

First Name: _____

Last Name: _____

1.14 - Worksheet - Mass-Spring Systems and Pendulums /18 marks

Part 1: Mass-Spring Systems

Use the following information to answer Q1-Q2:

A mass of 200g is attached to a spring. The spring is held stretched at 50cm by a force of 200N. When the spring is released, it will oscillate back and forth.

Q1: What is the period of the oscillation?

- a. 1.57×10^{-6} s
- b. 2.51×10^{-5} s
- c. 1.40×10^{-1} s
- d. 2.81×10^{-1} s

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{0.2}{400}}$$

$$T = 0.140496 \approx 1.40 \times 10^{-1} \text{ s}$$

$$F = kx$$

$$200 = k(0.5)$$

$$k = 400 \text{ N/m}$$

Q2: How much potential energy is stored in the spring the moment before it is released?

- a. 50 J
- b. 100 J
- c. 200 J
- d. 400 J

$$E_p = \frac{1}{2} kx^2$$

$$= \frac{1}{2} (400)(0.5)^2$$

$$= 50 \text{ J}$$

Q3: The maximum speed of the mass as it oscillates is $a.bc \times 10^d$ m/s, where a , b , c , and d are __, __, __, and __.

(Record your four digit answer in the Numerical Response boxes below)

2	2	4	1
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$$E_p \rightarrow E_k$$

$$50 = \frac{1}{2} m v^2$$

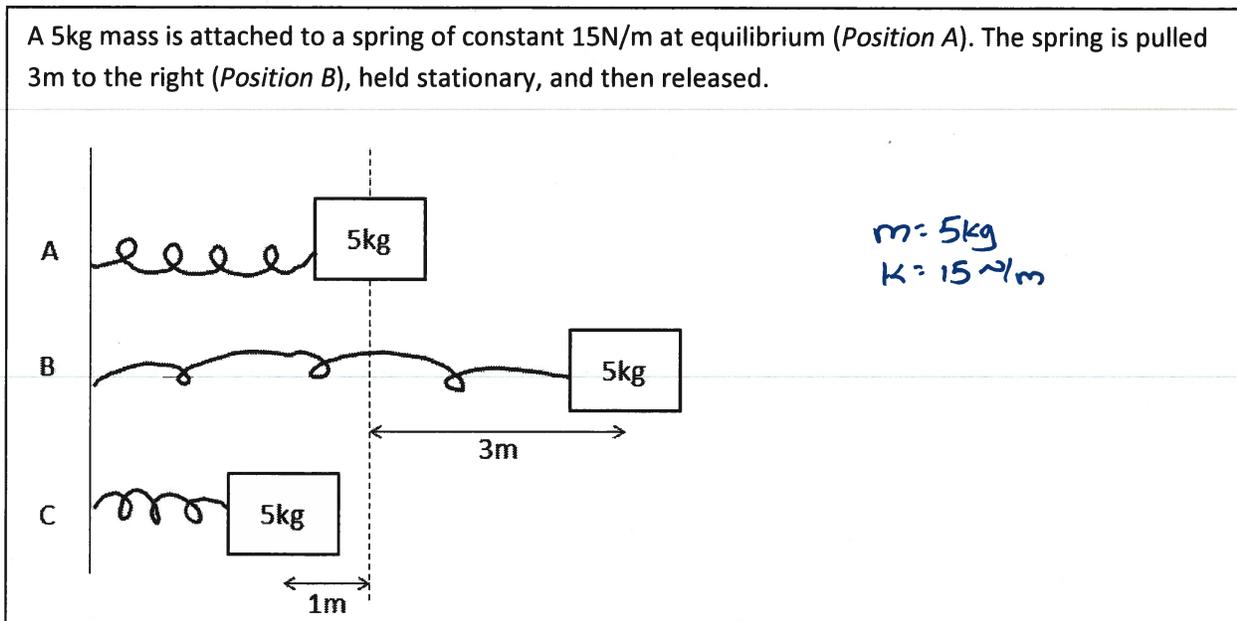
$$50 = \frac{1}{2} (0.2) v^2$$

$$v^2 = 500$$

$$v = 22.36$$

$$v \approx 2.24 \times 10^1 \text{ m/s}$$

Use the following information to answer Q4-Q5:



Q4: How much force is required to keep the mass held at *Position B*?

- a. 5.0 N
- b. 15.0 N
- c. 45.0 N
- d. 67.5 N

$$F = kx$$

$$= (15)(3)$$

$$= 45\text{ N}$$

Q5: What is the speed of the 5kg mass when 1m to the left of equilibrium (*Position C*), in m/s?

(Record your **three digit** answer in the Numerical Response boxes below)

4.90

$$E_i \longrightarrow E_f$$

$$\frac{1}{2}kx_i^2 = \frac{1}{2}kx_f^2 + \frac{1}{2}mv_f^2$$

$$\frac{1}{2}(15)(3)^2 = \frac{1}{2}(15)(1)^2 + \frac{1}{2}(5)v_f^2$$

$$67.5 = 7.5 + 2.5v_f^2$$

$$60 = 2.5v_f^2$$

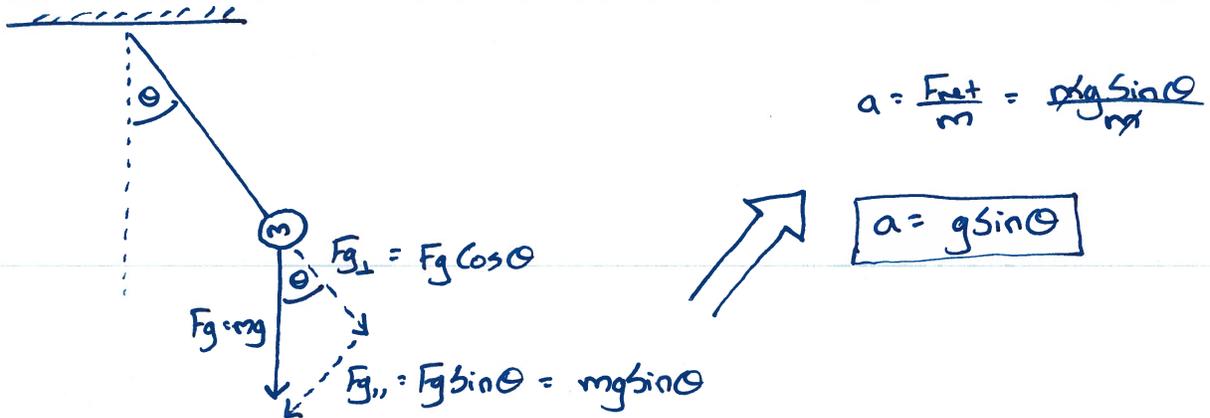
$$24 = v_f^2$$

$$v_f = 4.89897 \dots \text{ m/s}$$

$$v_f \approx 4.90 \text{ m/s}$$

Part 2: Pendulums

Q6: Mathematically prove that the instantaneous acceleration of a pendulum is independent of the mass on the end of the pendulum arm. (2 marks)



Q7: An astronaut with a mass of 80kg travels to Pluto, and builds a pendulum of arm length 0.8m. If the pendulum oscillates with a period of 8.89 seconds. The Force of Gravity that the astronaut is experiencing is $a.bc \times 10^d$ N, where $a, b, c,$ and d are , , , and .

(Record your four digit answer in the Numerical Response boxes below)

3	2	0	1
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Astronaut
 $m = 80 \text{ kg}$
 $F_g = ?$

Pendulum
 $L = 0.8 \text{ m}$
 $T = 8.89 \text{ s}$

$$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{0.8}{g}}$$

$$8.89 = 2\pi \sqrt{\frac{0.8}{g}}$$

$$1.414887 = \sqrt{\frac{0.8}{g}}$$

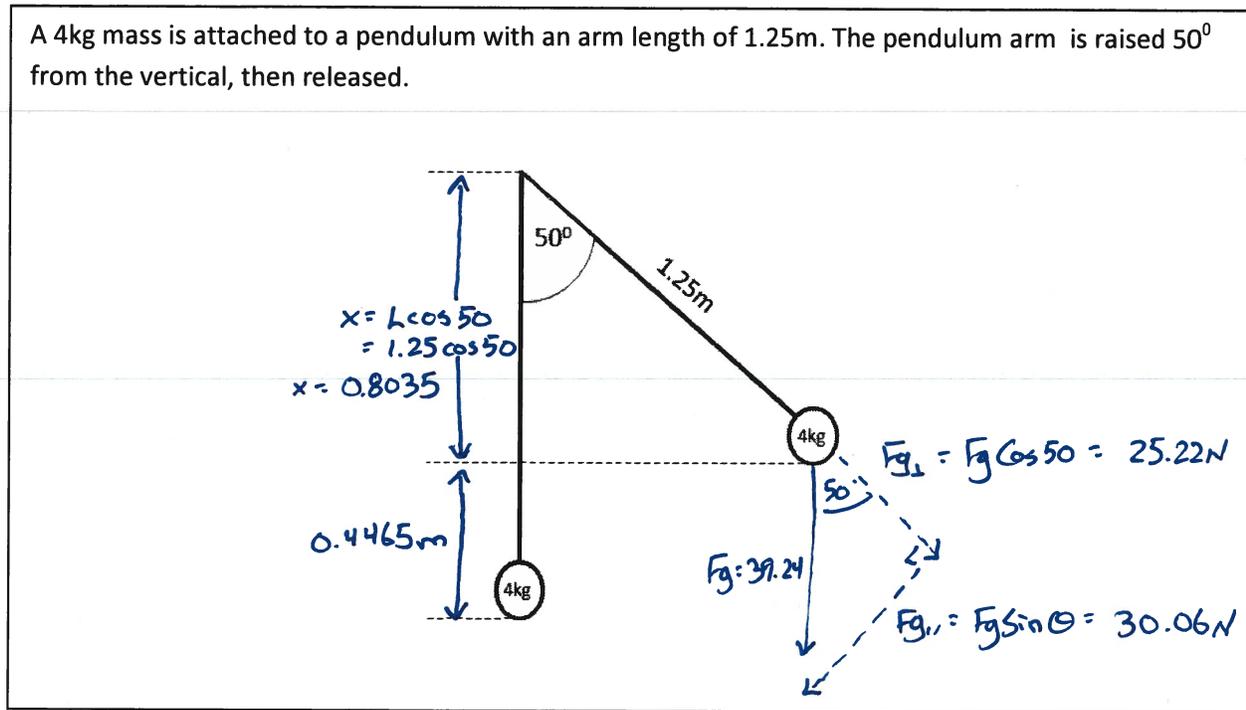
$$2.0019 = \frac{0.8}{g}$$

$$(2.0019)g = 0.8$$

$$g = 0.399619 \text{ N/kg}$$

$$\begin{aligned}
 F_g &= mg \\
 &= (80)(0.399619) \\
 &= 31.9695 \text{ N} \\
 &\approx 3.19695 \times 10^1 \text{ N} \\
 &\approx 3.20 \times 10^1 \text{ N}
 \end{aligned}$$

Use the following information to answer Q8-Q10:



Q8: What is the restoring force acting on the pendulum the moment it is released?

- a. 25.22 N
- b. 30.06 N
- c. 39.24 N
- d. 46.76 N

Q9: The frequency of oscillation of the pendulum is $a.bc \times 10^d$ Hz, where a , b , c , and d are _____, _____, _____ and _____.

(Record your four digit answer in the Numerical Response boxes below)

4	4	6	1
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$$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{1.25}{9.81}} = 2.24285 \text{ s}$$

$$f = \frac{1}{T} = 0.44586 \text{ Hz}$$

$$f \approx 4.46 \times 10^{-1} \text{ Hz}$$

Q10: What is the maximum speed of the pendulum during oscillation, in m/s? (2 marks)

$$E_p \rightarrow E_k$$

$$mgh = \frac{1}{2}mv^2$$

$$(9.81)(0.4465) = \frac{1}{2}v^2$$

$$v^2 = 8.7606$$

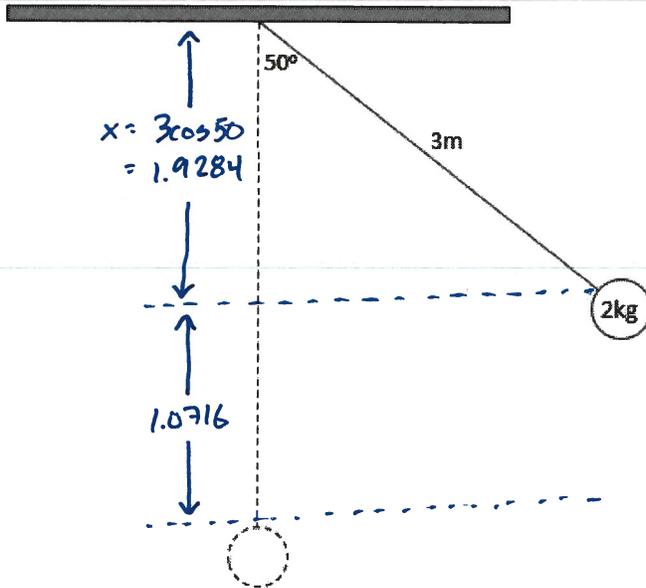
$$v = 2.9598 \text{ m/s}$$

$$v \approx 2.96 \text{ m/s}$$

Part 3: Combination Questions (Hard)

Q11: What is the tension in the pendulum arm at the bottom of the swing? (3 marks)

HINT: The answer is *not* 19.62N.



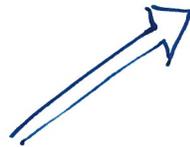
$$E_p \rightarrow E_k$$

$$mgh = \frac{1}{2}mv^2$$

$$(9.81)(1.0716) = \frac{1}{2}v^2$$

$$v^2 = 21.0255$$

$$v = 4.5854 \text{ m/s}$$



Over a very small distance at the bottom, approximately uniform circular motion.

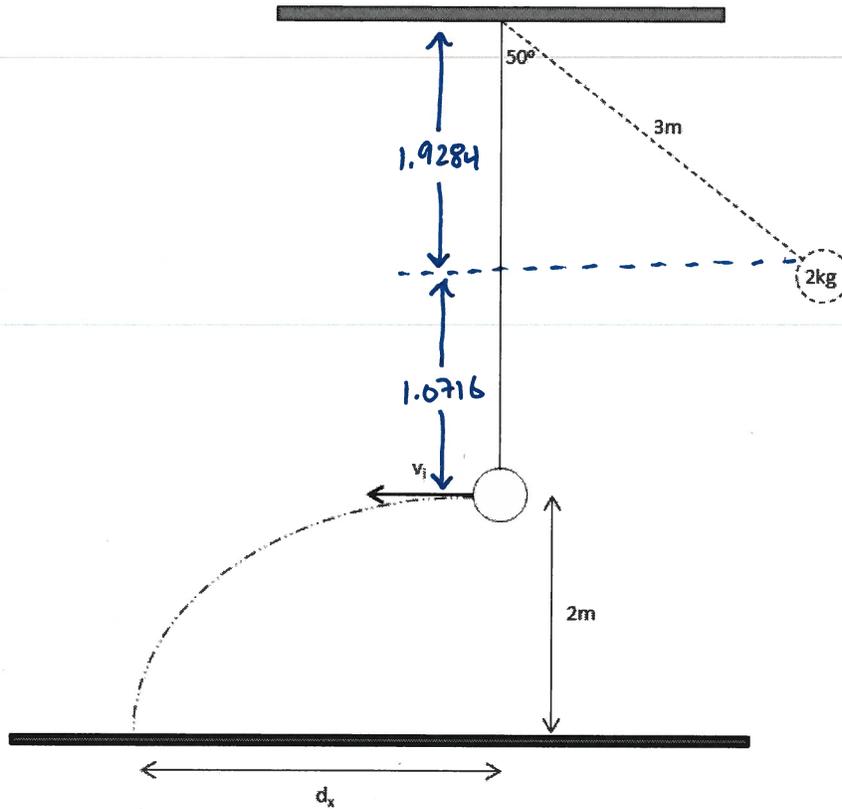
$$F_c = \frac{mv^2}{r} = \frac{(2)(4.5854)^2}{3} = 14.017 \text{ N}$$

$$\begin{matrix} \uparrow & \downarrow & \uparrow & \uparrow \\ F_c & = & F_g & + & F_T & & \downarrow \\ & & & & & & - \end{matrix}$$

$$(14.017) = (-19.62) + F_T$$

$$F_T = 33.64 \text{ N}$$

Q12: A pendulum arm is clipped when at equilibrium, as per the diagram below. What is the horizontal distance that the 2kg mass travels after being clipped from the pendulum arm? (3 marks)



Same diagram as last one.

$v_{\text{bottom}} = 4.5854 \text{ m/s}$

x-comp

$v_x = 4.5854 \text{ m/s}$
 $d_x = ?$
 $t = ?$

y-comp

$v_{iy} = 0 \text{ m/s}$
 $a_y = -9.81 \text{ m/s}^2$
 $dy = -2 \text{ m}$
 $t = ?$

$d = v_i t + \frac{1}{2} a t^2$
 $-2 = (0)t + \frac{1}{2} (-9.81)t^2$
 $-2 = -4.905t^2$
 $t^2 = 0.4077$
 $t = 0.6386 \text{ s}$



$v_x = \frac{d_x}{t}$

$4.5854 = \frac{d_x}{0.6386}$

$d_x = 2.928 \text{ m}$

$d_x \approx 2.93 \text{ m}$