L11 – Oscillatory Motion of a Mass-Spring System

https://www.youtube.com/watch?v=lHaX9asEXlo

Terminology

Oscillation: Repetitive back-and-forth motion.

Oscillatory Motion: Motion with a constant period (T).
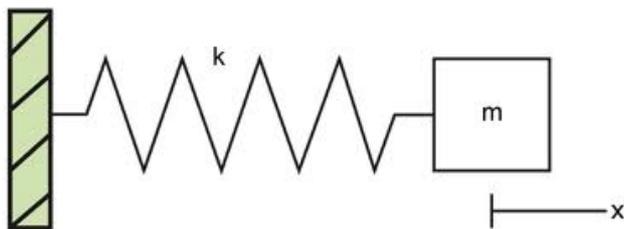
Period: Time required for an object to make one complete oscillation (seconds).

Frequency: Number of cycles per second (Hz).

Hooke's Law (Applied and Restoring Forces)

When a spring with a specific spring constant (k) is stretched a distance (x), the force required to stretch it is equal to the distance stretched multiplied by the spring constant.
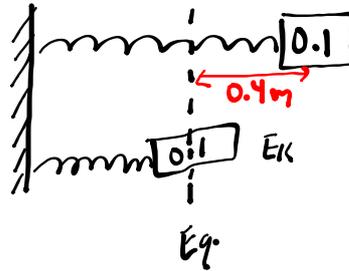
$$F = kx$$

Review

Q1: A 100 g mass is attached to horizontal spring on a frictionless surface. The spring constant is 1.014 N/m. The instructor pulls the mass a displacement of 40 cm from the equilibrium and releases it. Determine the maximum speed of the mass.

$$k = 1.014 \text{ N/m}$$
$$x = 0.40 \text{ m}$$
$$v = ?$$
$$m = 0.1 \text{ kg}$$

E_p $E_p \rightarrow E_K$

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2$$
$$(1.014)(0.4)^2 = (0.1)v^2$$
$$1.6224 = v^2$$
$$\boxed{v \approx 1.27 \text{ m/s}}$$

E_q

Q2: Mr. Campbell draws an arrow back 50 cm with a force of 400 N of force.

a) Calculate the spring constant

$$x = 0.5 \text{ m} \qquad F = kx$$
$$F = 400 \text{ N} \qquad (400) = k(0.5)$$
$$k = ?$$
$$\qquad\qquad k = 800 \text{ N/m}$$

b) Calculate the elastic potential energy of the arrow.

$$E_p = \tfrac{1}{2}kx^2$$
$$= \tfrac{1}{2}(800)(0.5)^2$$
$$= 100 \text{ J}$$

c) If arrow has a mass of 0.025 kg, calculate the speed the arrow has leaving the bow.

$$E_p \rightarrow E_K$$
$$100 = \tfrac{1}{2}mv^2$$
$$100 = \tfrac{1}{2}(0.025)v^2$$
$$v^2 = 8000$$
$$v = 89.4 \text{ m/s}$$

d) If the arrow was fired straight up, calculate the maximum height of the arrow.

$$E_{p_{elastic}} \rightarrow E_K \rightarrow E_{p_{grav}}$$
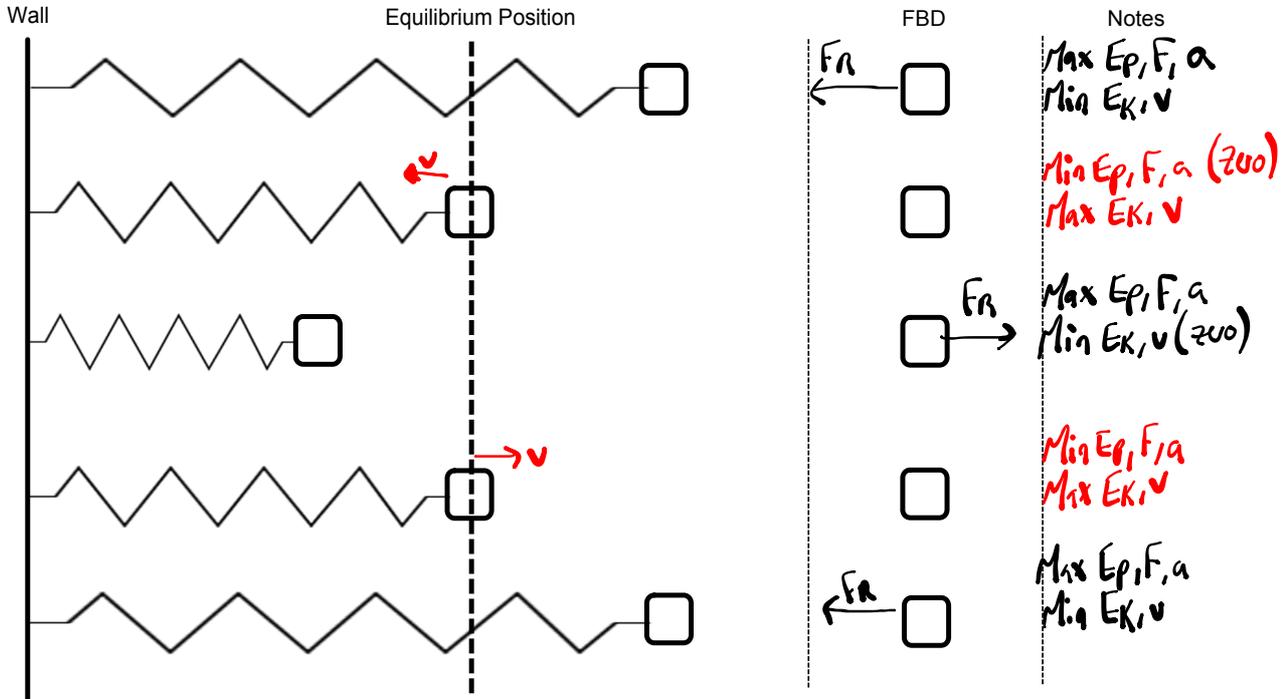$$100 \rightarrow 100 \rightarrow mgh$$
$$100 = (0.025)(9.81)h$$
$$\boxed{h = 407.74 \text{ m}}$$

2

*Oscillatory Motion and Free-Body Diagrams* **Important**

Q3: Describe how the force, acceleration and energy changes as the mass oscillates on the spring?

$E_p = \frac{1}{2}kx^2 \Rightarrow$ Max when furthest. $a = \frac{F_{net}}{m} \Rightarrow$ Max when furthest

$F = kx \Rightarrow$ Max when furthest.

Wall	Equilibrium Position	FBD	Notes

$\leftarrow F_R$ Max E_P, F, a
Min E_K, v

Min E_p, F, a (zero)
Max E_K, v

$F_R \rightarrow$ Max E_P, F, a
Min E_K, v (zero)

Min E_p, F, a
Max E_K, v

$\leftarrow F_R$ Max E_p, F, a
Min E_K, v

Q4: How do we derive the equation $v_{max} = A\sqrt{\dfrac{k}{m}}$?

A = Amplitude (max stretch)

Max Stretch = A

x

x

Equilibrium
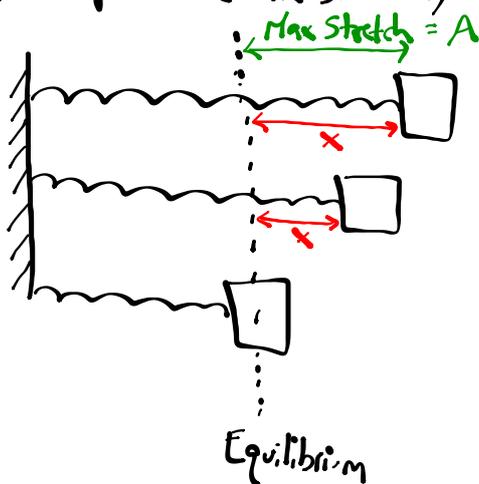
$E_P \longrightarrow E_K$

$\boxed{\frac{1}{2}kx^2 \longrightarrow \frac{1}{2}mv^2}$

$\frac{1}{2}kA^2 \longrightarrow \frac{1}{2}mv_{max}^2$

$kA^2 = mv^2$
$\div m \qquad \div m$

$\dfrac{kA^2}{m} = v^2$

$v = \sqrt{\dfrac{kA^2}{m}}$

$\boxed{v = A\sqrt{\dfrac{k}{m}}}$

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Period of the Mass-Spring System

$$T = 2\pi\sqrt{\frac{m}{k}}$$

T = Period (sec)
m = mass (kg)
K = Spring Constant (N/m)

Springs
$F = kx$ $E_p = \frac{1}{2}kx^2$

$T = 2\pi\sqrt{\frac{m}{K}}$

Q5: A 0.200 kg mass is attached to a horizontal spring. What is the spring constant if it has a 2.0 s period?

$m = 0.2$
$T = 2$
$k = ?$

$2 = 2\pi\sqrt{\frac{0.2}{K}}$

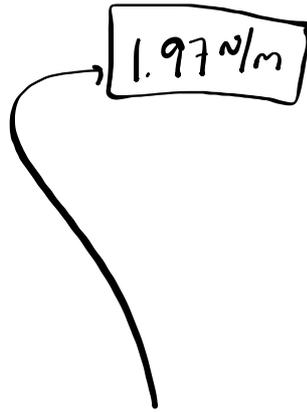$\div 2\pi \qquad \div 2\pi$

$0.3183... = \sqrt{\frac{0.2}{K}}$

$0.10132... = \frac{0.2}{K}$

$\cdot K \qquad \cdot K$

$(0.10132...)k = 0.2$
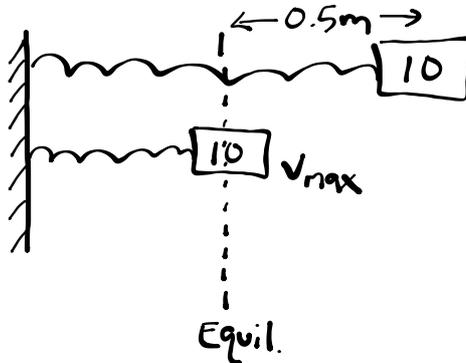
$\div 0.10132... \qquad \div 0.10132...$

$\boxed{K = 1.973920888022... \text{ N/m}}$

$\boxed{1.97 \text{ N/m}}$

Q6: A 10kg mass is attached to a spring and stretched 50cm to the right before being released. It oscillates with a period of 4.0 seconds. What is the maximum speed of the mass?

$m = 10 \text{ kg}$
$x = 0.5 \text{ m}$
$T = 4s$
$V_{max} = ?$

$\leftarrow 0.5m \rightarrow$ 10

10 V_{max}

Equil.

$F = kx$
$E_p = \frac{1}{2}kx^2$
$T = 2\pi\sqrt{\frac{m}{k}}$

$4 = 2\pi\sqrt{\frac{10}{k}}$

$0.6366... = \sqrt{\frac{10}{k}}$

$0.40528... = \frac{10}{k}$

$k(0.40528...) = 10$

$\boxed{k = 24.674 \text{ N/m}}$

$E_p \rightarrow E_k$
$\frac{1}{2}kx^2 \rightarrow \frac{1}{2}mv^2$
$(24.674)(0.5)^2 = (10)v^2$

$0.61685... = v^2$

$\boxed{0.785 \text{ m/s} = v}$

4

Practice

Pg 372 #1-3

Pg 376 #1-4

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