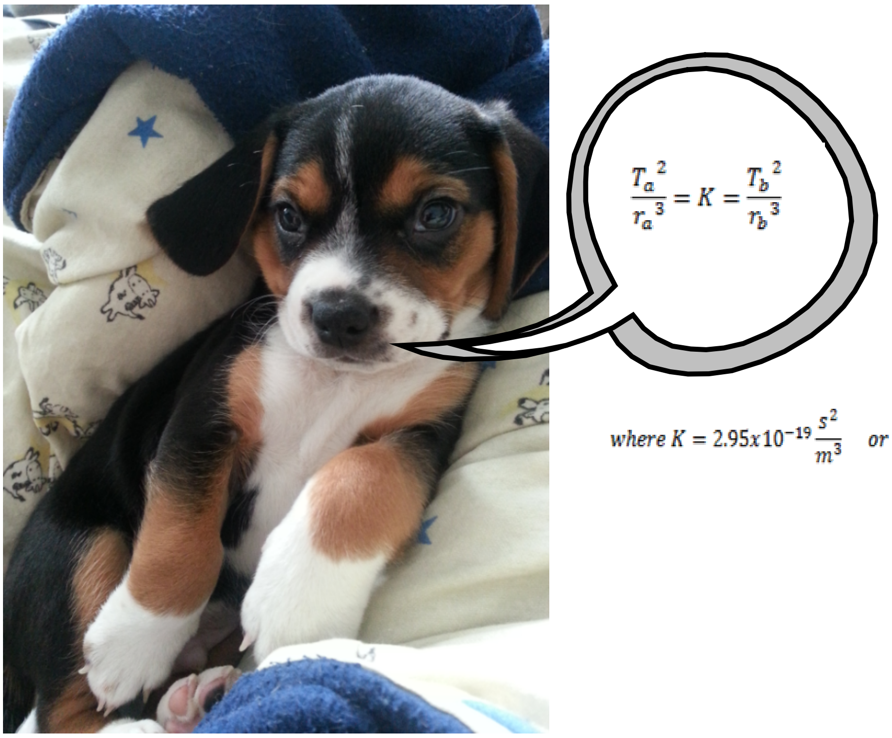
L12 - Kepler's Laws



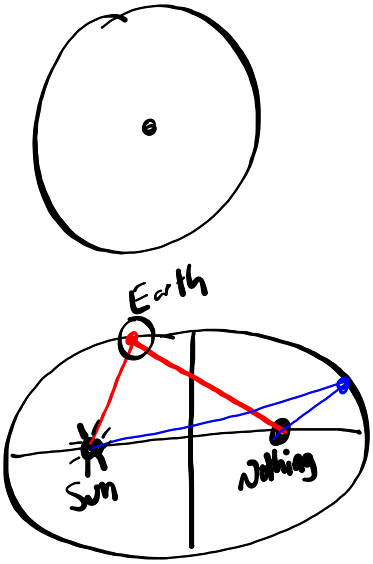
$$\frac{T_a^{\,2}}{r_a^{\,3}} = K = \frac{T_b^{\,2}}{r_b^{\,3}}$$

$$where\ K = 2.95x10^{-19}\frac{s^2}{m^3} \quad or \quad K = 1\frac{a^2}{AU^3}$$

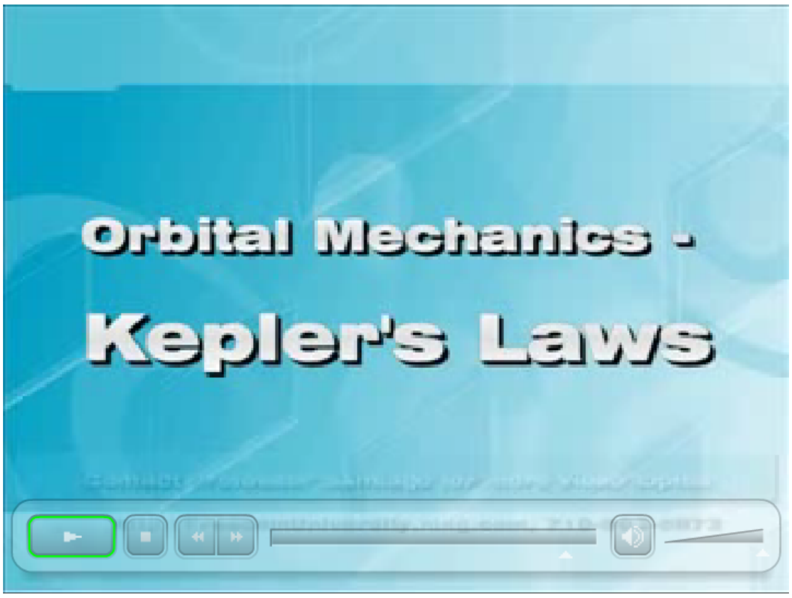
Agenda:

- Formative Quiz
- Lesson: Kepler's Law
- Practice

Johannes Kepler's Three Laws of Motion

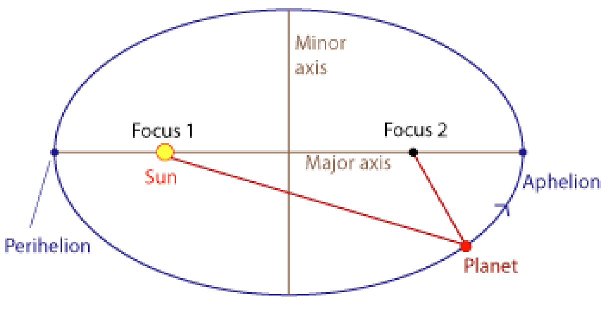


Satellite Orbital Mechanics - Kepler's Laws



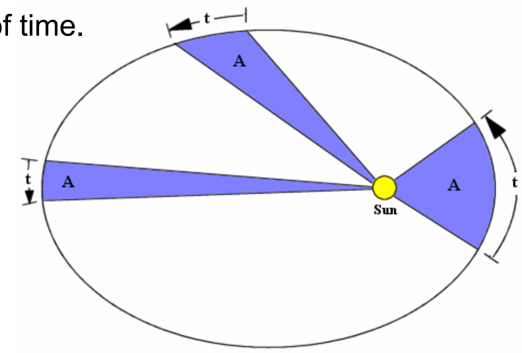
Kepler's Laws

First Law: The orbit of every planet is an ellipse with the Sun at one of the two foci.



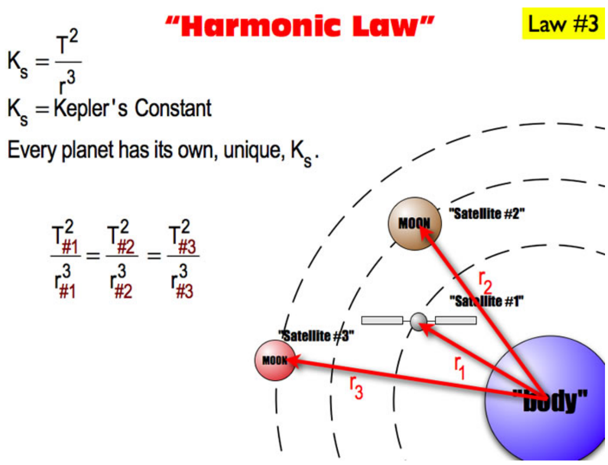
An elliptical orbit of a planet
(greatly exaggerated)

Second Law: A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.



Click me for an Animation

Third Law: The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.



Orbit Sun

Mercury	Earth
T_m	T_E
r_m	r_E

$$\frac{T_m^2}{r_m^3} = \frac{T_E^2}{r_E^3}$$

Kepler's Third Law

Unit 3 – Circular Motion, Work, and Energy

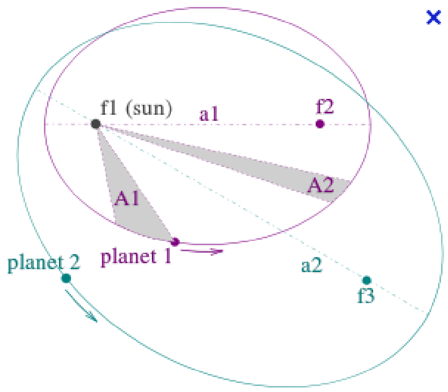
$$T = \frac{1}{f} \qquad f = \frac{1}{T} \qquad v = \frac{2\pi r}{T} \qquad v = 2\pi r f \qquad a_c = \frac{v^2}{r} \qquad F_c = \frac{mv^2}{r}$$

$$\frac{T_a^{\ 2}}{r_a^{\ 3}} = K = \frac{T_b^{\ 2}}{r_b^{\ 3}} \quad \text{where } K = 2.95x10^{-19}\frac{s^2}{m^3} \quad or \quad K = 1\frac{a^2}{AU^3}$$

$$W = F\Delta d \qquad E_p = mgh \qquad E_k = \frac{1}{2}mv^2 \qquad E_m = E_k + E_p \qquad P = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t} = Fv_{avg}$$

Let ***T*** represent the orbital period for a planet. Let ***r*** represent the orbital radius for a planet.

The ratio of the planet's orbital period squared to its orbital radius cubed is a constant. All objects (Planet 1, Planet 2) orbiting the same focus (eg: the Sun) have the same constant.



$$\frac{T_1^{\ 2}}{r_1^{\ 3}} = K \quad \textit{and} \quad \frac{T_2^{\ 2}}{r_2^{\ 3}} = K \quad \textit{so} \quad \frac{T_1^{\ 2}}{r_1^{\ 3}} = K = \frac{T_2^{\ 2}}{r_2^{\ 3}}$$

For objects orbiting our Sun, K = 2.95 x10^{-19} s^2/m^3

Ugly units are ugly.

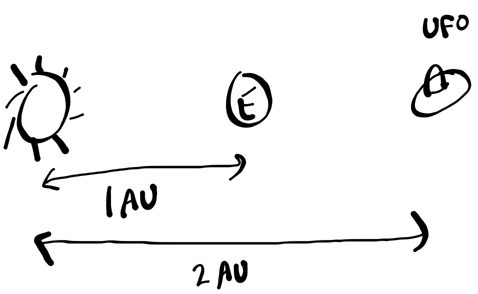
For objects orbiting our Sun, K = 2.95 x10^{-19} s^2/m^3

Better units?

How long does it take our planet to orbit the sun? One year [a].

What is the mean distance of our planet from the sun? One astronomical unit [AU].

$$K = \frac{(1\ a)^2}{(1\ AU)^3} = 1\ \frac{a^2}{AU^3}$$



Examples

Q1: Mars has an orbital radius of 1.52 AU. What is its orbital period?

Mars: $r = 1.52$ AU, $T = ?$

Earth: $r = 1$ AU, $T = 1$ year

$$\frac{T^2}{r^3} = \frac{T^2}{r^3}$$

$$\frac{T^2}{1.52^3} = \frac{1^2}{1^3}$$

$$(1.52^3)(1^2) = (1^3)(T^2)$$

$T_{Mars} = 1.87$ years

Q2: Mars has two moons, Deimos and Phobos. Phobos has an orbital radius of 9378km and an orbital period of 0.3189 Earth days. Deimos has an orbital period of 1.262 Earth days. What is the orbital radius of Deimos?

Phobos: $r = 9378$ km, $T = 0.3189$ days

Deimos: $r = ?$, $T = 1.262$ days

$$\frac{T^2}{r^3} = \frac{T^2}{r^3}$$

$$\frac{(0.3189)^2}{(9378)^3} = \frac{(1.262)^2}{r^3}$$

$$(9378)^3 \cdot (1.262)^2 = (0.3189^2)\, r^3$$

$$1.3133558... \times 10^{12} = (0.10169721)\, r^3$$

$$1.2916366... \times 10^{13} = r^3$$

$$23{,}462.8 \text{ km} = r$$

$$\sqrt[3]{x} = x^{1/3}$$

Practice

Pg 272 #1-3

Pg 275 #1-3

Attachments

Kepler Picture