sides?

   $5(1600) = 8000\ cm^3$

   $\sqrt[3]{8000} = 20cm$

4.2   Integral Exponents

28. Simplify each expression.

   a) $(x^8)(x^8)(x^8)$

   X^{24}

   b) $(x^6)(x)(x^3)$

   X^{10}

   c) $(-2a)(-a^5b)(a^3b^7)$

   $2a^9b^8$

   d) $(7x^7y^5)(-2x^4y^8)$

   $-14x^{11}y^{13}$

   e) $(xyz^5)(x^5y^8z)$

   $x^6y^9z^6$

   f) $(2^5)(2^8)(2^3)$

   2^{16}

   g) $(-7a^7b^6)(2a^5b)$

   $-14a^{12}b^7$

   h) $(4x^6y)(\dfrac{1}{2}x^7y^8)$

   $2x^{13}y^9$

   i) $(-3xy^5z^6)(xyz^7)$

   $-3x^2y^6z^{13}$

29. Simplify each expression.

a) $\dfrac{a^8b^4c^3}{a^5b^3c} = a^3b^1c^2$

b) $\dfrac{a^9b}{ab} = a^8$

c) $\dfrac{a^8b^{10}}{a^2b^6} = a^6b^4$

d) $\dfrac{6a^2b}{-ab} = -6a$

e) $\dfrac{-72a^3b^8}{-8ab^6} = 9a^2b^2$

g) $\dfrac{-54a^6b^4}{9a^3b} = -6a3b^3$

30. Evaluate each expression.

a) $\dfrac{(4)^5(4)^8(4)^{10}}{(4)^6(4)^{15}}$

$= \dfrac{4^{23}}{4^{21}} = 4^2$

b) $\dfrac{(10)^{10}(10)^{100}}{(10)^{108}} = \dfrac{10^{110}}{10^{108}} = 10^2$

c) $\dfrac{(-2)^8(-2)^9(-2)^{12}}{(-2)^3(-2)^6(-2)^{11}}$

$= \dfrac{(-2)^{29}}{(-2)^{20}} = (-2)^9$

d) $\dfrac{(7)^2(7)^3(7)^{10}}{(7)^{14}}$

$= \dfrac{7^{15}}{7^{14}} = 7^1$

31. Evaluate. Leave all of your answers as powers using positive exponents.

a) $2^{-6} \times 2^2$

$$\frac{2^2}{2^6} = \frac{1}{2^4}$$

b) $5^{-1} \times 5^{-2}$

$$\frac{1}{5} \cdot \frac{1}{5^2} = \frac{1}{5^3}$$

c) $3^2 \div 3^{-2}$

d) $\frac{(-3)^{-4}}{(-3)^{-2}}$

$$\frac{(-3)^2}{(-3)^4} = \frac{1}{(-3)^2}$$

e) $3^{-3} \times 3^2 \times 3^{-1}$

$$\frac{1}{3^3} \cdot \frac{3^2}{1} \cdot \frac{1}{3}$$

$$\frac{3^2}{3^4} = \frac{1}{3^2}$$

f) $\frac{4}{4^{-2}} \times \frac{2^{-2}}{2^2} = \frac{2^2}{(2^2)^{-2}} \cdot \frac{2^{-2}}{2^2}$

$$\frac{2^2}{2^{-4}} \cdot \frac{2^{-2}}{2^2} = \frac{(2^2)(2^4)}{1} \cdot \frac{(2^2)(2^2)}{1}$$

$$= 2^6 \cdot 2^4 = 2^{10}$$

g) $\frac{(-3)^0}{(-3)^{-1}}$

$$\frac{1}{(-3)^{-1}} = (-3)^1$$

h) $\frac{3^2}{3^{-1}} \times \frac{3^{-4}}{3^0}$

$$(3^2)(3^1) \cdot \frac{1}{3^4}$$

$$= \frac{3^3}{3^4} = \frac{1}{3}$$

i) $\frac{2^5}{2^3} \times \frac{4^1}{4^{-2}} = \frac{2^5}{2^3} \cdot \frac{(2^2)^{-1}}{(2^2)^{-2}}$

$$= \frac{2^5}{2^3} \cdot \frac{2^{-2}}{2^{-4}} = \frac{2^3}{2^{-1}} = 2^4$$

j) $\frac{8^5 \times 8^{-11}}{8^{-3}}$

$$\frac{8^{-6}}{8^{-3}} = \frac{8^3}{8^6} = \frac{1}{8^3}$$

k) $\frac{(-2)^9 \times (-2)^{-6}}{(-2)^2}$

$$\frac{(-2)^3}{(-2)^2} = (-2)^1$$

l) $\frac{2^7}{2^{-2}} \times \frac{2^{-4}}{2^3} = \frac{2^3}{2^1} = 2^2$

15

32. Simplify each of the following as far as possible. Leave your answer with positive exponents.

a) $\left(a^2b^4\right)\left(a^2b^5\right)$

a^4b^{-1}

$\dfrac{a^4}{b}$

b) $\dfrac{x^{-6}}{x^{-6}} = 1$

c) $\dfrac{x^2y^{-2}}{y^{-1}} = \dfrac{x^2y^1}{y^2} = \dfrac{x^2}{y}$

d) $\left(x^3y^2\right)\left(x^2y^3\right)$

x^5y^5

e) $\left(x^{-1}y^{-2}\right)\left(x^{-2}y^{-3}\right)$

$x^{-3}y^{-5}$

$\dfrac{1}{x^3y^5}$

f) $\dfrac{x^{-3}y^{-2}}{x^2y^{-6}} = \dfrac{y^6}{x^2x^3y^2} = \dfrac{y^6}{x^5y^2}$

$= \dfrac{y^4}{x^5}$

4.3   Rational Exponents

33. Use the laws of exponents to simplify. Leave your answers with positive exponents.

a) $5^{\frac{3}{4}}\times5^{\frac{1}{8}}$

$5^{6/8}\cdot5^{1/8}$

$5^{7/8}$

b) $\dfrac{3^{\frac{5}{8}}}{3^{-\frac{1}{8}}} = 3^{5/8}\cdot3^{1/8}$

$= 3^{6/8}\cdot3^{3/4}$

c) $\left(10^{\frac{3}{5}}\right)^{\frac{2}{5}} = 10^{6/25}$

d) $a^{\frac{2}{3}}\times a^{\frac{5}{4}}$

$a^{8/12}\cdot a^{15/12}$

$a^{23/12}$

e) $\left(27^{-\frac{2}{3}}\right)^{\frac{3}{2}}$

$27^{-6/6}$

27^{-1}

$\dfrac{1}{27}$

f) $\left(m^{\frac{2}{3}}n^{-\frac{1}{4}}\right)^{\frac{1}{2}}$

$m^{2/6}n^{-1/8}$

$\dfrac{m^{1/3}}{n^{1/8}}$

34. The function $f = k(2)^{\frac{x}{12}}$ can be used to determine the frequency, in cycles per second of a musical note that is x half-steps above a note with a frequency k. The frequency of a middle C on a piano is 262 cycles per second. Find the frequency of the C that is one octave (12 half-steps) higher.

$x = 12$

$K = 262$

$f = ?$

$f = k(2)^{x/12}$

$= 262(2)^{12/12}$

$= 262(2^1)$

$= 524 \; \textcircled{Hz} \rightarrow$ Cycles per second.

35. The astronomer Johann Kepler found a formula which can be used to determine the number of Earth-days it takes each planet to travel once around the sun. The formula is:

$N = 0.2R^{\frac{3}{2}}$, where R is the mean distance from the planet to the sun in millions of kilometers and N is the number of Earth-days. Determine the number of Earth-days in the year of Saturn if Saturn is 1420 million kilometers away from the sun.

$R = 1420$

$N = ?$

$N = 0.2(1420)^{3/2}$

$= 0.2(53,509.7)$

$= 10,701.94 \; days$

4.4 Irrational Numbers

36. Express each power as an equivalent radical.

a) $2^{\frac{1}{3}}$

$\sqrt[3]{2}$

b) $x^{\frac{1}{2}}$

$\sqrt{x}$

c) $7^{-\frac{1}{2}} = \dfrac{1}{7^{1/2}} = \dfrac{1}{\sqrt{7}}$

d) $x^{-\frac{3}{7}} = \dfrac{1}{x^{3/7}}$

$= \dfrac{1}{\sqrt[7]{x^3}}$

e) $(3x)^{\frac{1}{2}}$

$\sqrt{3x}$

f) $3x^{\frac{1}{2}}$

$3\sqrt{x}$

37. Express each radical as a power.

a) $\sqrt{7} = 7^{1/2}$

b) $\sqrt[3]{-11}$

$(-11)^{1/3}$

c) $\sqrt[3]{6^4}$

$6^{4/3}$

d) $\dfrac{1}{\left(\sqrt[5]{x}\right)^4}$

$\dfrac{1}{x^{4/5}}$

$\underline{\underline{or}}$

$x^{-4/5}$

e) $\sqrt[3]{2b^3}$

$(2b^3)^{1/3}$

$2^{1/3}b$

f) $\sqrt[n]{27}$

$27^{1/n}$

18

38. Express each mixed radical as an equivalent entire radical.

a) $3\sqrt{2}$

$\sqrt{3^2}\sqrt{2}$

$\sqrt{9}\sqrt{2}$

$\sqrt{18}$

b) $-4\sqrt{3}$

$\sqrt{(-4)^2}\sqrt{3}$

$\sqrt{16}\sqrt{3}$

$\sqrt{48}$

c) $5\sqrt{27}$

$\sqrt{5^2}\sqrt{27}$

$\sqrt{25}\sqrt{27}$

$\sqrt{675}$

d) $6\sqrt{8}$

$\sqrt{6^2}\sqrt{8}$

$\sqrt{36}\sqrt{8}$

$\sqrt{288}$

e) $2\sqrt[3]{3}$

$\sqrt[3]{2^3}\sqrt[3]{3}$

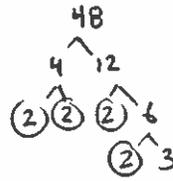$\sqrt[3]{8}\sqrt[3]{3}$

$\sqrt[3]{24}$

f) $2\sqrt[4]{27}$

$\sqrt[4]{2^4}\sqrt[4]{27}$

$\sqrt[4]{16}\sqrt[4]{27}$

$\sqrt[4]{432}$

39. Express each entire radical as an equivalent mixed radical.

a) $\sqrt{32}$ 32
                      $16\overset{\frown}{\ }2$

$\sqrt{16}\sqrt{2}$

$4\sqrt{2}$

b) $\sqrt{48}$ 48
                  $4\overset{\frown}{\ }12$

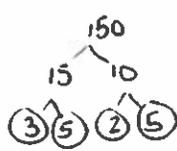$\sqrt{2^2\cdot2^2\cdot3}$ ②②②6

$2\cdot2\sqrt{3}$ ②3

$4\sqrt{3}$

c) $-3\sqrt{27}$ 27
                    ③9
                      ③③

$-3\sqrt{3^2\cdot3}$

$-3\cdot3\sqrt{3}$

$-9\sqrt{3}$

d) $-6\sqrt{150}$ 150
                    $15\ \ 10$

$-6\sqrt{5^2\cdot2\cdot3}$ ③⑤②⑤

$-6\cdot5\sqrt{2\cdot3}$

$-30\sqrt{6}$

e) $\sqrt[3]{128}$ 128
                     $4\overset{\frown}{\ }32$

$\sqrt[3]{2^3\cdot2^3\cdot2}$ ②②②16

$2\cdot2\cdot\sqrt[3]{2}$ ②8

$4\sqrt[3]{2}$ ②4
                               ②②

f) $3\sqrt[3]{135}$ 135
                    ⑤27
                      ③9
                        ③③
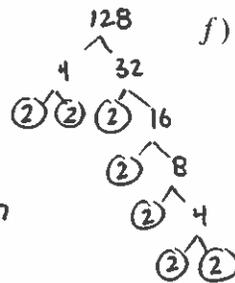
$3\sqrt[3]{3^3\cdot5}$

$3\cdot3\sqrt[3]{5}$

$9\sqrt[3]{5}$
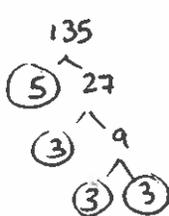
40. Arrange the following from least to greatest: $3\sqrt{6}$, $5\sqrt{2}$, $2\sqrt{15}$, $4\sqrt{3}$

7.35   7.07   7.75   6.93
 ③   ②   ④   ①
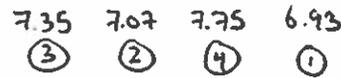
$4\sqrt{3}$, $5\sqrt{2}$, $3\sqrt{6}$, $2\sqrt{15}$

Chapter 5 – Polynomials

5.1 Multiplying Polynomials

41. Expand and collect like terms.

a) $(a+1)(a+2)$

$a^2 + 2a + 1a + 2$

$a^2 + 3a + 2$

b) $(n-3)(n-2)$

$n^2 - 2n - 3n + 6$

$n^2 - 5n + 6$

c) $(x+9)(x+7)$

$x^2 + 7x + 9x + 63$

$x^2 + 16x + 63$

d) $(a-2)^2$

$(a-2)(a-2)$

$a^2 - 2a - 2a + 4$

$a^2 - 4a + 4$

e) $(2x-3)(x-2)$

$2x^2 - 4x - 3x + 6$

$2x^2 - 7x + 6$

f) $(2a-3)(3a+2)$

$6a^2 + 4a - 9a - 6$

$6a^2 - 5a - 6$

42. Use the distributive property to determine each product.

a) $n(5n^2 - n + 4)$

$5n^3 - n^2 + 4n$

b) $-k(k^2 - 5k + 1)$

$-k^3 + 5k^2 - k$

c) $(a-2)(a^2 + 2a + 4)$

$a^3 + 2a^2 + 4a - 2a^2 - 4a - 8$

$a^3 - 8$

d) $(3p+2)(5p^2 - 6p + 2)$

$3p^3 - 18p^2 + 6p + 10p^2 - 12p + 4$

$3p^3 - 8p^2 - 6p + 4$

e) $(2x^2 + 3x - 2)(5x^2 + x + 6)$

$10x^4 + 2x^3 + 12x^2$

$ + 15x^3 + 3x^2 + 18x$

$ -10x^2 - 2x - 12$

$\overline{10x^4 + 17x^3 + 5x^2 + 16x - 12}$

43. Multiply and then collect like terms.

a) $(x-3)(x+2)+(2x-5)$

$(x^2+2x-3x-6) + (2x-5)$

$x^2 \bullet -1x-6 + 2x-5$

$x^2 + x - 11$

b) $(2a-5b)(3a+b)-(6a-b)(4a+7b)$

$(6a^2+2ab-15ab-5b^2) - (24a^2+42ab-4ab-7b^2)$

~~$6a^2+2ab-15ab-5$~~

$(6a^2-13ab-5b^2) - (24a^2+38ab-7b^2)$

$6a^2-13ab-5b^2 - 24a^2 + 38ab+7b^2$

$-18a^2-51ab+2b^2$

c) $3(2a-3b)(a+2b)-2(3a-b)^2$

$3(2a^2+4ab-3ab-6b^2) - 2(3a-b)(3a-b)$

$3(2a^2+1ab-6b^2) - 2(9a^2-3ab-3ab+b^2)$

$(6a^2+3ab-18b^2) - (18a^2-6ab-6ab+2b^2)$

$6a^2+3ab-18b^2-18a^2+12ab-2b^2$

$-12a^2+15ab-20b^2$

d) $(2x+3)(3x^2-5x+4)+(x-4)(2x^2-7)$

$(6x^3-10x^2+8x-9x^2-15x+12)+(2x^3-7x-8x^2+28)$

$6x^3-19x^2-7x+12 + 2x^3-8x^2-7x+28$

$8x^3-27x^2-14x+40$

5.2   Common Factors

44. Identify the GCF of the following sets of terms.

a) $21x^2$, $28x^4$, and $-14x$

$7x$

b) $3y^4$, $8y^6$, and $-6y^9$

y^4

c) $12a^3b$, $-16a^2b^2$, and $-24a^2b^3$

$4a^2b$

45. Factor the following polynomials.

a) $5y - 10$

$$5(y-2)$$

b) $51x^2y + 39xy^2 - 72xy$

$$3xy(17x + 13y - 24)$$

c) $35z^2 - 14z^6$

$$\cancel{35\left(\frac{z^2}{7z^2}\right)}$$

$$7z^2(5 - 2z^4)$$

d) $3x^2 + 5x^3 + x$

$$5x^3 + 3x^2 + 1x$$

$$x(5x^2 + 3x + 1)$$

$\square + \square = 3$

$\square \cdot \square = 5 \quad \rightarrow$ Can't factor further.

e) $8x^2y - 32xy^2 + 16x^2y^2$

$$8xy(x - 4y + 2xy)$$

46. Factor the following polynomials.

a) $2x(\underline{\underline{y-2}})-3(\underline{\underline{y-2}})$

$$(y-2)(2x-3)$$

b) $a^2+2a+ab+2b$

$$(a^2+2a)+(ab+2b)$$
$$a(a+2)+b(a+2)$$
$$(a+2)(a+b)$$

c) $2p-2q+pq-p^2$

$$(2p-2q)+(pq-p^2)$$
$$2(p-q)+p(q-p) \rightarrow \text{Factor out a -1 from second term.}$$
$$2(p-2)-p(p-2)$$
$$(p-2)(2-p)$$

d) $a^2+6a+7a+42$

$$(a^2+6a)+(7a+42)$$
$$a(a+6)+7(a+6)$$
$$(a+6)(a+7)$$

5.3   Factoring Trinomials

47. Factor, if possible.

a) $x^2 + 14x + 40$

$\square + \square = 14$

$\square \cdot \square = 40$

$10 \quad 4$

$(x + 10)(x + 4)$

b) $g^2 - 4g - 77$

$\square + \square = -4$

$\square \cdot \square = -77$

$7 \quad -11$

$(g + 7)(g - 11)$

c) $x^2 - 10x - 24$

$\square + \square = -10$

$\square \cdot \square = -24$

$-12 \quad 2$

$(x - 12)(x + 2)$

d) $k^2 + 21k + 90$

$\square + \square = 21$

$\square \cdot \square = 90$

$6 \quad 15$

$(k + 6)(k + 15)$

1, 90
2, 45
3, 30
5, 18
6, 15

e) $p^2 - 17p - 60$

$\square + \square = -17$

$\square \cdot \square = -60$

$-20 \quad 3$

$(p - 20)(p + 3)$

1, 60
2, 30
3, 20
4, 15
5, 12
6, 10

f) $x^2 + 2x - 15$

$\square + \square = 2$

$\square \cdot \square = -15$

$5 \quad -3$

$(x + 5)(x - 3)$

g) $2y^2 - 6y + 4$

$2(y^2 - 3y + 2)$

$\square + \square = -3$

$\square \cdot \square = 2$

$-1 \quad -2$

$2(y - 1)(y - 2)$

h) $6m^2 + 18m - 24$

$6(m^2 + 3m - 4)$

$\square + \square = 3$

$\square \cdot \square = -4$

$6(m + 4)(m - 1)$

$4 \quad -1$

24

48. Factor each of the following, if possible.

a) $2x^2 + 3x + 1$

$\square + \square = 3$

$\underset{1}{\square} \cdot \underset{2}{\square} = 2$

$2x^2 + 1x + 2x + 1$

$(2x^2 + 1x) + (2x + 1)$

$x(2x+1) + 1(2x+1)$

$(2x+1)(x+1)$

b) $3y^2 - 2y - 1$

$\square + \square = -2$

$\underset{1}{\square} \cdot \underset{-3}{\square} = -3$

$3y^2 + 1y - 3y - 1$

$(3y^2 + 1y) + (-3y - 1)$

$y(3y+1) - 1(3y+1)$

$(3y+1)(y-1)$

c) $12m^2 - 16m - 4$

$4(3m^2 - 4m - 1)$

$\square + \square = -4$

$\square \cdot \square = -3$ → Not factorable.

d) $16x^2 + 56xy + 49y^2$

$\sqrt{16} = 4$

$\sqrt{49} = 7$

$(4x + 7y)(4x + 7y)$ Double check! Yep, it works!

e) $8x^2 - 29x - 12$

$\square + \square = -29$

$\square \cdot \square = -96$

3   -32

$8x^2 + 3x - 32x - 12$

$(8x^2 + 3x) + (32x - 12)$

$x(8x+3) - 4(8x+3)$

$(8x+3)(x-4)$

1, 96
2, 48
3, 32
4, 24
6, 16
8, 12

f) $6y^2 - 11y - 10$

$\square + \square = -11$

$\square \cdot \square = -60$

4   -15

$6y^2 + 4y - 15y - 10$

$(6y^2 + 4y) + (-15y - 10)$

$2y(3y+2) - 5(3y+2)$

$(3y+2)(2y-5)$

1, 60
2, 30
3, 20
4, 15

g) $12m^2 + 19m - 10$

$\square + \square = 19$

$\square \cdot \square = -120$

24   -5

$12m^2 + 24m - 5m - 10$

$(12m^2 + 24m) + (-5m - 10)$

$12m(m+2) - 5(m+2)$

$(m+2)(12m - 5)$

1, 120
2, 60
3, 40
4, 30
5, 24

h) $2x^2 - 5 + 9x$

$2x^2 + 9x - 5$

$\square + \square = 9$

$\square \cdot \square = -10$

10   -1

$(2x^2 + 10x) + (-1x - 5)$

$2x(x+5) - 1(x+5)$

$(x+5)(2x-1)$

5.4   Factoring Special Trinomials

49. Factor each binomial, if possible.

a) $x^2 - 81$

$$(x+9)(x-9)$$

b) $4x^2 - 25y^2$

$$(2x+5y)(2x-5y)$$

c) $25x^2 - 121$

$$(5x+11)(5x-11)$$

d) $81 + x^2$

$$(9+x)(9-x)$$

50. Factor each trinomial, if possible.

a) $x^2 - 18x + 81$

$$(x-9)(x-9)$$
$$(x-9)^2$$

b) $x^2 + 14x + 49$

$$(x+7)(x+7)$$
$$(x+7)^2$$

c) $5x^2 - 10x + 5$

$$5(x^2 - 2x + 1)$$
$$5(x-1)(x-1)$$
$$5(x-1)^2$$

d) $x^2 + 16x + 64$

$$(x+8)(x+8)$$
$$(x+8)^2$$

51. Evaluate the following by using the difference of squares method.

a) $53^2 - 47^2$

$$(53 + 47)(53 - 47)$$
$$(100)(6)$$
$$600$$

b) $45^2 - 25^2$

$$(45 + 25)(45 - 25)$$
$$(70)(20)$$
$$1400$$

c) $82^2 - 18^2$

$$(82 + 18)(82 - 18)$$
$$(100)(64)$$
$$6400$$