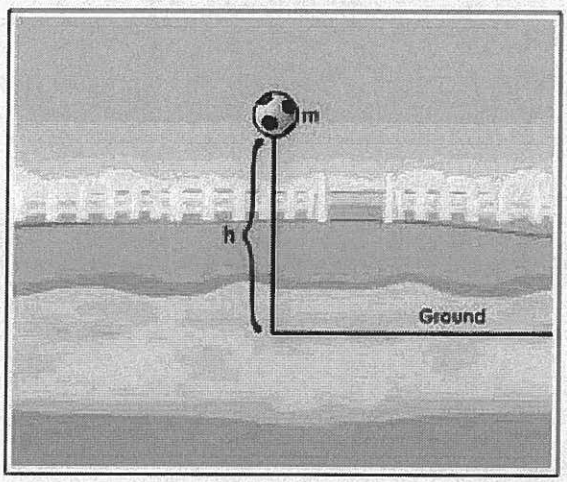
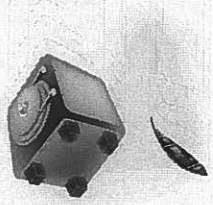
L02 - Conservation of Mechanical Energy

Gravitational Potential Energy: The energy of an object because of its position above the surface of Earth.





$\Delta E = W \quad where\ W = Fd$

$E_p = Fd \quad where \quad F_g = mg \quad and \quad d = height$

$E_p = mgh$

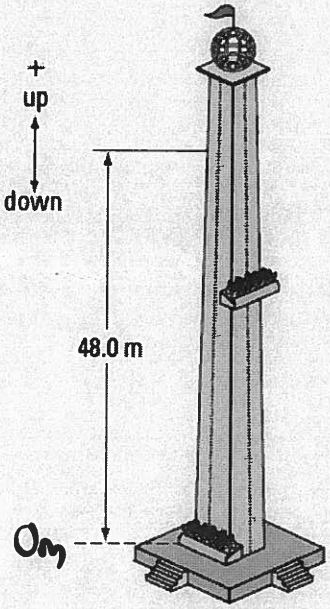
Changes in gravitational potential energy result from changes in vertical distance.

Objects only have gravitational potential energy in relation to a specific reference height.

$E_p = mgh \Rightarrow \Delta E_p = mg\Delta h$

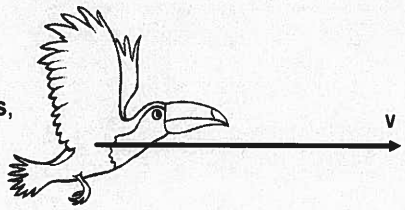
Q1: An amusement park car and its passengers have a mass of 500kg. What is their gravitational potential energy, relative to the ground, when they are lifted through a height of 48.0 meters?

$E_p = mgh$
$= (500)(9.81)(48)$
$= 235,440 J$



Kinetic Energy: Energy of a moving object.

$E_k = \frac{1}{2}mv^2$



Q2: If a toucan of mass 5kg is flying at a speed of 4m/s, how much kinetic energy does the bird have?

$E_K = \frac{1}{2}mv^2$
$= \frac{1}{2}(5)(4)^2$
$= 40\ J$

Q3: If a toucan of mass 5kg has 200J of kinetic energy, how fast is the toucan flying?

$E_K = \frac{1}{2}mv^2$
$200 = \frac{1}{2}(5)v^2$
$v^2 = 80$
$v = 8.94\ m/s$

Q4: An object has a kinetic energy of 100J. Another object of the same mass is travelling at four times the speed. What is its kinetic energy?

$E_K = 100\ J$

$E_{new} = \frac{1}{2}(m)(4v)^2$
$= \frac{1}{2}(m)(16v^2)$
$= 16(\frac{1}{2}mv^2)$
$= 16(100)$
$= 1600\ J$

Q5: An object has a kinetic energy of 500J. Another object has three times the mass but is only travelling half the speed. What is its kinetic energy?

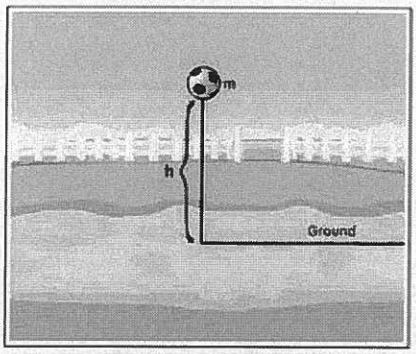
$E_K = 500\ J$

$E_{new} = \frac{1}{2}(3m)(\frac{1}{2}v)^2$
$= \frac{1}{2}(3m)(\frac{1}{4}v^2)$
$= \frac{3}{4}(\frac{1}{2}mv^2)$
$= \frac{3}{4}(500)$
$= 375\ J$

Conservation of Mechanical Energy

Gravitational Potential Energy

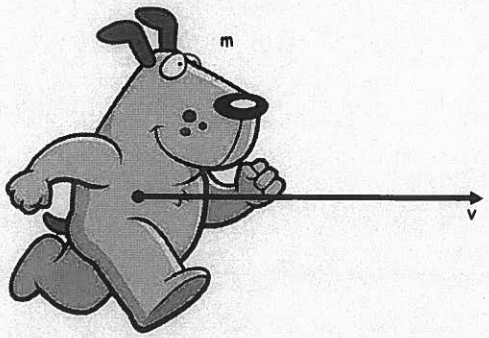
$\Delta E_p = mg\Delta h$



Gravitational Potential Energy: The energy of an object because of its position above the surface of Earth.

Kinetic Energy

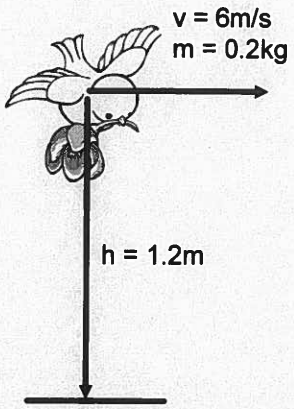
$E_k = \frac{1}{2} mv^2$



Kinetic Energy: Energy of a moving object.

Q6: What is the gravitational potential energy of the bird, in Joules?

$E_p = mgh$
$= (0.2)(9.81)(1.2)$
$= 2.3544\ J$



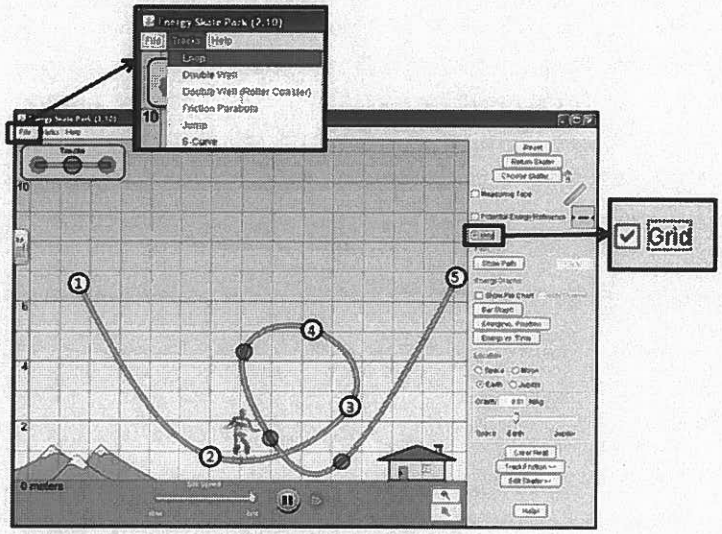
Q7: What is the kinetic energy of the bird, in Joules?

$E_K = \frac{1}{2}mv^2$
$= \frac{1}{2}(0.2)(6)^2$
$= 3.6\ J$

Q8: What is the total mechanical energy of the bird, in Joules?

$E_m = E_p + E_k$
$= 2.3544 + 3.6$
$= 5.9544\ J$

Demo - Colorado PhET: Energy Skate Park



Q9: Can we ever reach position #5? Why or why not?

No. Not enough energy.

Q10: In a frictionless system, what do you notice about the mechanical energy at each position?

E_m is constant/conserved

$E_m = E_p + E_k$ (↑ ↓)

Same

Q11: With friction, where does the lost energy go?

Thermal Energy

Conservation of Energy: Falling Object

$E_P \rightarrow E_K$

Q12: A 2kg object is held 5m above the ground before being dropped. Calculate the speed of the object the instant before it strikes the ground, using (a) Conservation of Energy, and (b) Kinematics.

(A) $Em_i = Em_f$

$Ep_i + \cancel{Ek_i}^{0} \rightarrow \cancel{Ep_f}^{0} + Ek_f$

$Ep \rightarrow Ek$

$mgh \rightarrow \frac{1}{2}mv^2$

$(9.81)(5) = \frac{1}{2}v^2$

$(2)(9.81)(5) = v^2$

$v = 9.90 m/s$

(B) $v_f^2 = \cancel{v_i^2}^{0} + 2ad$

$v_f^2 = (2)(9.81)(5)$

$v_f = 9.90 m/s$

Objects Thrown to Maximum Height

Q13: A 5kg object is thrown vertically at an initial velocity of 3m/s. Calculate its maximum height using (a) Conservation of Energy, and (b) Kinematics.

(A) $Em_i \rightarrow Em_f$

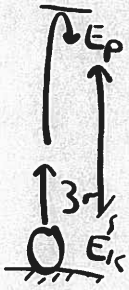
$\cancel{Ep_i}^{0} + Ek_i \rightarrow Ep_f + \cancel{Ek_f}^{0}$

$Ek \rightarrow Ep$

$\frac{1}{2}mv^2 \rightarrow mgh$

$\frac{1}{2}(3)^2 = (9.81)h$

$h = 0.459 m$

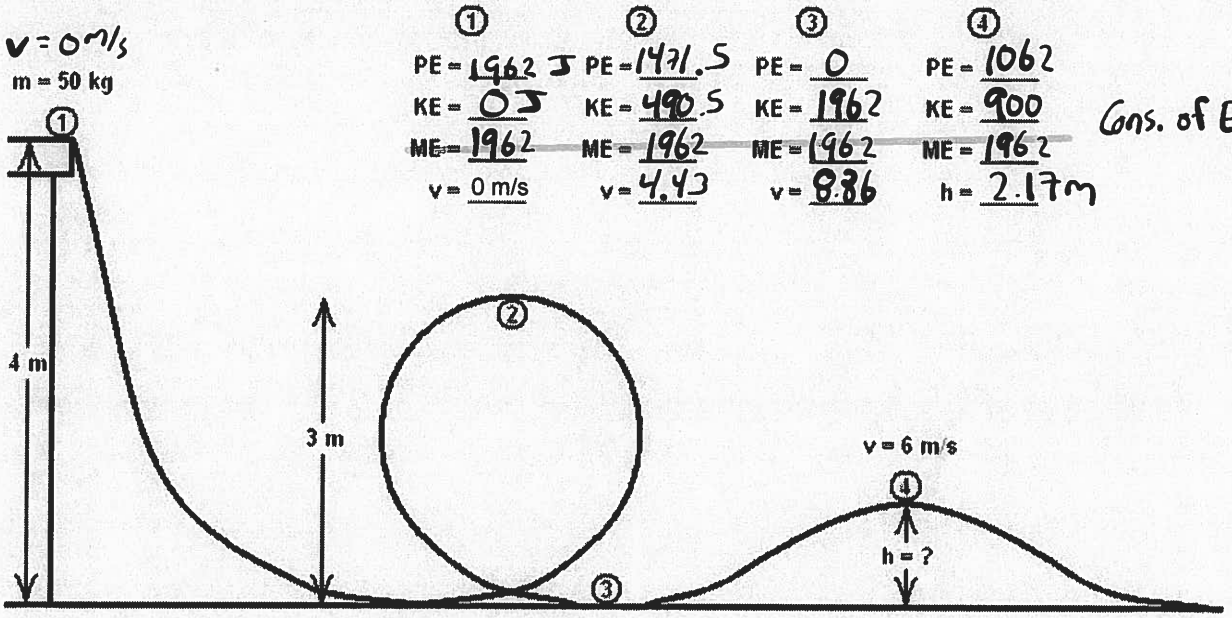


(B) $v_f^2 = v_i^2 + 2ad$

$0^2 = (3)^2 + 2(-9.81)d$

$d = 0.459 m$

Energy in a Frictionless Rollercoaster



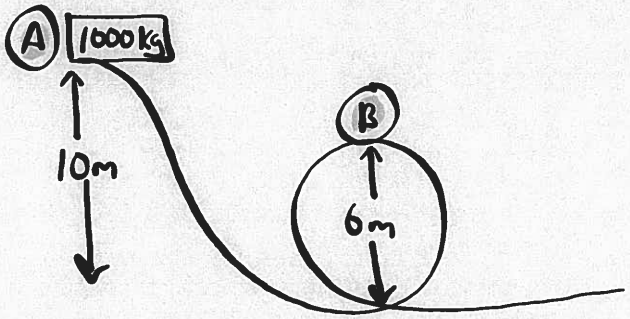
①	②	③	④	
PE = 1962 J	PE = 1471.5	PE = 0	PE = 1062	
KE = 0 J	KE = 490.5	KE = 1962	KE = 900	Cons. of E_m
ME = 1962	ME = 1962	ME = 1962	ME = 1962	
v = 0 m/s	v = 4.43	v = 8.86	h = 2.17 m	

① $E_k = \frac{1}{2}mv^2$
$= \frac{1}{2}(50)(0)^2$
$= 0$ J
$E_p = mgh$
$= (50)(9.81)(4)$
$= 1962$ J

② $E_m = 1962$ J
$E_p = mgh$
$= (50)(9.81)(3)$
$= 1471.5$ J
$E_m = E_p + E_k$
$1962 = 1471.5 + E_k$
$E_k = 490.5$ J
$E_k = \frac{1}{2}mv^2$
$490.5 = \frac{1}{2}(50)v^2$
$v = 4.43$ m/s

③ $E_m = 1962$ J
$E_p = 0$ J
$E_k = 1962$ J
$E_k = \frac{1}{2}mv^2$
$1962 = \frac{1}{2}(50)v^2$
$v = 8.86$ m/s

Energy in a Frictionless Rollercoaster (Continued)



Q14: What is the total mechanical energy of the cart while at *Position A*?

$E_p = mgh$
$= (1000)(9.81)(10)$
$= 98,100\,J$

$E_K = \frac{1}{2}mv^2$
$= \frac{1}{2}(1000)(0)^2$
$= 0\,J$

$E_m = E_p + E_K$
$= 98,100\,J$

Q15: What is the kinetic energy of the cart while at *Position B*?

$E_m = 98,100\,J$

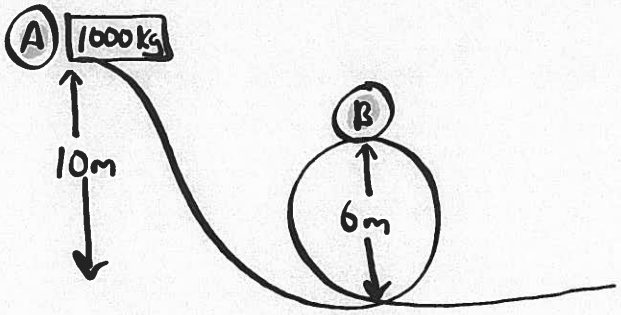
$E_p = mgh$
$= (1000)(9.81)(6)$
$= 58,860\,J$

$E_m = E_p + E_K$
$98,100 = 58,860 + E_K$
$E_K = 39,240\,J$

Q16: What is the speed of the cart while at *Position B*?

$E_K = \frac{1}{2}mv^2$
$39,240 = \frac{1}{2}(1000)v^2$
$v = 8.86\,m/s$

Energy in a Frictionless Rollercoaster (One Step)



Q17: What is the speed of the cart while at *Position B*?

$E_P \rightarrow E_P + E_K$

$mgh_i \rightarrow mgh_f + \frac{1}{2}mv_f^2$

$(9.81)(10) \rightarrow (9.81)(6) + \frac{1}{2}v^2$

$98.1 \rightarrow 58.86 + \frac{1}{2}v^2$

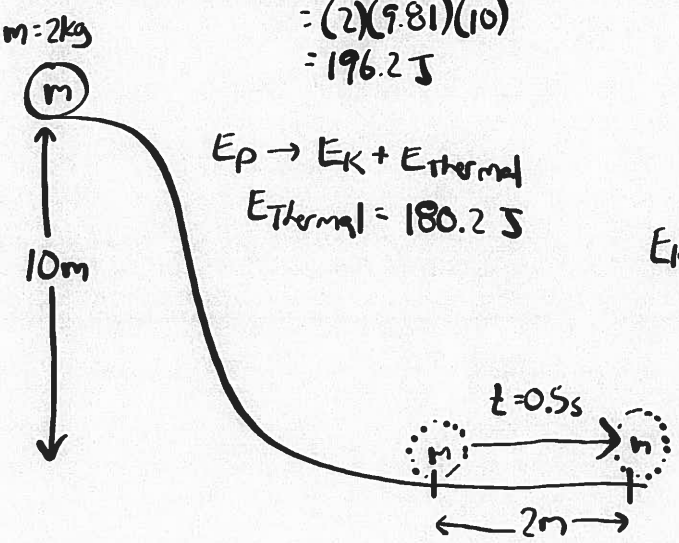
$39.24 = \frac{1}{2}v^2$

$v = 8.86 m/s$

Thermal Energy Loss due to Friction

Q18: What is the thermal energy loss due to friction, in Joules?

$E_P = mgh$
$= (2)(9.81)(10)$
$= 196.2\ J$



$E_P \rightarrow E_K + E_{Thermal}$
$E_{Thermal} = 180.2\ J$

$E_K = \frac{1}{2}mv^2$
$= \frac{1}{2}(2)(4)^2$
$= 16\ J$

$v = \frac{d}{t} = \frac{2m}{0.5s}$
$= 4\ m/s$